AZUMAYA ALGEBRAS OVER HENSEL RINGS

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In this paper we prove the following theorem.
Let \((R, \alpha)\) be an henselian couple and let \(\mathcal{P}(R)\) be the set of isomorphism classes of Azumaya \(R\)-algebras; then the canonical map

\[ \mathcal{P}(R) \longrightarrow \mathcal{P}(R/\alpha) \]

is bijective.

As a corollary we obtain that, if \((R, \alpha)\) is an henselian couple, then the canonical homomorphism

\[ \mathcal{B}_e(R) \longrightarrow \mathcal{B}_e(R/\alpha) \]

between the Brauer groups, is an isomorphism.

Introduction. The corollary mentioned in the abstract generalizes a theorem of Azumaya ([2], Th. 31). The proof is similar to the one used by Grothendieck in proving the above theorem in case that \(R\) is a local ring and \(\alpha\) is its maximal ideal ([6], Th. 6.1).

Concerning the definition of henselian couple and Azumaya algebra we refer to [10] and [9] respectively.

All the rings and algebras are supposed to have unity.

In §1 we recall some properties of representable functors and smooth morphisms we shall need later.

In §§2, 3 we study two particular functors \(F_1, F_2\) from the category of commutative \(R\)-algebras to the category of sets and we prove that \(F_i\) and \(F_2\) are represented by smooth commutative \(R\)-algebras. These functors will be used to prove the theorem.

In §4, applying a known property of henselian couples, we obtain the theorem stated before and deduce some corollaries.

1. In this section we give some properties of representable functors and smooth morphisms.

Let \(R\) be a commutative ring; if \(F\) : \((\text{comm. } R\text{-alg.}) \rightarrow (\text{sets})\) is a functor we will say shortly that \(F\) is a sheaf if \(F\) is a sheaf of sets on the category of affine schemes over \(\text{Spec } R\) in the Zariski topology ([1] Def. 0.1 and 0.2).

**Proposition 1.** Let \(F\) : \((\text{comm. } R\text{-alg.}) \rightarrow (\text{sets})\) be a functor and suppose that \(F\) is a sheaf. Suppose that there exists a family \([f_i]_{i \in I}\) of elements of \(R\) generating the unity ideal in \(R\), such that the functor \(F_{f_i}\) : \((\text{comm. } R_{f_i}\text{-alg.}) \rightarrow (\text{sets})\) induced by \(F\) is representable for all \(i \in I\); then \(F\) is representable.
Proof. The proof is straightforward and we omit it.

Now we recall the definition of smooth \( R \)-algebra.

**Definition 1.** Let \( U \) be a commutative \( R \)-algebra. We say that \( U \) is smooth if

(a) \( U \) is of finite presentation.

(b) \( U \) is formally smooth, i.e. for every commutative \( R \)-algebra \( S \), for every nilpotent ideal \( I \) of \( S \), and for every \( R \)-homomorphism \( U \rightarrow S/I \), there exists an \( R \)-homomorphism \( U \rightarrow S \) such that the diagram

\[
\begin{array}{ccc}
S & \rightarrow & S/I \\
\downarrow & & \downarrow \\
U & \rightarrow & S/I \\
\end{array}
\]

commutes.

**Proposition 2.** Let \( U \) be a commutative \( R \)-algebra of finite presentation and \( S \) a faithfully flat commutative \( R \)-algebra; then \( U \) is a smooth \( R \)-algebra if and only if \( U \otimes S \) is a smooth \( S \)-algebra.

**Proof.** See [5] Corollary 17.7.2.

**Proposition 3.** Let \( U \) be a commutative \( R \)-algebra of finite presentation; if for every prime ideal \( p \) of \( R \), \( U_p \) is a smooth \( R_p \)-algebra, then \( U \) is a smooth \( R \)-algebra.

**Proof.** Let \( \mathfrak{p} \) be a prime ideal of \( U \) and let \( p = \mathfrak{p} \cap R \). \( U_p \) is a smooth \( R_p \)-algebra by hypothesis and it is easy to prove that \( U_p \) is a formally smooth \( U_p \)-algebra. Hence \( U_p \) is a formally smooth \( R_p \)-algebra and, by [5] Th. 17.5.1, \( U \) is a smooth \( R \)-algebra.

2. In this section we consider the functor \( F_1 \) defined as follows. Let \( A \) and \( A' \) be two Azumaya \( R \)-algebras; let \( a \) be an ideal of \( R \) and suppose that

\[
A/aA \approx A'/aA'.
\]

For every commutative \( R \)-algebra \( S \), define

\[
F_1(S) = \text{Isom}_{S\text{-alg}}(A \otimes S, A' \otimes S)
\]

i.e. \( F_1(S) \) is the set of isomorphisms of the \( S \)-algebra \( A \otimes S \) onto \( A' \otimes S \). It is easy to see that \( F_1 \) is a sheaf. The functor \( F_1 \) satisfies the following properties.

(1) \( F_1 \) is representable.
By Proposition 1 we can suppose that $A$ and $A'$ are free as $R$-modules and with the same rank $n$, because of the hypothesis $A/aA \approx A'/aA'$. Let $\{e_i\}$ and $\{e'_i\}$, $i = 1, \ldots, n$, be bases for $A$ and $A'$ respectively and let

$$e_i e_j = \sum_k m_{ijk} e_k, \quad e'_i e'_j = \sum_k m'_{ijk} e'_k$$

be the multiplication laws in $A$ and $A'$ respectively. Let $\varphi: A \otimes S \rightarrow A' \otimes S$ be an isomorphism; we can write

$$\varphi(e_i) = \sum_j x_{ij} e'_j, \quad x_{ij} \in S$$

where the $x_{ij}$'s must satisfy the following conditions:

(a) since $\varphi$ must satisfy $\varphi(e_i e_j) = \varphi(e_i) \varphi(e_j)$ we have

$$\sum_k m_{ijk} x_{kt} = \sum_k m'_{ijk} x_{ik} x_{jt}$$

for all $i, j, t = 1, \ldots, n$.

(b) $\det (x_{ij})$ is invertible in $S$.

Then consider the ring $R[\cdots, X_{ij}, \cdots]$ where the $X_{ij}$'s ($i, j = 1, \ldots, n$) are indeterminate and let

$$f_{ij} = \sum_k m'_{ijk} X_{kt} - \sum_k m_{ijk} X_{ik} X_{jt}$$

and

$$d = \det (X_{ij}) .$$

We set

$$U_1 = \left( \frac{R[\cdots, X_{ij}, \cdots]}{\cdots, f_{ij}, \cdots} \right)_d$$

and define the isomorphism

$$\varphi: A \otimes U_1 \longrightarrow A' \otimes U_1$$

by

$$\varphi(e_i) = \sum_j X_{ij} e'_j .$$

It is immediate to see that the couple $(U_1, \varphi)$ represents the functor $F_1$.

(2) The $R$-algebra $U_1$ which represents $F_1$ is smooth.

(a) By the definition of $U_1$ we have that $U_1$ is locally of finite presentation, hence $U_1$ is of finite presentation ([4] Prop. 1.4.6).

(b) To prove that $U_1$ is formally smooth, by Prop. 3 we can
suppose \( R \) local ring. Consider the strict henselization \( \widetilde{R} \) of \( R \); it is known that, if \( m \) is the maximal ideal of \( R \), then \( m\widetilde{R} \) is the maximal ideal of \( \widetilde{R} \) and the residue field \( \Omega \) of \( \widetilde{R} \) is a separable closure of the residue field \( k \) of \( R \) ([11], Chap. VIII § 2). We have \( A \otimes \Omega \simeq M_r(\Omega) \), i.e. the full matrix algebra of rank \( r \) over \( \Omega \) ([9], Chap. III, Cor. 6.3); by this we have

\[
A \otimes \widetilde{R} \simeq M_r(\widetilde{R})
\]

([3] Cor. 5.6).

By Proposition 2 we can suppose that

\[
A \simeq M_r(R) \simeq A'
\]

then \( U_1 \) represents the functor

\[
Aut(M_r) : (\text{comm. } R\text{-alg.}) \longrightarrow (\text{sets})
\]

defined by

\[
Aut(M_r)(S) = Aut_{S\cdot \text{alg}}(M_r(S))
\]

We must prove that, if \( I \) is a nilpotent ideal of \( S \), the map

\[
Aut_{S\cdot \text{alg}} ((M_r(S)) \longrightarrow Aut_{S/I\cdot \text{alg}} (M_r(S/I))
\]

is surjective.

This is an immediate consequence of the following proposition, because there is a bijection between

\[
Aut_{S\cdot \text{alg}} (M_r(S))
\]

and the set of all systems \( \{e_{ij}\} \) \((i, j = 1, \ldots, r)\) of matrix units in \( M_r(S) \).

**Proposition 4.** Let \((S, I)\) be an henselian couple and \( C \) a finite \( S \)-algebra. If \( \{\widetilde{e}_{ij}\} \) \((i, j = 1, \ldots, r)\) is a system of matrix units in \( C/I\Sigma \), then \( \{\widetilde{e}_{ij}\} \) can be lifted to a system \( \{e_{ij}\} \) of matrix units in \( C \).

**Proof.** The proof is the same as in [3] Th. 3.3.

3. In this section we consider the functor \( F_2 \) defined as follows. Let \( P \) be a finite projective \( R \)-module and, for every commutative \( R \)-algebra \( S \), define \( F_2(S) = \) set of multiplication laws \( m \) which can be defined on \( S \otimes P \) such that \( (S \otimes P, m) \) is an Azumaya \( S \)-algebra. Note that \( F_2 \) is a sheaf: this is an easy consequence of the fact that the property of being an Azumaya \( R \)-algebra is a local property on \( \text{Spec } R([9], \text{ Chap. III, Th. 6.6}) \). The functor \( F_2 \) satisfies the following properties.
(1) $F_2$ is representable.

By Proposition 1 we can suppose that $P$ is a free $R$-module of rank $n$. Let $\{e_i\} (i = 1, \cdots, n)$ be a basis for $P$. A multiplication law on $P \otimes S$ is defined by

$$e_ie_j = \sum_k m_{ijk}e_k, \quad m_{ijk} \in S$$

where the elements $m_{ijk}$ must satisfy the following properties. By the associative law $(e_ie_j)e_k = e_i(e_je_k)$ we have

$$\sum_l (m_{ijl}m_{lk} - m_{jkl}m_{ilk})$$

for all $i, j, k, t = 1, \cdots, n$.

Let $1 = \sum_i x_ie_i$ be the identity element; we have

$$\sum_i x_im_{ijk} = \sum_i x_im_{jik} = \delta_{ik}$$

for all $i, k = 1, \cdots, n$.

In order to express the condition that $(P \otimes S, m)$ is an Azumaya $S$-algebra, we recall the following proposition.

**Proposition 5.** Let $A$ be an $R$-algebra and suppose that, as $R$-module, $A$ is free of rank $n$; let $\{e_i\} (i = 1, \cdots, n)$ be a basis. Then $A$ is an Azumaya $R$-algebra if and only if the matrix $(a_{ij})$, defined by $a_{ij} = e_ie_i$, is an invertible matrix in the ring $M_n(A)$.


Then if we denote by $(b_{kl}) = (\sum_l m_{kl}e_i)$ the inverse matrix of $(a_{ij}) = (\sum_s m_{js}e_s)$, we have

$$\sum_{jkl} m_{jik}m_{kl}m_{jlt} = \delta_{il}x_l$$

for all $i, l, s = 1, \cdots, n$.

Then consider the ring

$$R[\cdots, X_i, \cdots; \cdots, Y_{ij}, \cdots; \cdots, Y'_{ij}, \cdots]$$

where the $X_i$'s, $Y_{ij}$'s, $Y'_{ij}$'s are indeterminate $(i, j, k = 1, \cdots, n)$. Set $f_{ijk} = \sum_i (Y_{ijl}Y_{ikl} - Y_{ijkl}Y_{ilt})$

$$g_{jk} = \sum_i X_i Y_{ijk} - \delta_{jk}, \quad g'_{jk} = \sum_i X_i Y_{jlk} - \delta_{jk}$$

$$h_{ilt} = \sum_{jkl} Y_{jkl} Y_{kls} Y'_{ilt} - \delta_{it}X_s$$

and set

$$U_2 = \frac{R[\cdots, X_i, \cdots; \cdots, Y_{ij}, \cdots; \cdots, Y'_{ij}, \cdots]}{(\cdots, f_{ijk}, \cdots; \cdots, g_{jk}, \cdots; \cdots, g'_{jk}, \cdots, h_{ilt}, \cdots)}.$$
Define on $P \otimes U_2$ a multiplication law $m$ by

$$e_i e_j = \sum_k X_{i j k} e_k ;$$

then it is easy to see that $(U_2, m)$ represents $F_2$

(2) The $R$-algebra $U_2$ which represents $F_2$ is smooth.

(a) As with the algebra $U_1$, $U_2$ is of finite presentation.

(b) To see that $U_2$ is formally smooth, consider the following proposition.

**Proposition 6.** Let $S$ be a commutative $R$-algebra and $I$ a nilpotent ideal of $S$; then if $\bar{A}$ is an Azumaya $S/I$-algebra, there exists an Azumaya $S$-algebra $A$ such that $A/IA \simeq \bar{A}$.

First we prove that the proposition implies $U_2$ formally smooth, i.e. the map $F_2(S) \to F_2(S/I)$ surjective. Let $\bar{m} \in F_2(S/I)$; call $\bar{A}$ the algebra $(P \otimes S/I, \bar{m})$. By Prop. 6 there exists an Azumaya $S$-algebra $A$ such that $A/IA \simeq \bar{A}$. Call $Q$ the $S$-module underlying to $A$; $Q$ is finite and projective and $Q/IQ \simeq P \otimes S/I$. Since $Q$ is projective the above isomorphism lifts to an $S$-module homomorphism $\varphi: Q \to P \otimes S$ and it is easy to prove that $\varphi$ is an isomorphism. Hence the multiplicative structure on $A$ is carried by $\varphi$ to a multiplication $m$ on $P \otimes S$ whose image in $F_2(S/I)$ is $\bar{m}$.

**Proof of Proposition 6.** We can suppose that $\bar{A}$, as a projective $S/I$-module has constant rank $n$ (by [9] Chap. I. Lemma 6.3 and [3] Cor. 3.2). It is known that there exists a faithfully flat étale extension $\bar{S}$ of $\bar{S} = S/I$ such that

$$\bar{A} \otimes \bar{S} \simeq M_*(\bar{S})$$

with $r^2 = n$ ([9] Chap. III Cor. 6.3).

By a known theorem ([11] Chap. V, Th. 4) there exists an étale $S$-algebra $S'$ such that $S'/IS' \simeq \bar{S}'$ and it is easy to see that $S'$ is faithfully flat $S$-algebra. Now recall that, if $S'$ is a faithfully flat extension of $S$, the isomorphism classes of Azumaya $S$-algebras $A$ such that

$$A \otimes S' \simeq M_*(S')$$

are classified by

$$H^1(S'/S, \text{Aut}(M_*))$$

where $\text{Aut}(M_*)$: (comm. $S$-alg.) $\to$ (groups) is the functor defined before ([9] Chap. II, Rem. 8.2). Then the Proposition 6 follows from the lemma.
**Lemma.** Let $S'$ be a faithfully flat extension of $S$, $I$ a nilpotent ideal of $S$, $F: \text{comm. } S\text{-alg.} \rightarrow \text{groups}$ a functor represented by a smooth $S$-algebra. Let $\bar{S} = S/I$, $\bar{S}' = S'/IS'$ and $\bar{F}: \text{comm. } S/I\text{-alg.} \rightarrow \text{groups}$ be the functor induced by $F$. Then the canonical map

$$H^i(S'/S, F) \longrightarrow H^i(\bar{S}'/\bar{S}, \bar{F})$$

is bijective.


4. In this section we prove the theorem enunciated in the introduction and deduce some corollaries.

First we recall a result on henselian couples.

**Theorem 1.** Let $(R, \alpha)$ be an henselian couple and $U$ a smooth $R$-algebra; then the canonical map

$$\text{Hom}_{R\text{-alg}}(U, R) \longrightarrow \text{Hom}_{R\text{-alg}}(U, R/\alpha)$$

is surjective.

**Proof.** See [8] Theorem 1.8.

Now we are able to prove the following propositions.

**Proposition 7.** Let $(R, \alpha)$ be an henselian couple and $A, A'$ two Azumaya $R$-algebras such that $A/\alpha A \simeq A'/\alpha A$; then $A \simeq A'$.

**Proof.** By Theorem 1 and §2.

**Proposition 8.** Let $(R, \alpha)$ be an henselian couple and $\bar{A}$ an Azumaya $R/\alpha$-algebra; then there exists an Azumaya $R$-algebra $A$ such that $A/\alpha A \simeq \bar{A}$.

**Proof.** Let $\bar{P}$ be the finite projective $R/\alpha$-module underlying to $\bar{A}$; then by [3] Theorem 4.1 there exists a finite projective $R$-module $P$ such that $P/\alpha P \simeq \bar{P}$. Then the proposition follows from Theorem 1 and §3.

**Theorem 2.** Let $(R, \alpha)$ be an henselian couple and let $\mathcal{P}(R)$ be the set of isomorphism classes of Azumaya $R$-algebras. Then the canonical map

$$\mathcal{P}(R) \longrightarrow \mathcal{P}(R/\alpha)$$

is bijective.
Proof. By Propositions 7 and 8.

COROLLARY 1. Let \((R, \alpha)\) be an henselian couple; then the canonical homomorphism

\[ \mathcal{B}_s(R) \longrightarrow \mathcal{B}_s(R/\alpha) \]

between the Brauer groups is an isomorphism.

Proof. The injectivity is in [3] Proposition 5.7; the surjectivity follows from Theorem 2.

COROLLARY 2. Let \((R, \alpha)\) be an henselian couple and let

\[ G: (\text{Azumaya } R\text{-alg.}) \longrightarrow (\text{Azumaya } R/\alpha\text{-alg.}) \]

be the functor defined by \(G(A) = A/\alpha A\) for every Azumaya \(R\)-algebra \(A\). Then \(G\) is essentially bijective and full, but, if \(\alpha \neq (0)\), is not faithful.

Proof. \(G\) is essentially bijective means exactly what we proved in Theorem 2. In order to prove that \(G\) is full consider two Azumaya \(R\)-algebras \(A\) and \(A'\) and define the functor

\[ F'': (\text{comm. } R\text{-alg.}) \longrightarrow (\text{sets}) \]

by

\[ F''(S) = \text{Hom}_{S\text{-alg}}(A \otimes S, A' \otimes S). \]

As with the functor \(F_1\) we can prove that \(F''\) is represented by an \(R\)-algebra \(U''\) of finite presentation.

To prove that \(U''\) is a smooth \(R\)-algebra we can suppose, as with the algebra \(U_1, A \simeq M_2(R)\) and \(A' \simeq M_2(R)\). Now observe that, if \(\psi \in F''(S)\) and \(\{e_{ij}\} (i, j = 1, \ldots, n)\) is a system of matrix units in \(A\), then \(\{\psi(e_{ij})\}\) is a system of matrix units in \(A'\), hence we have

\[ F''(S) = \emptyset \quad \text{if } m \neq n. \]

\[ F''(S) = \text{Aut}_{S\text{-alg}}(M_n(S)) \quad \text{if } m = n. \]

Hence \(U''\) is a smooth \(R\)-algebra and by Theorem 1 we have that \(G\) is full.

Now let \(a \in \alpha, a \neq 0\). Consider the inner automorphism \(\alpha\) of \(M_2(R)\) given by the element

\[ \begin{pmatrix} 1 + a & 0 \\ 0 & 1 \end{pmatrix} \in M_2(R); \]

the induced automorphism \(\bar{\alpha}\) of \(M_2(R/\alpha)\) is the identity automorphism.
while \( \alpha \) is not the identity automorphism of \( M_2(R) \). This proves that \( G \) is not faithful.

Now suppose \( R \) connected and recall that two Azumaya \( R \)-algebras \( A \) and \( A' \) are said to be stable isomorphic if there exist integers \( m \) and \( n \) such that

\[
M_n(A) \simeq M_m(A')
\]

Denote by \( \mathcal{A}(R) \) the set of stable isomorphism classes of Azumaya \( R \)-algebras ([6] Remark 1.8).

**Corollary 3.** Let \( (R, \alpha) \) be an henselian couple and suppose that \( R/\alpha \) is connected. Then the canonical map

\[
\mathcal{A}(R) \longrightarrow \mathcal{A}(R/\alpha)
\]

is bijective.

**Proof.** First we observe that if \( R/\alpha \) is connected then \( R \) is connected. Now we show that \( M_n(A) \) is an Azumaya \( R \)-algebra, if \( A \) is an Azumaya \( R \)-algebra: in fact we know that there exists a faithfully flat extension \( S \) of \( R \) such that \( A \otimes S \simeq M_\pi(S) \); then \( M_n(A) \otimes S \simeq M_{nx}(S) \), i.e. \( M_n(A) \) is an Azumaya \( R \)-algebra. Then the Corollary 3 follows from the Propositions 7 and 8.

**REFERENCES**


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