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AZUMAYA ALGEBRAS OVER HENSEL RINGS

ROSARIO STRANO

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In this paper we prove the following theorem.

Let (R, \mathfrak{a}) be an henselian couple and let $\mathcal{P}(R)$ be the set of isomorphism classes of Azumaya R -algebras; then the canonical map

$$\mathcal{P}(R) \longrightarrow \mathcal{P}(R/\mathfrak{a})$$

is bijective.

As a corollary we obtain that, if (R, \mathfrak{a}) is an henselian couple, then the canonical homomorphism

$$\mathcal{B}_e(R) \longrightarrow \mathcal{B}_e(R/\mathfrak{a})$$

between the Brauer groups, is an isomorphism.

Introduction. The corollary mentioned in the abstract generalizes a theorem of Azumaya ([2], Th. 31). The proof is similar to the one used by Grothendieck in proving the above theorem in case that R is a local ring and \mathfrak{a} is its maximal ideal ([6], Th. 6.1).

Concerning the definition of henselian couple and Azumaya algebra we refer to [10] and [9] respectively.

All the rings and algebras are supposed to have unity.

In §1 we recall some properties of representable functors and smooth morphisms we shall need later.

In §§2, 3 we study two particular functors F_1, F_2 from the category of commutative R -algebras to the category of sets and we prove that F_1 and F_2 are represented by smooth commutative R -algebras. These functors will be used to prove the theorem.

In §4, applying a known property of henselian couples, we obtain the theorem stated before and deduce some corollaries.

1. In this section we give some properties of representable functors and smooth morphisms.

Let R be a commutative ring; if $F: (\text{comm. } R\text{-alg.}) \rightarrow (\text{sets})$ is a functor we will say shortly that F is a sheaf if F is a sheaf of sets on the category of affine schemes over $\text{Spec } R$ in the Zariski topology ([1] Def. 0.1 and 0.2).

PROPOSITION 1. *Let $F: (\text{comm. } R\text{-alg.}) \rightarrow (\text{sets})$ be a functor and suppose that F is a sheaf. Suppose that there exists a family $[f_i]_{i \in I}$ of elements of R generating the unity ideal in R , such that the functor $F_i: (\text{comm. } R_{f_i}\text{-alg.}) \rightarrow (\text{sets})$ induced by F is representable for all $i \in I$; then F is representable.*

Proof. The proof is straightforward and we omit it.

Now we recall the definition of smooth R -algebra.

DEFINITION 1. Let U be a commutative R -algebra. We say that U is *smooth* if

(a) U is of finite presentation.

(b) U is *formally smooth*, i.e. for every commutative R -algebra S , for every nilpotent ideal I of S , and for every R -homomorphism $U \rightarrow S/I$, there exists an R -homomorphism $U \rightarrow S$ such that the diagram

$$\begin{array}{ccc}
 & & S \\
 & \nearrow & \downarrow \\
 U & & S/I
 \end{array}$$

commutes.

PROPOSITION 2. Let U be a commutative R -algebra of finite presentation and S a faithfully flat commutative R -algebra; then U is a smooth R -algebra if and only if $U \otimes S$ is a smooth S -algebra.

Proof. See [5] Corollary 17.7.2.

PROPOSITION 3. Let U be a commutative R -algebra of finite presentation; if for every prime ideal \mathfrak{p} of R , $U_{\mathfrak{p}}$ is a smooth $R_{\mathfrak{p}}$ -algebra, then U is a smooth R -algebra.

Proof. Let \mathfrak{P} be a prime ideal of U and let $\mathfrak{p} = \mathfrak{P} \cap R$. $U_{\mathfrak{p}}$ is a smooth $R_{\mathfrak{p}}$ -algebra by hypothesis and it is easy to prove that $U_{\mathfrak{p}}$ is a formally smooth $U_{\mathfrak{p}}$ -algebra. Hence $U_{\mathfrak{p}}$ is a formally smooth $R_{\mathfrak{p}}$ -algebra and, by [5] Th. 17.5.1, U is a smooth R -algebra.

2. In this section we consider the functor F_1 defined as follows. Let A and A' be two Azumaya R -algebras; let \mathfrak{a} be an ideal of R and suppose that

$$A/\mathfrak{a}A \approx A'/\mathfrak{a}A'.$$

For every commutative R -algebra S , define

$$F_1(S) = \text{Isom}_{S\text{-alg}}(A \otimes S, A' \otimes S)$$

i.e. $F_1(S)$ is the set of isomorphisms of the S -algebra $A \otimes S$ onto $A' \otimes S$. It is easy to see that F_1 is a sheaf. The functor F_1 satisfies the following properties.

(1) F_1 is representable.

By Proposition 1 we can suppose that A and A' are free as R -modules and with the same rank n , because of the hypothesis $A/\alpha A \approx A'/\alpha A'$. Let $\{e_i\}$ and $\{e'_i\}$, $i = 1, \dots, n$, be bases for A and A' respectively and let

$$e_i e_j = \sum_k m_{ijk} e_k, \quad e'_i e'_j = \sum_k m'_{ijk} e'_k$$

be the multiplication laws in A and A' respectively. Let $\varphi: A \otimes S \rightarrow A' \otimes S$ be an isomorphism; we can write

$$\varphi(e_i) = \sum_j x_{ij} e'_j, \quad x_{ij} \in S$$

where the x_{ij} 's must satisfy the following conditions:

(a) since φ must satisfy $\varphi(e_i e_j) = \varphi(e_i) \varphi(e_j)$ we have

$$\sum_k m_{ijk} x_{kt} = \sum_{kl} m'_{klt} x_{ii} x_{jl}$$

for all $i, j, t = 1, \dots, n$.

(b) $\det(x_{ij})$ is invertible in S .

Then consider the ring $R[\dots, X_{ij}, \dots]$ where the X_{ij} 's ($i, j = 1, \dots, n$) are indeterminate and let

$$f_{ijt} = \sum_k m_{ijk} X_{kt} - \sum_{kl} m'_{klt} X_{ik} X_{jl}$$

and

$$d = \det(X_{ij}).$$

We set

$$U_1 = \left(\frac{R[\dots, X_{ij}, \dots]}{(\dots, f_{ijt}, \dots)} \right)_d$$

and define the isomorphism

$$\varphi: A \otimes U_1 \longrightarrow A' \otimes U_1$$

by

$$\varphi(e_i) = \sum_j X_{ij} e'_j.$$

It is immediate to see that the couple (U_1, φ) represents the functor F_1 .

(2) *The R -algebra U_1 which represents F_1 is smooth.*

(a) By the definition of U_1 we have that U_1 is locally of finite presentation, hence U_1 is of finite presentation ([4] Prop. 1.4.6).

(b) To prove that U_1 is formally smooth, by Prop. 3 we can

suppose R local ring. Consider the strict henselization \tilde{R} of R ; it is known that, if \mathfrak{m} is the maximal ideal of R , then $\mathfrak{m}\tilde{R}$ is the maximal ideal of \tilde{R} and the residue field Ω of \tilde{R} is a separable closure of the residue field k of R ([11], Chap. VIII § 2). We have $A \otimes \Omega \simeq M_r(\Omega)$, i.e. the full matrix algebra of rank r over Ω ([9], Chap. III, Cor. 6.3); by this we have

$$A \otimes \tilde{R} \simeq M_r(\tilde{R})$$

([3] Cor. 5.6).

By Proposition 2 we can suppose that

$$A \simeq M_r(R) \simeq A'$$

then U_1 represents the functor

$$\underline{\text{Aut}}(M_r): (\text{comm. } R\text{-alg.}) \longrightarrow (\text{sets})$$

defined by

$$\underline{\text{Aut}}(M_r)(S) = \text{Aut}_{S\text{-alg}}(M_r(S)).$$

We must prove that, if I is a nilpotent ideal of S , the map

$$\text{Aut}_{S\text{-alg}}(M_r(S)) \longrightarrow \text{Aut}_{S/I\text{-alg}}(M_r(S/I))$$

is surjective.

This is an immediate consequence of the following proposition, because there is a bijection between

$$\text{Aut}_{S\text{-alg}}(M_r(S))$$

and the set of all systems $\{e_{ij}\}$ ($i, j = 1, \dots, r$) of matrix units in $M_r(S)$.

PROPOSITION 4. *Let (S, I) be an henselian couple and C a finite S -algebra. If $\{\bar{e}_{ij}\}$ ($i, j = 1, \dots, r$) is a system of matrix units in C/IC , then $\{\bar{e}_{ij}\}$ can be lifted to a system $\{e_{ij}\}$ of matrix units in C .*

Proof. The proof is the same as in [3] Th. 3.3.

3. In this section we consider the functor F_2 defined as follows. Let P be a finite projective R -module and, for every commutative R -algebra S , define $F_2(S) =$ set of multiplication laws m which can be defined on $S \otimes P$ such that $(S \otimes P, m)$ is an Azumaya S -algebra. Note that F_2 is a sheaf: this is an easy consequence of the fact that the property of being an Azumaya R -algebra is a local property on $\text{Spec } R$ ([9], Chap. III, Th. 6.6). The functor F_2 satisfies the following properties.

(1) F_2 is representable.

By Proposition 1 we can suppose that P is a free R -module of rank n . Let $\{e_i\}$ ($i = 1, \dots, n$) be a basis for P . A multiplication law on $P \otimes S$ is defined by

$$e_i e_j = \sum_k m_{ijk} e_k, \quad m_{ijk} \in S$$

where the elements m_{ijk} must satisfy the following properties. By the associative law $(e_i e_j) e_k = e_i (e_j e_k)$ we have

$$\sum_l (m_{ijl} m_{lkt} - m_{jkl} m_{ilt})$$

for all $i, j, k, t = 1, \dots, n$.

Let $1 = \sum_i x_i e_i$ be the identity element; we have

$$\sum_i x_i m_{ijk} = \sum_i x_i m_{jik} = \delta_{ik}$$

for all $i, k = 1, \dots, n$.

In order to express the condition that $(P \otimes S, m)$ is an Azumaya S -algebra, we recall the following proposition.

PROPOSITION 5. *Let A be an R -algebra and suppose that, as R -module, A is free of rank n ; let $\{e_i\}$ ($i = 1, \dots, n$) be a basis. Then A is an Azumaya R -algebra if and only if the matrix (a_{ij}) , defined by $a_{ij} = e_j e_i$, is an invertible matrix in the ring $M_n(A)$.*

Proof. See [2] Theorem 12.

Then if we denote by $(b_{kl}) = (\sum_t m'_{klt} e_t)$ the inverse matrix of $(a_{ij}) = (\sum_s m_{jis} e_s)$, we have

$$\sum_{jkt} m_{jik} m_{kts} m'_{jlt} = \delta_{is}$$

for all $i, l, s = 1, \dots, n$.

Then consider the ring

$$R[\dots, X_i, \dots; \dots, Y_{ijk}, \dots; \dots, Y'_{ijk}, \dots]$$

where the X_i 's, Y_{ijk} 's, Y'_{ijk} 's are indeterminate ($i, j, k = 1, \dots, n$).

Set $f_{ijkt} = \sum_l (Y_{ijl} Y_{lkt} - Y_{jkl} Y_{ilt})$

$$g_{jk} = \sum_i X_i Y_{ijk} - \delta_{jk}, \quad g'_{jk} = \sum_i X_i Y'_{ijk} - \delta_{jk}$$

$$h_{ils} = \sum_{jkt} Y_{jik} Y_{kts} Y'_{jlt} - \delta_{il} X_s$$

and set

$$U_2 = \frac{R[\dots, X_i, \dots; \dots, Y_{ijk}, \dots; \dots, Y'_{ijk}, \dots]}{(\dots, f_{ijkt}, \dots; \dots, g_{jk}, \dots; \dots, g'_{jk}, \dots, h_{ils}, \dots)}.$$

Define on $P \otimes U_2$ a multiplication law m by

$$e_i e_j = \sum_k X_{ijk} e_k;$$

then it is easy to see that (U_2, m) represents F_2

(2) *The R -algebra U_2 which represents F_2 is smooth.*

(a) As with the algebra U_1 , U_2 is of finite presentation.

(b) To see that U_2 is formally smooth, consider the following proposition.

PROPOSITION 6. *Let S be a commutative R -algebra and I a nilpotent ideal of S ; then if \bar{A} is an Azumaya S/I -algebra, there exists an Azumaya S -algebra A such that $A/IA \simeq \bar{A}$.*

First we prove that the proposition implies U_2 formally smooth, i.e. the map $F_2(S) \rightarrow F_2(S/I)$ surjective. Let $\bar{m} \in F_2(S/I)$; call \bar{A} the algebra $(P \otimes S/I, \bar{m})$. By Prop. 6 there exists an Azumaya S -algebra A such that $A/IA \simeq \bar{A}$. Call Q the S -module underlying to A ; Q is finite and projective and $Q/IQ \simeq P \otimes S/I$. Since Q is projective the above isomorphism lifts to an S -module homomorphism $\varphi: Q \rightarrow P \otimes S$ and it is easy to prove that φ is an isomorphism. Hence the multiplicative structure on A is carried by φ to a multiplication m on $P \otimes S$ whose image in $F_2(S/I)$ is \bar{m} .

Proof of Proposition 6. We can suppose that \bar{A} , as a projective S/I -module has constant rank n (by [9] Chap. I. Lemma 6.3 and [3] Cor. 3.2). It is known that there exists a faithfully flat étale extension \bar{S}' of $\bar{S} = S/I$ such that

$$\bar{A} \otimes \bar{S}' \simeq M_r(\bar{S}')$$

with $r^2 = n$ ([9] Chap. III Cor. 6.3).

By a known theorem ([11] Chap. V, Th. 4) there exists an étale S -algebra S' such that $S'/IS' \simeq \bar{S}'$ and it is easy to see that S' is faithfully flat S -algebra. Now recall that, if S' is a faithfully flat extension of S , the isomorphism classes of Azumaya S -algebras A such that

$$A \otimes S' \simeq M_r(S')$$

are classified by

$$H^1(S'/S, \underline{\text{Aut}}(M_r))$$

where $\underline{\text{Aut}}(M_r): (\text{comm. } S\text{-alg.}) \rightarrow (\text{groups})$ is the functor defined before ([9] Chap. II, Rem. 8.2). Then the Proposition 6 follows from the lemma.

LEMMA. *Let S' be a faithfully flat extension of S , I a nilpotent ideal of S , $F: (\text{comm. } S\text{-alg.}) \rightarrow (\text{groups})$ a functor represented by a smooth S -algebra. Let $\bar{S} = S/I$, $\bar{S}' = S'/IS'$ and $\bar{F}: (\text{comm. } S/I\text{-alg.}) \rightarrow (\text{groups})$ be the functor induced by F . Then the canonical map*

$$H^1(S'/S, F) \longrightarrow H^1(\bar{S}'/\bar{S}, \bar{F})$$

is bijective.

Proof. [7] Lemma 8.1.8, page 404.

4. In this section we prove the theorem enunciated in the introduction and deduce some corollaries.

First we recall a result on henselian couples.

THEOREM 1. *Let (R, α) be an henselian couple and U a smooth R -algebra; then the canonical map*

$$\text{Hom}_{R\text{-alg}}(U, R) \longrightarrow \text{Hom}_{R\text{-alg}}(U, R/\alpha)$$

is surjective.

Proof. See [8] Theorem 1.8.

Now we are able to prove the following propositions.

PROPOSITION 7. *Let (R, α) be an henselian couple and A, A' two Azumaya R -algebras such that $A/\alpha A \simeq A'/\alpha A$; then $A \simeq A'$.*

Proof. By Theorem 1 and § 2.

PROPOSITION 8. *Let (R, α) be an henselian couple and \bar{A} an Azumaya R/α -algebra; then there exists an Azumaya R -algebra A such that $A/\alpha A \simeq \bar{A}$.*

Proof. Let \bar{P} be the finite projective R/α -module underlying to \bar{A} ; then by [3] Theorem 4.1 there exists a finite projective R -module P such that $P/\alpha P \simeq \bar{P}$. Then the proposition follows from Theorem 1 and § 3.

THEOREM 2. *Let (R, α) be an henselian couple and let $\mathcal{P}(R)$ be the set of isomorphism classes of Azumaya R -algebras. Then the canonical map*

$$\mathcal{P}(R) \longrightarrow \mathcal{P}(R/\alpha)$$

is bijective.

Proof. By Propositions 7 and 8.

COROLLARY 1. *Let (R, \mathfrak{a}) be an henselian couple; then the canonical homomorphism*

$$\mathcal{B}_z(R) \longrightarrow \mathcal{B}_z(R/\mathfrak{a})$$

between the Brauer groups is an isomorphism.

Proof. The injectivity is in [3] Proposition 5.7; the surjectivity follows from Theorem 2.

COROLLARY 2. *Let (R, \mathfrak{a}) be an henselian couple and let*

$$G: (\text{Azumaya } R\text{-alg.}) \longrightarrow (\text{Azumaya } R/\mathfrak{a}\text{-alg.})$$

be the functor defined by $G(A) = A/\mathfrak{a}A$ for every Azumaya R -algebra A . Then G is essentially bijective and full, but, if $\mathfrak{a} \neq (0)$, is not faithful.

Proof. G is essentially bijective means exactly what we proved in Theorem 2. In order to prove that G is full consider two Azumaya R -algebras A and A' and define the functor

$$F': (\text{comm. } R\text{-alg.}) \longrightarrow (\text{sets})$$

by

$$F'(S) = \text{Hom}_{S\text{-alg}}(A \otimes S, A' \otimes S).$$

As with the functor F_1 we can prove that F' is represented by an R -algebra U' of finite presentation.

To prove that U' is a smooth R -algebra we can suppose, as with the algebra U_1 , $A \simeq M_n(R)$ and $A' \simeq M_m(R)$. Now observe that, if $\varphi \in F'(S)$ and $\{e_{ij}\}$ ($i, j = 1, \dots, n$) is a system of matrix units in A , then $\{\varphi(e_{ij})\}$ is a system of matrix units in A' , hence we have

$$F'(S) = \emptyset \quad \text{if } m \neq n.$$

$$F'(S) = \text{Aut}_{S\text{-alg}}(M_n(S)) \quad \text{if } m = n.$$

Hence U' is a smooth R -algebra and by Theorem 1 we have that G is full.

Now let $a \in \mathfrak{a}$, $a \neq 0$. Consider the inner automorphism α of $M_2(R)$ given by the element

$$\begin{pmatrix} 1 + a & 0 \\ 0 & 1 \end{pmatrix} \in M_2(R);$$

the induced automorphism $\bar{\alpha}$ of $M_2(R/\mathfrak{a})$ is the identity automorphism

while α is not the identity automorphism of $M_2(R)$. This proves that G is not faithful.

Now suppose R connected and recall that two Azumaya R -algebras A and A' are said to be *stable isomorphic* if there exist integers m and n such that

$$M_n(A) \simeq M_m(A').$$

Denote by $\mathcal{H}\mathcal{P}(R)$ the set of stable isomorphism classes of Azumaya R -algebras ([6] Remark 1.8).

COROLLARY 3. *Let (R, \mathfrak{a}) be an henselian couple and suppose that R/\mathfrak{a} is connected. Then the canonical map*

$$\mathcal{H}\mathcal{P}(R) \longrightarrow \mathcal{H}\mathcal{P}(R/\mathfrak{a})$$

is bijective.

Proof. First we observe that if R/\mathfrak{a} is connected then R is connected. Now we show that $M_n(A)$ is an Azumaya R -algebra, if A is an Azumaya R -algebra: in fact we know that there exists a faithfully flat extension S of R such that $A \otimes S \simeq M_r(S)$; then $M_n(A) \otimes S \simeq M_{n \times r}(S)$, i.e. $M_n(A)$ is an Azumaya R -algebra. Then the Corollary 3 follows from the Propositions 7 and 8.

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