GENERATING LARGE INDECOMPOSABLE CONTINUA

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It has been shown by D. P. Bellamy that every metric continuum is homeomorphic to a retract of some metric indecomposable continuum. This result was later extended by G. R. Gordh who proved a similar theorem in the non-metric case. In the present paper a different technique is used to generate such continua.

It is shown that if \( \alpha \) is an infinite cardinal number then there is an indecomposable continuum with \( 2^\alpha \) composants and if \( I \) a (non-metric) continuum then \( I \) is homeomorphic to a retract of such a continuum. An indecomposable continuum is constructed such that if \( C \) is a composant of it and \( H \) is an infinite subset of \( C \) then \( C \) contains a limit point of \( H \). Finally a non-metric continuum is found so that each proper subcontinuum of it is metric.

**Definitions and notations.** A continuum is a compact connected Hausdorff space. Suppose \( A \) is a well ordered set, for each \( a \in A \) \( M_a \) is a topological space, and if \( a < b \) in \( A \), \( \theta_b^a \) is a continuous function from \( M_b \) onto \( M_a \) so that if \( a < b < c \) in \( A \) then \( \theta_b^a \circ \theta_c^b = \theta_c^a \). The space \( M \) is the inverse limit \( M = \lim_{\alpha \in A} \{ M_\alpha, \theta \} \) means that \( M \) is the topological space to which the point \( P \) belongs if and only if \( P \) is a function from \( A \) into \( \bigcup_{a \in A} \{ M_a \} \) so that \( P_a \in M_a \) and if \( a < b \) in \( A \) then \( \theta_b^a(P_b) = P_a \). \( R \) is a region in \( M \) means that there is an element \( a \in A \) and an open set \( S \subseteq M_a \) so that \( R = \{ P \mid P_a \in S \} \). \( P_a \) denotes the function from \( M \) into \( M_a \) so that \( P_a(P) = P_a \). If \( S = \prod_{a \in A} \{ S_a \} \) is a product space then \( x = \{ x_a \}_{a \in A} \) denotes the point of \( S \) so that \( x_a \in S_a \), and \( \pi_a \) denotes the function from \( S \) into \( S_a \) so that \( \pi_a(x) = x_a \). If \( \alpha \) is an ordinal number \( \Pi_{i<a} [0, 1]_i \) denotes the cartesian product of \( \alpha \) copies of the interval \( [0, 1] \). If \( M = \lim_{\alpha \in A} \{ M_\alpha, \theta \} \) and for each \( a \in A \) \( M_a \) is a continuum then \( M \) is a continuum. Also if for each \( a \in A \), \( M_a \) is an indecomposable continuum then so is \( M \). For theorems concerning inverse limits the reader should consult [2].

**Theorem 1.** Suppose \( M \) is a compact continuum, \( \alpha \) is a well ordered set with no last element, \( M \) is the inverse limit \( M = \lim_{\alpha \in A} \{ M_\alpha, \theta \} \) of a collection of Hausdorff continua, and for each \( a \in A \) \( M_a \) is a continuum then \( M \) is a continuum. Also if for each \( a \in A \), \( M_a \) is an indecomposable continuum then so is \( M \). For theorems concerning inverse limits the reader should consult [2].
(2) if $I$ is a subcontinuum of $M_{\alpha}$ intersecting $I_{\alpha}$ and $M_{\alpha} - I_{\alpha}$ then $I$ contains $I_{\alpha}$.

Then $M$ is indecomposable.

Proof. Suppose $a \in \alpha$ and $P$ is a point of $M_{\alpha} - I_{\alpha}$. Then there is a subcontinuum $V$ of $M_{\alpha}$ which is irreducible from the point $P$ to $I_{\alpha}$. The set $V - I_{\alpha} \cap V$ is connected and $V - I_{\alpha} \cap V = V$. From condition (2) it follows that $I_{\alpha} \subseteq V$, so $I_{\alpha} \subset V$. From condition (2) it follows that $I_{\alpha} \subseteq V$, so $I_{\alpha} \subset V = V - I_{\alpha} \cap V \subset M - I_{\alpha}$.

Now suppose $M$ is the union of two proper subcontinua $H$ and $K$. Let $P$ be a point of $H$ not in $K$ and let $Q$ be a point of $K$ not in $H$. There exists an element $a \in \alpha$ and mutually exclusive regions $R_{a}$ and $S_{a}$ of $M_{\alpha}$ containing $P_{a}$ and $Q_{a}$ respectively so that $R = \{ x \mid x_{a} \in R_{a} \}$
does not intersect \(K\) and \(S = \{x \mid x_a \in S_a\}\) does not intersect \(H\). Thus \(R\) and \(S\) are mutually exclusive open sets in \(M\) containing \(P\) and \(Q\) respectively. It follows from the above and condition (1) that \(\theta_{a+1}(R_a)\) and \(\theta_{a+1}^{-1}(S_a)\) are mutually exclusive open sets in \(M_{a+1}\) and each intersects both \(I_{a+1}\) and \(M_{a+1} - I_{a+1}\). So \(P_{a+1}(H)\) and \(P_{a+1}(K)\) both intersect \(I_{a+1}\) and \(M_{a+1} - I_{a+1}\). So by condition (2) \(I_{a+1}\) is a subset of both \(P_{a+1}(H)\) and \(P_{a+1}(K)\). But then \(P_a(K) = P_a = P_a(H)\), since \(P_a = \theta_a^{a+1} \circ P_a\), which is a contradiction. Thus \(M\) is indecomposable.

**Theorem 2.** If \(q\) is an infinite cardinal number, there is an indecomposable continuum \(M\) with \(2^q\) composants.

**Proof.** Let \(\alpha\) be the first ordinal number so that \(|\alpha| = q\). The continuum \(M\) will be constructed as an inverse limit of \(\alpha\) irreducible continua. Let \(M_0 = [0, 1]\). Let \(M_i\) be the subcontinuum of \([0, 1] \times [0, 1]\) so that

\[
M_1 = (M_0 \times \{0\}) \cup \left( \bigcup_{i=0}^{\infty} \left( \left( M_0 \times \left[ \frac{1}{2i+1}, \frac{1}{2} \right] \right) \cup \left( \{0\} \times \left[ \frac{1}{2i+2}, \frac{1}{2i+1} \right] \right) \right) \right).
\]

The continuum \(M_1\) is the union of countably many copies of \(M_0\) and countably many arcs. If \(A_1 = (0, 0)\) and \(B_1 = (1, 1)\) then \(M_1\) is irreducible from \(A_1\) to \(B_1\). Let \(\theta_0\) be the function from \(M_1\) onto \(M_0\) so that \(\theta_0(P_1, P_2) = P_1\).

Suppose that \(b < \alpha\) and that \(M_a\) and \(\theta_c^\alpha\) have been defined for \(c < a < b\) so that \(M_a\) is a subcontinuum of \(\Pi_{i \leq a} [0, 1]i\), which is irreducible from the point \(A_a = \{0\}_{i \leq a}\) to the point \(B_a = \{1\}_{i \leq a}\), and \(\theta_c^\alpha\) is a function from \(M_a\) onto \(M_c\) so that \(\theta_c^\alpha(\{x_i\}_{i \leq a}) = \{x_i\}_{i \leq c}\). Suppose that \(b\) is not a limit ordinal, \(b = a + 1\) for some \(a < \alpha\). Let \(M_b\) be the subcontinuum of \(\Pi_{i \leq b} [0, 1]i\), so that

\[
M_b = (M_a \times \{0\}) \cup \left( \bigcup_{i=0}^{\infty} \left( \left( M_a \times \left[ \frac{1}{2i+1}, \frac{1}{2i+2} \right] \right) \cup \left( \{0\} \times \left[ \frac{1}{2i+1}, \frac{1}{2i+2} \right] \right) \right) \right).
\]

The continuum \(M_b\) is the union of countably many copies of \(M_a\) and countably many arcs. \(M_b\) is irreducible from any point of \((M_a \times \{0\})\) to
the point \((B_{b} \times \{1\})\). Let \(A_{b} = A_{a} \times \{0\}\) and \(B_{b} = B_{a} \times \{1\}\). Let \(\theta_{p}^{b}\) be the function from \(M_{b}\) onto \(M_{a}\) so that if \(\{x_{i}\}_{i \leq b} \in M_{b}\) then \(\theta_{p}^{b}(\{x_{i}\}_{i \leq b}) = \{x_{i}\}_{i \leq a}\). If \(c < a\) define \(\theta_{p}^{c}\) to be the function \(\theta_{p}^{b} \circ \theta_{p}^{c}\).

Suppose that \(b\) is a limit ordinal. Let \(M_{b}'\) be the \(\cdot\) continuum \(M_{b}' = \lim \{M_{a}, \theta_{a}\}_{a < b}\). Let \(A_{b}'\) denote the point \(P\) so that \(P_{a}(P) = A_{a}\) and let \(B_{b}'\) denote the point \(P\) so that \(P_{a}(P) = B_{a}\). Then \(M_{b}'\) is irreducible from \(A_{b}'\) to \(B_{b}'\) since for each \(a < b\) \(M_{a}\) is irreducible from \(P_{a}(A_{a}')\) to \(P_{a}(B_{a}')\). Let \(L_{b}\) denote the function from \(M_{b}'\) into \(\Pi_{i < b} [0, 1]\) so that if \(P \in M_{b}'\) then \(L_{b}(P) = \{\pi_{i}(P)\}_{i < b}\) where \(P_{i}\) is the \(i\)th coordinate of the point \(P\), \(P_{i} = P_{i}(P)\). Note that \(P_{i}(P) \in M_{i} \subset \Pi_{i \leq b} [0, 1]_{b}\). \(L_{b}\) is a homeomorphism because if \(P\) is a point of \(M_{b}'\) and \(i < j < b\) then \(\pi_{a}(P_{i}(P)) = \pi_{a}(P_{j}(P))\) for all \(a \leq i\); in other words the \(a\)th coordinate in the cartesian product \(\Pi_{i \leq b} [0, 1]_{b}\) is the same as the \(a\)th coordinate in \(\Pi_{i \leq b} [0, 1]_{b}\) of \(P_{i}(P)\). Then \(L_{b}(M_{b}') \subset \Pi_{i < b} [0, 1]_{b}\). \(M_{b}\) is defined by replacing \(M_{a}\) by \(L_{b}(M_{a})\) in \([*]\) above and \(A_{a}\) by \(L_{b}(A_{a})\) and \(B_{a}\) by \(L_{b}(B_{a})\). So \(M_{b}\) is irreducible from any point of \((L_{b}(M_{a})) \times \{0\}\) to the point \((L_{b}(B_{a})) \times \{1\}\). Let \(A_{b} = (L_{b}(A_{a}) \times \{0\})\) and \(B_{b} = (L_{b}(B_{a}) \times \{1\})\). If \(a < b\) let \(\theta_{a}^{b}\) be the function from \(M_{b}\) onto \(M_{a}\) so that if \(\{x_{i}\}_{i \leq b} \in M_{b}\) then \(\theta_{a}^{b}(\{x_{i}\}_{i \leq b}) = \{x_{i}\}_{i \leq a}\). For notational convenience, if \(b\) is a limit ordinal let \(M_{b-1}\) denote the space \(L_{b}(M_{b}')\) and let \(P_{b-1}\) denote the function \(f \circ P_{b}\) where \(f\) projects \(L_{b}(M_{b}') \times \{0, 1\}\) onto \(L_{b}(M_{b}') \times \{0\}\).

Let \(M = \lim \{M_{a}, \theta_{a}\}_{a < \alpha}\). If for each \(a\), \(I_{a} = M_{a-1} \times \{0\}\) then \(M\) and the collection \(\{I_{a}\}_{a < \alpha}\) satisfy the hypothesis of Theorem 1 because \(M_{a}\) is irreducible from the point \(B_{a}\) to each point of \(I_{a}\). Thus \(M\) is indecomposable. If \(P \in M\) let \(P_{\gamma}\) denote \(P_{\gamma}(P)\). Let \(L\) denote the projection \(L_{\alpha}\) as defined above.

Suppose \(x\) is a point of \(M\) and \(w_{\gamma}\) is the set to which \(P\) belongs if and only if there exists a \(\beta < \alpha\) so that if \(\beta < \gamma < \alpha\) then \(\pi_{\alpha}(P) = \pi_{\alpha}(x)\) for all \(a\) so that \(\beta < a \leq \gamma\). Equivalently: \(w_{\gamma}\) is the point set to which \(P\) belongs if and only if there exists a \(\beta < \alpha\) so that \(\pi_{\alpha}(L(P)) = \pi_{\alpha}(L(x))\) for all \(a > \beta\). The set \(w_{\gamma}\) will be shown to be the composant of \(M\) containing \(x\).

Suppose \(P \in w_{\gamma}\). Then there exists a \(\beta < \alpha\) so that \(\pi_{\alpha}(L(P)) = \pi_{\alpha}(L(x))\) for all \(a > \beta\). Then \(\{y \mid y \in M\text{ and } (y_{a})_{a} = (x_{a})_{a}\text{ for all }a\text{ such that }\beta < a \leq \gamma\}\) is a proper subcontinuum of \(M\) containing \(x\) and \(P\). The following lemma implies that \(w_{\gamma}\) is a composant.

**Lemma A.** If \(I\) is a proper subcontinuum of \(M\) containing the point \(x\) then there exists a \(\beta < \alpha\) so that if \(\beta < \gamma < \alpha\) then \(\pi_{\alpha}(P_{\gamma}(I)) = \pi_{\alpha}(x)\) for all \(a\) so that \(\beta < a \leq \gamma\); (or, there exists a \(\beta < \alpha\) so that \(\pi_{\alpha}(L(I)) = \pi_{\alpha}(L(x))\) for all \(a\) so that \(\beta < a < \alpha\).)

**Proof.** Suppose that \(I\) is a subcontinuum of \(M\) containing the point \(x\). Then there exists an element \(\beta < \alpha\) so that \(P_{\beta}(I) \neq M_{\beta}\). Suppose that
the lemma is false. Then there exists a first element $a_1 > \beta$ so that $\pi_{a_1}(L(I))$ is non-trivial. Likewise there is a first element $a_2$ after $a_1$ and a first element $a_3$ after $a_2$ so that $\pi_{a_1}(L(I))$ and $\pi_{a_2}(L(I))$ are non-trivial, $\beta < a_1 < a_2 < a_3$.

Let $\gamma > a_3$. Suppose $0 \in \pi_{a_1}(P_{\gamma}(I))$ for some $i = 1, 2, 3$. Then there is a number $t$ distinct from 0 in $\pi_{a_i}(P_{\gamma}(I))$. But $P_{\gamma}(I)$ intersects $M_{a_{i-1}} \times \{0\}$ and $M_{a_{i}} - (M_{a_{i-1}} \times \{0\})$, so $M_{a_{i-1}} \times \{0\} \subset P_{a_i}(I)$. Thus $M_{\beta} \subset P_{\beta}(I)$ which is a contradiction.

Suppose $1 \in \pi_{a_2}(P_{\gamma}(I))$. Then there is a number $1 < t < 1$ in $\pi_{a_3}(P_{\gamma}(I))$. But there is a number $r$ so that $\{A_i \times [r, 1) \subset P_{a_2}(I)\}$. Then $0 \in \pi_{a_1}(P_{a_2}(I))$ since $A_i$ is an element of $S_1$ not in $S_2$. Then $1 \in \pi_{a_3}(P_{\gamma}(I))$. Similarly $1 \in \pi_{a_2}(P_{\gamma}(I))$.

Suppose $0 < t_1 < t_2 < 1$ and $[t_1, t_2] \subset \pi_{a_3}(P_{\gamma}(I))$. But $P_{a_2}(I)$ does not intersect any of the sets $\{(A_{i-1}) \times [1/(2i + 2), 1/(2i + 1)]\}_{i=0}^\infty$ or any of the sets $\{(B_{i-1}) \times [1/(2i + 1), 1/2i]\}_{i=1}^\infty$, or else either 0 or 1 would belong to $\pi_{a_2}(P_{a_2}(I))$. Then $P_{a_2}(I)$ must be a subset of $M_{a_2} \times \{1/k\}$ for some integer $k > 1$. But $\pi_{a_2}(P_{a_2}(I)) = \pi_{a_2}(P_{\gamma}(I))$ for $a \leq a_3$ so $\pi_{a_2}(P_{\gamma}(I)) = 1/k$ which is a contradiction. So the lemma must be true.

**Lemma B.** Suppose $q$ is a cardinal number and $\alpha$ is the first ordinal number so that $q = |\alpha|$. Then there exists a collection $G$ of functions from $\alpha$ into the set $\{0, 1\}$ of cardinality $2^q$ so that if $f$ and $g$ belong to $G$ then the set $\{x \mid x \in \alpha \text{ and } f(x) \neq g(x)\}$ is cofinal in $\alpha$.

**Proof.** Let $T$ be a bijection from $\alpha \times \alpha$ onto $\alpha$. Suppose that $S$ is a subset of $\alpha$, let $f$ be the function from $\alpha$ into $\{0, 1\}$ so that $f(i) = 1$ if and only if $i \in T(S \times \alpha)$. Let $G = \{f_i \mid S \text{ is a subset of } \alpha\}$. Suppose $S_1$ and $S_2$ are two distinct subsets of $\alpha$ and $a$ is an element of $S_1$ not in $S_2$. Then $f(a)(T(S_1 \times \alpha)) = 1$ and $f(a)(T(S_2 \times \alpha)) = 0$ so $\{x \mid x \in \alpha \text{ and } f_a(x) \neq f_b(x)\}$ contains the set $T(S_1 \times \alpha)$ which is cofinal in $\alpha$. Thus $|G| = 2^q$ and the lemma is proven.

The continuum $M$ was constructed so that every function from $\alpha$ into the set $\{0, 1\}$ belongs to $L(M)$. If $q$ is a cardinal number and $\alpha$ is the first ordinal number so that $q = 2^{\alpha}$ then, by Lemma B, the number of composants of $M$ is at least $2^{\alpha}$. If $c$ denotes the cardinality of $[0, 1]$ then $M$ has cardinality at most $c^{\alpha}$. But $2^{\alpha} = c^{\alpha}$, so $M$ has $2^{\alpha}$ composants.

Notation: If $\lambda$ is a limit ordinal let $M_{\lambda}$ denote the indecomposable continuum obtained by the construction of Theorem 2 with $\lambda = \alpha$.

**Corollary 2.1.** If $X$ is a continuum then $X$ is homeomorphic to a retract of an indecomposable continuum with an arbitrarily large number of composants.
Proof. It follows from the construction in [3] that $X$ is homeomorphic to a retract of an irreducible continuum $Y$. Then if $Y$ is irreducible from the point $A$ to the point $B$ merely replace $M_0$ by $Y$ and $\{0\}$ and $\{1\}$ by $A$ and $B$ respectively in the above construction.

**Corollary 2.2.** There exists a non-metric continuum each proper subcontinuum of which is metric.

Proof. Consider $M_{\omega_1}$, where $\omega_1$ is the first uncountable ordinal. By Lemma A, if $I$ is a proper subcontinuum of $M$ there is a point $x \in M$ and an element $\beta < \omega_1$ so that $\pi_a(L(I)) = \pi_a(x)$ for all $a$ so that $\beta < a < \omega_1$. Thus $L(I)$ is embedded in $\prod_{a \leq \beta} [0, 1]_a \times \{ (\pi_a(L(x)))_{a < \beta} \}$. So $I$ is homeomorphic to a subset of the cartesian product of countably many intervals and hence is metric. For each $a < \omega_1$ let $x_a$ be the point of $\prod_{a < \alpha} [0, 1]$, which is 1 at the $a$th coordinate and is 0 elsewhere. Then the set $\{ x \mid x = x_a, a < \alpha \}$ is an uncountable set of points in $L(M)$ which contains none of its limit points. Thus $L(M)$ is not metric.

**Observation 1.** If $X$ is a non-metric continuum and every proper subcontinuum of $X$ is metric then $X$ is indecomposable.

**Observation 2.** The continuum $M_{\omega_1}$ has $2^{\omega_1}$ composants, and $c \leq 2^{\omega_1} \leq 2^c$. Thus the continuum could have $c$ or $2^c$ composants depending on which axioms of set theory are assumed. It is also possible that neither equality holds.

**Corollary 2.3.** There exists a continuum $M$ every proper subcontinuum of which is less numerous than $M$.

Proof. Let $\alpha$ be the first ordinal number so that $2^c < 2^{|\alpha|}$, where $c$ is the cardinality of the interval $[0, 1]$. Then if $\beta < \alpha$, $2^{\beta} < 2^{|\alpha|}$. Consider the continuum $M_\alpha$ constructed above. $M_\alpha$ contains at least $2^{|\alpha|}$ points. By Lemma A, if $I$ is a proper subcontinuum of $M$ there exists a point $x \in M$ and an element $\beta < \alpha$ so that $\pi_a(L(I)) = \pi_a(x)$ for all $a$ so that $\beta < a < \alpha$. Thus $L(I)$ is embedded in $\prod_{a \leq \beta} [0, 1]_a \times \{ (\pi_a(L(x)))_{a < \beta} \}$. So $I$ has at most $c^{\beta}$ points and $c^{\beta} \leq 2^c < 2^{|\alpha|}$. Again observe that any continuum having this property must be indecomposable.

**Theorem 3.** Suppose $q$ is a cardinal number, $\alpha$ is the first ordinal number so that $|\alpha| = q$, and $C$ is a composant of $M_\alpha$. If $H \subseteq C$ and $|H| < \alpha$ then $\overline{H} \subseteq C$.

Proof. Suppose $H \subseteq w_\alpha$. It follows from the definition of $w_\alpha$ that there exists a $\beta < \alpha$ so that if $P \in H$ then $\pi_a(L(P)) = \pi_a(L(x))$ for all $a$
so that $\beta < a < \alpha$. Suppose $Q \in M - w_r$. Then there exists a $\delta > \beta$ so that $\pi_\delta(L(Q)) \neq \pi_\delta(L(x))$. Let $S_\delta$ be a region in $[0, 1]_c$ containing $\pi_\delta(L(\theta))$ and not $\pi_\delta(L(x))$. Then $R = \{Z | \pi_\delta(Z) \in S\}$ is an open set in $L(M)$ containing $L(Q)$ but no point of $L(H)$. So $Q \not\in H$. So $H \subset \omega_x$.

**DEFINITION.** The subset $H$ of the Hausdorff space $X$ is said to be conditionally compact if and only if it is true that every infinite subset of $H$ has a limit point in $H$.

**COROLLARY 3.1.** There exists a conditionally compact indecomposable connected Hausdorff space with only one composant.

**Proof.** By Theorem 3 any composant of $M_\alpha$ is such a space.

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