ANALYTIC EXTENSIONS OF VECTOR-VALUED FUNCTIONS

Josip Globevnik
Let $\Delta$ be the open unit disc in $\mathbb{C}$, $\partial\Delta$ its boundary and $B \subset \partial\Delta$ a relatively open set. Let $X$ be a complex Banach space. Denote by $H_b(\Delta, X)$ the set of all continuous functions from $\Delta \cup B$ to $X$ which are analytic on $\Delta$. A set $P \subset X$ is said to have the analytic extension property with respect to $H_b(\Delta, X)$ if for each relatively closed set $F \subset B$ of Lebesgue measure 0 and for each continuous function $f : F \to P$ there exists $g \in H_b(\Delta, X)$ with $g|F = f$ and $g(\Delta \cup B) \subset P$.

**Theorem.** Let $P \subset X$ be an open set. Then $P$ has the analytic extension property with respect to $H_b(\Delta, X)$ for every relatively open $B \subset \partial\Delta$ if and only if $P$ is connected.

By a result of E. A. Heard and J. H. Wells any closed disc in $\mathbb{C}$ has the analytic extension property with respect to $H_b(\Delta, C)$ for every relatively open $B \subset \partial\Delta$ (see [9]). The special case $B = \partial\Delta$ is the well known Rudin-Carleson theorem (see [4], [10], [12]). This result was generalized to the vector case by proving that every closed ball in $X$ has the analytic extension property with respect to $H_b(\Delta, X)$ for every relatively open $B \subset \partial\Delta$ (see [6]), the special case $B = \partial\Delta$ is the Rudin-Carleson theorem for vector-valued functions (see [5], [11], [14]).

It is a natural question whether the balls above can be replaced by some other sets:

**Problem** (see [8]). Obtain a (geometrical, topological) characterization of the sets having the analytic extension property with respect to $H_b(\Delta, X)$ for every relatively open $B \subset \partial\Delta$.

It seems that this problem is not solved even for the subsets of $\mathbb{C}$.

Taking $B = \partial\Delta$, $F = \{-1, 1\}$ it is trivial to see that every set having the analytic extension property with respect to $H_b(\Delta, X)$ for every relatively open $B \subset \partial\Delta$, is pathwise connected. The converse is not true in general as shown by taking $P = \{t : 0 \leq t \leq 1\}$. However, the converse turns out to be true for open sets and this is the main result of the present paper.

Throughout, we denote by $\bar{\Delta}$ the closure of $\Delta$. Given $r > 0$ we denote by $B_r(X)$ the open ball in $X$ of radius $r$, centered at the origin. If $K$ is a compact Hausdorff space we denote by $C(K, X)$
the space of all continuous functions from $K$ to $X$. By $A(\Delta, X)$ we denote the Banach space of all continuous functions from $\overline{\Delta}$ to $X$, analytic on $\Delta$, with sup norm, and we write $A = A(\Delta, C)$. We write $I = \{t: 0 \leq t \leq 1\}$ and we denote the set of all positive integers by $N$.

For the proof of theorem we shall need four lemmas.

**Lemma 1.** Suppose that $G$ is a closed subset of $\partial \Delta$ of Lebesgue measure 0 and let $U(G) \subset \overline{\Delta}$ be a neighbourhood of $G$. Let $p: I \rightarrow X$ be a path in a complex Banach space $X$ and let $\varepsilon > 0$ be arbitrary. There exists $\phi \in A(\Delta, X)$ having the following properties:

- (i) $\|\phi(z) - p(1)\| < \varepsilon \ (z \in G)$
- (ii) $\|\phi(z) - p(0)\| < \varepsilon \ (z \in \overline{\Delta} - U(G))$
- (iii) $\phi(\overline{\Delta}) \subset p(I) + B_{\varepsilon}(X)$.

*Proof.* By the Mergelyan theorem for analytic functions into a Banach space (see [3]) there exists a polynomial $f: C \rightarrow X$ satisfying $\|f(z) - p(z)\| < \varepsilon \ (z \in I)$. By the continuity of $f$ there exists an open neighbourhood $V$ of $I$ such that $f(V) \subset p(I) + B_{\varepsilon}(X)$. Let $W \subset V$ be an open set, bordered by a Jordan curve, containing the point 1 in its boundary and satisfying $I - \{1\} \subset W$, $\overline{W} \subset V$. Let $T \subset W$ be a neighbourhood of the point 0 in $W$ such that $\|f(z) - p(0)\| < \varepsilon \ (z \in T)$. Assume for a moment that $\alpha \in A$ satisfies $\alpha(\overline{\Delta}) \subset \overline{W}$, $\alpha(G) = \{1\}$ and $\alpha(\overline{\Delta} - U(G)) \subset T$. Then it is easy to check that $\phi = f \circ \alpha$ has all the required properties. It remains to prove the existence of such an $\alpha$. By the Riemann mapping theorem (see [13]) there exists a homeomorphism $\beta$ from $\overline{\Delta}$ onto $\overline{W}$, analytic on $\Delta$ and satisfying $\beta(0) = 0$, $\beta(1) = 1$. Let $S \subset \Delta$ be a neighbourhood of 0 such that $\beta(S) \subset T$. By the Rudin-Carleson theorem (see [12], p. 205) there exists $\psi \in A$ satisfying $\psi(\overline{\Delta}) \subset \overline{W}$, $\psi(G) = \{1\}$, $|\psi(z)| < 1 \ (z \in \overline{\Delta} - G)$. Let $U_i \subset U(G)$ be an open subset of $\overline{\Delta}$ containing $G$. Now $\overline{\Delta} - U_i$ is a compact set disjoint from $G$ and it follows that for sufficiently large $n \in N$ we have $\psi^n(z) \cdot \gamma(z) \in S \ (z \in \overline{\Delta} - U_i)$. Now putting $\alpha(z) = \beta[\psi^n(z) \cdot \gamma(z)] \ (z \in \overline{\Delta})$ it is easy to see that $\alpha$ has all the required properties.

**Lemma 2.** Let $X$ be a complex Banach space and let $Q$ be an open connected subset of $X$. Given a compact subset $K$ of $Q$ and a point $x \in K$ there exists $\delta_0 > 0$ such that for every $\delta: 0 < \delta < \delta_0$ there exists a path $p: I \rightarrow X$ satisfying

- (i) $p(0) = x$
- (ii) $K \subset p(I) + B_\delta(X)$
- (iii) $p(I) + B_{\delta_0}(X) \subset Q$. 

For the proof we shall need four lemmas.
Proof. By the compactness of $K$ there exists an $\varepsilon > 0$ such that $K + B_\varepsilon(X) \subset Q$. Cover $K$ by a finite number of balls, say by $B_\varepsilon, B_2, \cdots, B_n$ of radii $\varepsilon$ whose centers lie in $K$. With no loss of generality assume that the center of $B_i$ is $x_i$. By the connectedness of $Q$ there exists a path $q : I \to X$, satisfying $q(I) \subset Q$, $q(0) = x$, and connecting the centers of all $B_i$. By the compactness of $q(I)$ there exists $\delta_0 > 0$ such that $q(I) + B_{\delta_0}(X) \subset Q$. Let $\delta$ satisfy $0 < \delta < \delta_0$ and cover $K$ by a finite number of balls $B_1, B_2, \cdots, B_n$ of radii $\delta$ whose centers lie in $K$. Let $1 \leq i \leq n$. Consider those balls $B_k$ whose centers lie in $B_i$. Connect all these centers by a path $p_i$ starting and ending at the center of $B_i$ and satisfying $p_i(I) \subset B_i$. Having done this for all $i$, denote by $q_i (1 \leq i \leq n - 1)$ the part of the path $q$ between the centers of $B_i, B_{i+1}$. Now define $p$ as the sum of the paths

$$p = \sum_{i=1}^{n-1} (p_i + q_i) + p_n.$$

If $s \in I$ is such that $p(s)$ is in none of the balls $B_i$ $(1 \leq i \leq n)$ then $p(s) \in q(I)$ and consequently $p(s) + B_{\delta_0}(X) \subset q(I) + B_{\delta_0}(X) \subset Q$. If $s \in I$ is such that $p(s)$ is in some $B_i$ then $p(s) + B_{\delta_0}(X) \subset p(I) + B_{\delta_0}(X) \subset K + B_{\delta_0}(X) \subset Q$. On the other hand, if $y \in K$ then $y \in D_k$ for some ball $D_k$ whose center is contained in $p(I)$ which means that $y \in p(I) + B_\varepsilon(X)$.

**Lemma 3.** Let $F \subset \partial \Delta$ be a closed set of Lebesgue measure 0 and let $U(F) \subset \Delta$ be a neighbourhood of $F$. Suppose that $Q$ is an open connected set in a complex Banach space $X$ containing the point 0. Let $\varepsilon > 0$ be arbitrary. Given $f \in C(F, X)$ satisfying $f(F) \subset Q$ there exists $\tilde{f} \in A(\Delta, X)$ satisfying

(i) $\tilde{f}|F = f$

(ii) $\tilde{f}(\Delta) \subset Q$

(iii) $\|\tilde{f}(z)\| < \varepsilon$ ($z \in \Delta - U(F)$).

Proof. $f(F) \cup \{0\}$ is a compact set contained in $Q$. By Lemma 2 there exists $\delta : 0 < \delta < \varepsilon/5$ and a path $p : I \to X$ satisfying $f(F) \subset p(I) + B_{\delta}(X)$, $p(I) + B_{\delta}(X) \subset Q$ and $p(0) = 0$. Since $F$ is a compact set the function $f$ is uniformly continuous on $F$. By the assumption $F$ is nowhere dense on $\partial \Delta$. It follows that

$$F = \bigcup_{i=1}^{\delta} F_i,$$

where $F_i \subset \partial \Delta$ are disjoint closed sets such that

$$\|f(\eta) - f(\zeta)\| < \delta \quad (\eta, \zeta \in F_i; \ 1 \leq i \leq n).$$
Let $U_i$ ($1 \leq i \leq n$) be disjoint open subsets of $\mathcal{I}$ satisfying $F_i \subset U_i \subset U(F)$ ($1 \leq i \leq n$). Since $f(F) \subset p(I) + B_s(X)$ there exist $t_i \in I$ and $z_i \in F_i$ ($1 \leq i \leq n$) such that

$$||p(t_i) - f(z_i)|| < \delta \quad (1 \leq i \leq n).$$

Applying Lemma 1 to the paths $t \mapsto p(t_i)$ ($1 \leq i \leq n$) there exist functions $\phi_i \in A(\mathcal{I}, X)$ ($1 \leq i \leq n$) satisfying

$$||\phi(z) - p(t_i)|| < \delta \quad (z \in F_i)$$
$$||\phi(z)|| < \delta/n \quad (z \in \mathcal{I} - U_i)$$
$$\phi_i(\mathcal{I}) \subset p(I) + B_s(X).$$

Now define $\Psi \in A(\mathcal{I}, X)$ by

$$\Psi = \sum_{i=1}^{n} \phi_i.$$

If $z \in \mathcal{I} \setminus \bigcup_{i=1}^{n} U_i$ then

$$||\Psi(z)|| \leq \sum_{i=1}^{n} ||\phi_i(z)|| < n. \quad \delta/n = \delta.$$

If $z \in U_i$ for some $i$ then $z \notin U_j$ ($i \neq j$) and

$$\Psi(z) = \phi_i(z) + \sum_{j=1 \neq i}^{n} \phi_j(z) \in p(I) + B_s(X) + B_s(X) \subset p(I) + B_{2\delta}(X).$$

Consequently $\Psi(\mathcal{I}) \subset p(I) + B_{2\delta}(X)$. Now define $\Theta \in C(F, X)$ by $\Theta(z) = \Psi(z) - f(z)$ ($z \in F$). If $z \in F$ then $z \in F_i$ for some $i$ and consequently

$$||\Theta(z)|| \leq ||\Psi(z) - p(t_i)|| + ||p(t_i) - f(z)|| + ||f(z_i) - f(z)||$$
$$\leq \sum_{j \neq i} ||\phi_j(z)|| + ||\phi_i(z) - p(t_i)|| + 2\delta.$$ 

By the Rudin-Carleson theorem for vector valued functions there exists $\tilde{\Theta} \in A(\mathcal{I}, X)$ satisfying $||\tilde{\Theta}|| < 4\delta$, $\tilde{\Theta} | F = \Theta$. Finally, define $\tilde{f}(z) = \Psi(z) - \tilde{\Theta}(z)$ ($z \in \mathcal{I}$). Clearly $\tilde{f} \in A(\mathcal{I}, X)$. Further, $\tilde{f}(\mathcal{I}) \subset p(I) + B_{2\delta}(X) + B_{2\delta}(X) \subset p(I) + B_{2\delta}(X) \subset Q$. Clearly $\tilde{f} | F = f$. Also, if $z \in \mathcal{I} \setminus U(F)$ then $z \in \mathcal{I} \setminus \bigcup_{i=1}^{n} U_i$ hence $||\tilde{f}(z)|| \leq ||\Psi(z)|| + ||\tilde{\Theta}(z)|| < \delta + 4\delta < \varepsilon$.

**Lemma 4.** Let $E$ be closed subset of $\partial \mathcal{I}$ and let $G \subset \partial \mathcal{I} - E$ be a relatively closed set of Lebesgue measure 0. Let $H \subset \partial \mathcal{I} - E$ be a compact set of Lebesgue measure 0, disjoint from $G$. Let $Q$ be an open connected set in a complex Banach space $X$ containing the point 0 and suppose that $f \in C(H, X)$ satisfies $f(H) \subset Q$.

There exists $\delta_0 > 0$ such that for every $\eta > 0$ the following holds for all $f_0 \in C(H, X)$ satisfying $f_0(H) \subset Q$ and $||f_0 - f|| < \eta$. There exists $\delta > 0$ such that for every $\delta < \delta_0$ and for all $f_0 \in C(H, X)$ satisfying $f_0(H) \subset Q$ and $||f_0 - f|| < \delta$ there exists $\tilde{f_0} \in C(H, X)$ satisfying $||\tilde{f_0} - f|| < \eta$.
every ε: 0 < ε < δ and for every neighbourhood \( U \subset \overline{A} - E \) of \( H \) there exists a continuous function \( \tilde{f}: \overline{A} - E \to X \), analytic on \( \Delta \) and satisfying

1. \( \tilde{f}|_H = f \)
2. \( \tilde{f}|_G = 0 \)
3. \( \| \tilde{f}(z) \| < \varepsilon \ (z \in (\overline{A} - E) - U) \)
4. \( f(\overline{A} - E) + B_\varepsilon(X) \subset Q \).

**Proof.** With no loss of generality we may assume that \( U \cap G = \emptyset \). By Lemma 2 there exists \( \delta_0 > 0 \) such that for every \( \delta: 0 < \delta < \delta_0 \) there exists a path \( p: I \to X \) satisfying \( p(0) = 0, f(H) \subset p(I) + B_\delta(X) \), \( p(I) + B_{\delta_0}(X) \subset Q \). Let \( 0 < \delta < \delta_0 \) and \( 0 < \varepsilon < \delta \). Applying Lemma 3 to the function \( f \) and to the (open connected) set \( p(I) + B_\varepsilon(X) \) there exists \( f_1 \in \mathcal{A}(\Delta, X) \) satisfying

\[
\tilde{f}_1|_H = f
\]

\[
\tilde{f}_1(\overline{A}) + B_{\delta_0}(X) \subset Q
\]

\[
\| \tilde{f}_1(z) \| < \varepsilon/2 \ (z \in \overline{A} - U).
\]

Define

\[
f_2(s) = \begin{cases} -\tilde{f}_1(s) & (s \in G) \\ 0 & (s \in H) \end{cases}.
\]

Then \( f_2 \) is continuous on \( G \cup H \) and satisfies \( \| f_2(s) \| < \varepsilon/2 \ (s \in G \cup H) \). By Theorem 2 in [6] there exists a continuous function \( \tilde{f}_2: \overline{A} - E \to X \), analytic on \( \Delta \), satisfying \( \tilde{f}_2|_G \cup H = f \) and \( \| \tilde{f}_2(z) \| \leq \varepsilon/2 \ (z \in \overline{A} - E) \). Put \( \tilde{f} = \tilde{f}_1 + \tilde{f}_2 \). It is easy to check that \( \tilde{f} \) has all the required properties.

**Proof of theorem.** Let \( Q \) be an open connected subset of a complex Banach space \( X \). Let \( E \subset \partial \Delta \) be a closed set and let \( F \subset \partial \Delta - E \) be a relatively closed set of Lebesgue measure 0. Suppose that \( f: F \to X \) is a continuous function satisfying \( f(F) \subset Q \). We will prove that there exists a continuous extension \( \tilde{f}: \overline{A} - E \to X \), \( \tilde{f}|_F = f \), which is analytic on \( \Delta \) and which satisfies \( f(\overline{A} - E) \subset Q \).

If \( E \) is empty then the statement of the theorem is proved by Lemma 3. So assume that \( E \) is not empty. With no loss of generality assume that \( 0 \in Q \). As in [6] write \( F = \bigcup_{n=1}^\infty F_n \) where \( F_n \subset \overline{A} - E \) are compact sets such that there exist disjoint open sets \( U_n \subset \overline{A} - E \) satisfying \( F_n \subset U_n \) for all \( n \).

Now we define inductively a sequence \( \{D_n\} \) of open subsets of \( \overline{A} - E \) satisfying \( F_n \subset D_n \subset U_n \) for all \( n \), a decreasing sequence \( \{\delta_n\} \) of positive numbers and a sequence \( \{\phi_n\} \) of functions from \( \overline{A} - E \) to \( X \) having the following properties:
for each $i \in \mathbb{N}$, $\phi_i$ is continuous on $\bar{A} - E$ and analytic on $A$.

(ii) $\phi_i \big|_{F_j} = 0$ ($i \neq j$; $i, j \in \mathbb{N}$)

(iii) $\phi_i \big|_{F_i} = f \big|_{F_i}$ ($i \in \mathbb{N}$)

(iv) $\phi_i(\bar{A} - E) + B_{\delta_i}(X) \subset Q$ ($i \in \mathbb{N}$)

(v) $\|\phi_i(z)\| < \delta_i/2^{i+1}$ ($z \in (\bar{A} - E) - D_i$; $i \in \mathbb{N}$)

(vi) $\|\sum_{j=1}^i \phi_j(z)\| < \delta_{i+1}/2$ ($z \in D_{i+1}; i \in \mathbb{N}$).

If $i = 1$, put $D_1 = U_1$ and apply Lemma 4 to the function $f \big|_{F_1}$ to obtain $\delta_i$ satisfying $B_{\delta_i}(X) \subset Q$ and $\phi_i$ which satisfies (i)-(v) above for $i = 1$. Now assume that $\delta_i, D_i, \phi_i$ ($1 \leq i \leq n$) are given satisfying (i)-(v) for $1 \leq i \leq n$ and (vi) $1 \leq i \leq n - 1$. Applying Lemma 4 to the function $f \big|_{F_{n+1}}$ there exists $\delta_{n+1} > 0 < \delta_n$ such that Lemma 4 holds for $\delta = \delta_{n+1}$. Since the function

$$z \mapsto \sum_{j=1}^n \phi_j(z)$$

is continuous on $\bar{A} - E$ and equal 0 on $F_{n+1}$ there exists a neighbourhood $D_{n+1} \subset \bar{A} - E$ of $F_{n+1}$ satisfying $D_{n+1} \subset U_{n+1}$ and such that (vi) is satisfied for $i = n$. Now, by Lemma 4 there exists $\phi_{n+1}$ satisfying (i)-(v) for $i = n + 1$.

Define

$$\tilde{f}(z) = \sum_{i=1}^n \phi_i(z) \quad (z \in \bar{A} - E).$$

If $z \in (\bar{A} - E) - \bigcup_{j=1}^n D_j$ then $\|\phi_i(z)\| < \delta_i/2^{i+1} < \delta_i/2^{i+1}$. Consequently the series converges uniformly for all such $z$. By

$$\sum_{i=1}^\infty \|\phi_i(z)\| < \delta_i/2$$

and by $B_{\delta_i}(X) \subset Q$ we have $f(z) \in Q$ for all such $z$. Suppose that $z \in D_k$ for some $k$. Then $z \in D_j$ for $j \neq k$ and by the above argument the series converges uniformly on $D_k$. Further, by (v) and (vi) we have

$$\left\|\sum_{j=k}^\infty \phi_j(z)\right\| \leq \left\|\sum_{j=1}^{k-1} \phi_j(z)\right\| + \sum_{j=k+1}^\infty \|\phi_j(z)\| < \delta_k/2 + \delta_k/2 = \delta_k.$$

Consequently by (iv) $f(z) \in \phi_k(\bar{A} - E) + B_{\delta_k}(X) \subset Q$. Since each compact subset of $\bar{A} - E$ misses all but a finite number of the sets $D_i$ the series converges uniformly on compact subsets of $\bar{A} - E$. Consequently $\tilde{f}$ is continuous on $\bar{A} - E$, analytic on $A$ and, as shown above, satisfies $\tilde{f}(\bar{A} - E) \subset Q$. By the properties of $\phi_i$ we have also $\tilde{f} \big|_{F} = f$.

**COROLLARY** (see [7]). Given any open connected subset $Q$ of a
separable complex Banach space $X$ there exists an analytic function $f: \Delta \to X$ whose range is contained and dense in $Q$.

**Proof.** Put $E = \{1\}$ and let $F = \{z_n\} \subset \partial \Delta - \{1\}$ be an injective sequence converging to 1. Let $f(z_n) = w_n$ where $\{w_n\} \subset Q$ is a sequence dense in $Q$ and then apply theorem.

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