MATRIX TRANSFORMATIONS AND ABSOLUTE SUMMABILITY

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The main results of this paper are two theorems which give necessary conditions for a matrix to map into \( \mathcal{J} \) the set of all subsequences (rearrangements) of a null sequence not in \( \mathcal{J} \). These results provide affirmative answers to the following questions proposed by J. A. Fridy. Is a null sequence \( x \) necessarily in \( \mathcal{J} \) if there exists a sum-preserving \( \mathcal{J} \rightarrow \mathcal{J} \) matrix \( A \) that maps all subsequences (rearrangements) of \( x \) into \( \mathcal{J} \)?

1. Introduction. Let \( s, m, c, c_0 \) and \( cs \) denote, respectively, the set of all complex sequences, the set of all bounded sequences in \( s \), the set of all convergent sequences in \( s \), the set of all null sequences in \( c \), and the set of all sequences in \( s \) with sequence of partial sums in \( c \). Let

\[
\mathcal{J} = \{ x \in s : \Sigma |x_p| < \infty \} \quad \text{and} \quad \mathcal{J}^c = \{ x \in s : \Sigma |x_p|^2 < \infty \}.
\]

A matrix \( A \) which maps each element of \( \mathcal{J} \) into \( \mathcal{J} \) is called an \( \mathcal{J} \rightarrow \mathcal{J} \) matrix and may be characterized [3] and [6] by the property:

\[
\left( \sum_{p=1}^{\infty} |a_{pq}| \right)_{q=1}^{\infty} \in m.
\]

If, in addition, \( \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} a_{pq} x_q = \sum_{q=1}^{\infty} x_q \), whenever \( x \in \mathcal{J} \), then \( A \) is a sum-preserving \( \mathcal{J} \rightarrow \mathcal{J} \) matrix; this is characterized by \( \sum_{p=1}^{\infty} a_{pq} = 1 \), for each \( q \).

In 1943, R. C. Buck [1] showed that a sequence \( x \) is convergent if some regular matrix sums every subsequence of \( x \). J. A. Fridy [5] has obtained an analog to Buck’s theorem in which “subsequence” is replaced by “rearrangement.” In addition, he has characterized \( \mathcal{J} \) by showing that \( x \in \mathcal{J} \) if there is a sum-preserving \( \mathcal{J} \rightarrow \mathcal{J} \) matrix that transforms every rearrangement of \( x \) into \( \mathcal{J} \). In §2 of the present paper, necessary conditions are obtained for a matrix to map into \( \mathcal{J} \) the set of all subsequences of a null sequence not in \( \mathcal{J} \). This result yields as a corollary the affirmative answer to the following question proposed by J. A. Fridy [5]. Is a null sequence \( x \) necessarily in \( \mathcal{J} \) if there exists a sum-preserving \( \mathcal{J} \rightarrow \mathcal{J} \) matrix that maps all subsequences of \( x \) into \( \mathcal{J} \)? In §3, necessary conditions are obtained for a matrix to map into \( \mathcal{J} \) all rearrangements of a null sequence not in \( \mathcal{J} \). This yields as a corollary Fridy’s characterization of \( \mathcal{J} \) mentioned above. Finally, §4 contains examples of matrix mappings involving both subsequences and rearrangements.

2. Subsequences. The following two lemmas will be instru-
LEMMA 1. Suppose x and a are sequences such that $\sum_{q=1}^{\infty} a_q y_q$ converges for every subsequence y of x. If $\varepsilon > 0$, then there exist $M > 0$ and a strictly increasing function $\delta: I^+ \to I^+$ such that if $t > M$, then $|\sum_{q=t}^{\infty} a_q y_q| \leq \varepsilon$ for every subsequence $(y'_q)_{q=t}^{\infty}$ of $(x_q)_{q=t}^{\infty}$.

LEMMA 2. If x is a null sequence not in $\mathcal{J}$ and a is a nonnull convergent sequence, then there exists a subsequence y of x such that $\lim_i |\sum_{q=1}^{\infty} y_q| = \infty$ and $(\sum_{q=1}^{\infty} a_q y_q)_{n=1}^{\infty}$ is not bounded.

THEOREM 1. Let x be a null sequence not in $\mathcal{J}$, and suppose A is a matrix such that $Ay \in \mathcal{J}$ for every subsequence y of x. Then

(i) $\sum_{p=1}^{\infty} |a_{pq}| < \infty$ for $q = 1, 2, 3, \ldots$; and

(ii) if $\lim_q \sum_{p=1}^{\infty} a_{pq} = L$, then $L = 0$.

Proof. To show (i), let $k$ be fixed and $j > i > k$ such that $x_i \neq x_j$. Let y be the subsequence of x such that $y_q = x_q$ for $q = 1, 2, \ldots, k - 1; y_k = x_i$; and $y_{k+t} = x_{j+t}$ for $t = 1, 2, 3, \ldots$. Let z be the subsequence of x such that $z_k = x_j$ and $z_q = y_q$ otherwise. Then

$$\infty > \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} a_{pq} y_q - \sum_{q=1}^{\infty} a_{pq} z_q = |x_i - x_j| \sum_{p=1}^{\infty} |a_{pq}| .$$

Therefore $\sum_{p=1}^{\infty} |a_{pq}| < \infty$.

Suppose $\lim_q \sum_{p=1}^{\infty} a_{pq} = L$ and $L \neq 0$. Let $(y_1, \ldots, y_{M-1})$ be a subsequence of x with $y_{M-1} = x_r$. Since $x \in \mathcal{J}$ there exists a subsequence $(w_q)_{q=r+1}^{\infty}$ of $(x_q)_{q=r+1}^{\infty}$ such that $\lim_q |\sum_{q=M}^{\infty} w_q| = \infty$. By Lemma 2 there exists a subsequence $(z_q)_{q=M}^{\infty}$ of $(w_q)_{q=M}^{\infty}$ such that $\lim_q |\sum_{q=M}^{\infty} z_q| = \infty$ and $\limsup_q |\sum_{q=M}^{\infty} z_q - \sum_{q=M}^{\infty} a_{pq}| = \infty$. Choose $k > M$ such that

$$|\sum_{q=M}^{k} z_q - \sum_{p=1}^{\infty} a_{pq}| > M + \sum_{q=1}^{M-1} |y_q| \sum_{p=1}^{\infty} |a_{pq}| + 3 .$$

Let $K > 0$ such that $|\sum_{p=K+1}^{\infty} a_{pq}| < 1/(k(|z_q| + 1))$ for $q = M, \ldots, k$. By Lemma 1, letting $\varepsilon = 1/K$, there exist $N'_p$ and $\delta'_p$ for $1 \leq p \leq K$, such that if $N = \max\{N'_1, \ldots, N'_K, k + 2\}$ and $\delta(i) = \max\{\delta'_p(i); p = 1, \ldots, K\}$, then $\sum_{p=1}^{K} |\sum_{q=N_p}^{\infty} a_{pq} y_q| < 1$ for every subsequence $(v_q)_{q=N}^{\infty}$ of $(x_q)_{q=N}^{\infty}$. Let $y_q = z_q$ for $M \leq q \leq k$, and choose $(y_{k+1}, \ldots, y_{N-1})$ a subsequence of $(w_q)_{q=1}^{\infty}$ such that $\sum_{q=k+1}^{N-1} |y_q| \sum_{p=1}^{\infty} |a_{pq}| < 1$. Note that the first $N - 1$ terms of a fixed sequence y have now been determined. If $y^*$ is any subsequence of x that agrees with y for these first $N - 1$ terms, then $\sum_{p=1}^{K} |\sum_{q=1}^{\infty} a_{pq} y^*_q| > M$.

This process for defining terms of y may be continued so that if $T > 0$, then there exist $M \leq T$ and $K > 0$ such that
Thus a subsequence $y$ of $x$ can be constructed such that $Ay \in \mathcal{I}$, a contradiction.

**Corollary 1.** A null sequence $x$ is in $\mathcal{I}$ if and only if there exists a sum-preserving $\mathcal{I} \to \mathcal{I}$ matrix $A$ such that $Ay \in \mathcal{I}$ for every subsequence $y$ of $x$.

3. **Rearrangements.** Following J. A. Fridy [5], the sequence $y$ is called a rearrangement of the sequence $x$ provided that there is a $1-1$ function $\pi$ from the positive integers onto themselves such that for each $k$, $x_k = y_{\pi(k)}$. The word "permutation" will be reserved to indicate the reordering of a finite sequence.

**Theorem.** If $x$ is a null sequence not in $\mathcal{I}$ and $A$ is a matrix such that $Ay \in \mathcal{I}$ for every rearrangement $y$ of $x$, then

$$
\lim_{q} \sum_{p=1}^{\infty} |a_{pq}| = 0.
$$

**Proof.** Let $x_i \neq x_j$ be nonzero elements of $x$. Suppose the $k$th column of $A$ is not in $\mathcal{I}$. Let $q \neq k$ and $y$ be a rearrangement of $x$ with $y_k = x_i$ and $y_q = x_j$. Let $z$ be the rearrangement of $x$ such that $z_k = x_j$, $z_q = x_i$, and $z_t = y_t$ otherwise. Then

$$
|x_i - x_j| \sum_{p=1}^{\infty} |a_{pk} - a_{pq}| = \sum_{p=1}^{\infty} |a_{pq}y_q - \sum_{q=1}^{\infty} a_{pq}x_q| < \infty.
$$

Therefore $\sum_{p=1}^{\infty} |a_{pk} - a_{pq}| < \infty$ for every $q \neq k$. Since $\sum_{p=1}^{\infty} |a_{pk}| = \infty$, it now follows that $\sum_{p=1}^{\infty} |a_{pq}| = \infty$ for $q \geq 1$. Suppose $N > 0$ and a permutation $(r_1, \cdots, r_M)$ of $M$ terms of $x$ has been chosen such that $\sum_{q=1}^{M} r_q \neq 0$. If $\lambda = \sum_{p=1}^{\infty} |\sum_{q=1}^{M} a_{pq}r_q| < \infty$, then

$$
\infty > \lambda + \sum_{q=2}^{N} |r_q| \sum_{p=1}^{\infty} |a_{p1} - a_{pq}| \equiv \left| \sum_{q=1}^{M} r_q \right| \sum_{p=1}^{\infty} |a_{p1}|,
$$

a contradiction. Therefore $\lambda = \infty$ and there exists $K > N$ such that $\sum_{p=M+1}^{K} |\sum_{q=1}^{M} a_{pq}r_q| > 2$. Let $i = \min \{q : x_q \in x(r_i, \cdots, r_M)\}$. J. A. Fridy [5] has shown that each row of $A$ is null. Therefore there exists $T > M + 1$ such that $|x_i| \sum_{p=1}^{K} |a_{pT}| < 2^{-(M+1)}$. Let $r_T = x_i$ and $(r_{M+1}, \cdots, r_{T-1})$ be a subsequence of $x(r_i, \cdots, r_M, r_T)$ such that $\sum_{p=1}^{K} \sum_{q=M+1}^{T-1} |a_{pq}||r_q| < 2^{-(M+2)}$ and $\sum_{q=1}^{T-1} r_q \neq 0$. Then

$$
\sum_{p=M+1}^{K} \sum_{q=1}^{T} |a_{pq}r_q| \equiv \sum_{p=M+1}^{T-1} \sum_{q=1}^{K} |a_{pq}r_q| - \sum_{p=M+1}^{K} \sum_{q=1}^{M} |a_{pq}r_q| - |r_T| \sum_{p=M+1}^{K} |a_{pT}| > 1.
$$
But this process may be continued. Therefore there exists a rearrangement \( r \) of \( x \) such that if \( L > 0 \), then there exist \( K > N \geq L \) such that \( \sum_{p=N}^{K} | \sum_{q=1}^{\infty} a_{pq} r_{q} | > 1 \), a contradiction. Hence each column of \( A \) is in \( \mathcal{C} \).

Now suppose there exists \( \varepsilon > 0 \) such that if \( N > 0 \), then there exists \( q > N \) such that \( \sum_{p=1}^{q} | a_{pq} | > \varepsilon \). Let \( z \in \mathcal{C} \) be a subsequence of \( x \) that includes all zero terms of \( x \). Let \( j_{i} = \min \{ q : x_{q} \neq 0 \} \). Let \( N_{1} > 0 \) such that \( \sum_{p=1}^{N_{1}} | a_{pq} | > \varepsilon \). Let \( r_{N_{1}} = x_{j_{2}} \). Also let \( (r_{1}, \ldots, r_{N_{1}-1}) \) be a subsequence of \( z \) such that \( \sum_{q=1}^{N_{1}-1} | r_{q} | \sum_{p=1}^{N_{1}} | a_{pq} | < 1/2 \) and \( z_{i} = r_{a} \) only if for each \( s < t \) such that \( z_{s} = 0 \) there exists \( b < a \) such that \( z_{a} = r_{b} \). Let \( M_{1} > 0 \) such that

\[
\sum_{p=1}^{M_{1}} | a_{pq} | > \frac{\varepsilon}{2} \quad \text{and} \quad r_{N_{1}} = \sum_{p=M_{1}+1}^{\infty} | a_{pq} | < \frac{1}{4}.
\]

Let \( j_{2} = \min \{ q : x_{q} \in x(\{r_{1}, \ldots, r_{N_{1}}\}, \text{and} \ x_{q} \neq 0 \} \). Since each row of \( A \) is null, there exists \( N_{2} > N_{1} + 1 \) such that \( \sum_{p=x_{1}+1}^{N_{2}} | a_{pq} | > \varepsilon/2 \) and \( | x_{j_{2}} | \sum_{p=1}^{N_{2}} | a_{pq} | < 1/8 \). Let \( r_{N_{2}} = x_{j_{2}} \). Also let \( (r_{N_{1}+1}, \ldots, r_{N_{2}-1}) \) be a subsequence of \( x(\{r_{1}, \ldots, r_{N_{1}}, r_{N_{2}}\}) \) such that \( \sum_{q=N_{1}+1}^{N_{2}-1} | r_{q} | \sum_{p=1}^{\infty} | a_{pq} | < 1/16 \) and \( z_{i} = r_{a} \) only if for each \( s < t \) such that \( z_{s} = 0 \) there exists \( b < a \) such that \( z_{a} = r_{b} \). Let \( M_{2} > M_{1} \) such that \( \sum_{p=M_{2}+1}^{\infty} | a_{pq} | > \varepsilon/2 \) and \( | r_{N_{2}} | \sum_{p=M_{2}+1}^{\infty} | a_{pq} | < 1/32 \). This selection process may be continued so that if \( k \) is fixed, then

\[
\sum_{p=1}^{M_{k}} | \sum_{q=1}^{\infty} a_{pq} r_{q} | \geq \left( \sum_{p=1}^{M_{k}} | a_{pq} r_{N_{1}} | + \cdots + \sum_{p=M_{k}+1}^{M_{k}} | a_{pq} r_{N_{k}} | \right)
- \left( \sum_{q=1}^{N_{k}-1} | r_{q} | \sum_{p=1}^{M_{k}} | a_{pq} | + \sum_{p=M_{k}+1}^{M_{k}} | a_{pq} r_{N_{1}} | \right)
+ \sum_{q=N_{k}+1}^{N_{k}-1} | r_{q} | \sum_{p=1}^{M_{k}} | a_{pq} | + | r_{N_{2}} | \sum_{p=1}^{M_{k}} | a_{pq} |
+ \sum_{p=M_{k}+1}^{M_{k}} | a_{pq} r_{N_{2}} | + \cdots \geq \frac{\varepsilon}{2} \sum_{i=1}^{k} | r_{N_{i}} | - 1.
\]

But \( r \) has been selected so that \( \lim_{k} \sum_{i=1}^{k} | r_{N_{i}} | = \infty \). Therefore \( Ar \notin \mathcal{C} \), a contradiction. Hence \( \lim_{q} \sum_{p=1}^{\infty} | a_{pq} | = 0 \).

The proof of Theorem 2 is now complete, and Corollary 2, which was first proved by J. A. Fridy [5], follows directly.

**Corollary 2.** The null sequence \( x \in \mathcal{C} \) is in \( \mathcal{C} \) if and only if there exists a sum-preserving \( \mathcal{C} \rightarrow \mathcal{C} \) matrix \( A \) such that \( Ay \in \mathcal{C} \) for every rearrangement \( y \) of \( x \).

**4. Examples.** By Theorem 2 a matrix \( A \) that maps all rearrangements of a sequence \( x \in c_{0} / \mathcal{C} \) into \( \mathcal{C} \) must be an \( \mathcal{C} \rightarrow \mathcal{C} \) matrix.
But Theorem 1 gives little insight into the question of whether $A$ must be $\prec \prec$ if it maps all subsequences of $x$ into $\prec$. The following example shows that $A$ need not be $\prec \prec$ in this case. Let $x_n = 1/n$ for $n = 1, 2, 3, \ldots$; $a_{nq} = q^{1/3}$ for $q = 1, 8, 27, 64, \ldots$; and $a_{nq} = 0$ otherwise. If $y$ is a subsequence of $x$ and $Ay = z$, then $|z_q| \leq q^{-2/3}$ for $q = 1, 8, 27, \ldots$ and $z_q = 0$ otherwise. Thus $z \in \prec$, but clearly $x \in c_0 \prec$ and $A$ is not $\prec \prec$.

I. J. Maddox [7] showed that a matrix $A$ is Schur if it maps all subsequences of some divergent sequence $x$ into $c$. This might cause one to suspect that if $A$ maps all subsequences (rearrangements) of a sequence $x \in c_0 \prec$ into $\prec$, then $Az \in \prec$ for every $z \in cs$. The following example shows that this is not true. (The author wishes to thank the referee for his comments which aided in the simplification of this example.) Let $x_n = 1/n$ for $n = 1, 2, 3, \ldots$; $a_{nq} = (-1)^q/q$ for $q \geq 1$ and $a_{nq} = 0$ otherwise. Since $(a_{nq})_{q=1}^\infty$ and $x$ are both in $\prec$, each subsequence (rearrangement) $y$ of $x$ is also in $\prec$; hence, $Ay \in \prec$. But if $z$ is defined by $z_q = (-1)^q/(\log (q + 1))$ for each $q$, then $z \in cs$ and $(a_{nq}z_q)_{q=1}^\infty \in cs$; thus, $Az \notin \prec$.

REFERENCES


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