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**ANALYTIC MAPS OF THE OPEN UNIT DISK ONTO A  
GLEASON PART**

KAZUO KISHI

## ANALYTIC MAPS OF THE OPEN UNIT DISK ONTO A GLEASON PART

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**The purpose of this paper is to show that in certain uniform algebras all analytic maps (for the definition see §2) of the open unit disk onto a nontrivial Gleason part are mutually closely related (Theorem 2), and that these maps are isometries of the open unit disk with pseudo-hyperbolic metric onto a nontrivial Gleason part with part metric (Theorem 3).**

The results of this paper are contained in §2. Some necessary preliminaries are given in §1.

1. Preliminaries. A *uniform algebra*  $A$  on a compact Hausdorff space  $X$  is a uniformly closed subalgebra of the algebra  $C(X)$  of complex valued continuous functions on  $X$  which contains the constants and separates the points of  $X$ . Let  $\mathcal{M}(A)$  denote the maximal ideal space of  $A$  which has the Gelfand topology. Let  $\hat{f}$  be the Gelfand transform of  $f$  in  $A$  and let  $\hat{A} = \{\hat{f}: f \in A\}$ .

For  $\varphi$  and  $\theta$  in  $\mathcal{M}(A)$  we define

$$(1.1) \quad G: G(\varphi, \theta) = \sup \{|\varphi(f) - \theta(f)|: f \in A, \|f\| \leq 1\},$$

$$(1.2) \quad \sigma: \sigma(\varphi, \theta) = \sup \{|\varphi(f)|: f \in A, \|f\| \leq 1, \theta(f) = 0\},$$

where  $\|f\| = \sup \{|f(x)|: x \in X\}$ , and we write  $\varphi \sim \theta$  when  $G(\varphi, \theta) < 2$  (or, equivalently,  $\sigma(\varphi, \theta) < 1$ ). Then  $\sim$  is an equivalence relation in  $\mathcal{M}(A)$ , and an equivalence class  $P(m) = \{\varphi: \varphi \in \mathcal{M}(A), \varphi \sim m\} (\cong \{m\})$  is called the (*nontrivial*) *Gleason part* of  $m \in \mathcal{M}(A)$  (cf. Gleason [2]).  $G(\varphi, \theta)$  and  $\sigma(\varphi, \theta)$  are metrics on  $P(m)$  (cf. König [5]).

If  $m \in \mathcal{M}(A)$  has a unique representing measure  $\mu_m$ , i.e., if  $m$  has a unique probability measure  $\mu_m$  on  $X$  such that  $m(f) = \int f d\mu_m$  for every  $f \in A$ , then every  $\varphi$  in  $P(m)$  also has a unique representing measure  $\mu_\varphi$ . It is also known that  $\varphi$  in  $M_A$  belongs to  $P(m)$  if and only if there exist mutually absolutely continuous representing measures  $\mu_\varphi, \mu_m$  for  $\varphi, m$  respectively; indeed, there exists a constant  $c$  ( $0 < c < 1$ ) such that  $c\mu_\varphi \leq \mu_m$  and  $c\mu_m \leq \mu_\varphi$ .

For example, let  $A(D)$  denote the disk algebra of all continuous functions on the closed unit disk  $\bar{D} = \{s: |s| \leq 1\}$  in the plane which are analytic in the open unit disk  $D$ . Then  $\mathcal{M}(A(D))$  can be identified with  $\bar{D}$ , and the open unit disk  $D$  is one part. For  $t, s \in D$ , we see that

$$(1.3) \quad G(t, s) = \sup \{ |f(t) - f(s)| : f \in A(D), \|f\| \leq 1 \},$$

$$(1.4) \quad \sigma(t, s) = \sup \{ |f(t)| : f \in A(D), \|f\| \leq 1, f(s) = 0 \} \\ = \left| \frac{t - s}{1 - \bar{s}t} \right| \quad (\text{pseudo-hyperbolic metric}).$$

Throughout the rest of this paper, we do not distinguish in notations  $\varphi \in \mathcal{M}(A)$  from its representing measure when  $\varphi$  has a unique representing measure, and we suppose that  $m \in \mathcal{M}(A)$  has a unique representing measure  $m$ . Let  $A_m = \{f \in A; m(f) = 0\}$ , the kernel of a complex homomorphism  $m$ . Let  $H^\infty(m), H_m^\infty$  be the weak-star closures of  $A, A_m$  in  $L^\infty(m)$  respectively, and for  $1 \leq p < \infty$  let  $H^p(m), H_m^p$  be the closures of  $A, A_m$  in  $L^p(m)$  norm respectively. If  $\varphi$  belongs to  $P(m) (\cong \{m\})$ , then for  $1 \leq p < \infty$  the spaces  $H^p(m)$  and  $H^p(\varphi)$  are identical as sets of (equivalence classes of) functions; as Banach spaces, they have distinct but equivalent norms. Under the same hypothesis, the Banach algebras  $H^\infty(m)$  and  $H^\infty(\varphi)$  are identical.

For a Dirichlet algebra Wermer [7] showed the following theorem, and Hoffman [3] generalized Wermer's result to a logmodular algebra (cf. Browder [1], Chap. IV). Functions in  $H^\infty(m)$  of unit modulus are called *inner functions*.

**THEOREM 1 (WERMER'S EMBEDDING THEOREM).** *Let  $A$  be a uniform algebra on a compact space  $X$ . Suppose that  $m \in \mathcal{M}(A)$  has a unique representing measure  $m$  on  $X$ , and that the part  $P$  of  $m$  consists of more than one point. Then we have the following.*

(1) *There is an inner function  $Z$  (= Wermer's embedding function) such that  $ZH^2(m) = H_m^2$ .*

(2) *If  $\varphi \in P$ , define  $\hat{Z}(\varphi) = \int Z d\varphi$ . Then  $\hat{Z}$  is a one-one map of the part  $P$  onto the open unit disk  $D$  in the plane. The inverse map  $\tau$  of  $\hat{Z}$  is a one-one continuous map of  $D$  onto  $P$  (with the Gelfand topology).*

(3) *For every  $f$  in  $A$  the composed function  $\hat{f} \circ \tau$  is analytic on  $D$ .*

Let  $\varphi$  be an element of the Gleason part  $P(m)$  of  $m$  in  $\mathcal{M}(A)$ . Then there is a function  $h$  in  $L^\infty(m)$  such that  $\varphi(f) = \int f d\varphi = \int fh dm$  for all  $f \in A$ , so  $\varphi$  has a unique extension to a linear functional  $\tilde{\varphi}$  on  $H^\infty(m)$  which is both multiplicative and weak-star continuous. For any  $f \in H^\infty(m)$   $\tilde{\varphi}$  has the form

$$\tilde{\varphi}(f) = \int f d\varphi = \int fh dm.$$

We call  $\tilde{\varphi}$  the *measure extension* of  $\varphi$  in  $P(m)$ .

**PROPOSITION.** *Let  $A, m, P(m)$  and  $Z$  be as in Wermer's embedding theorem. Let  $\mathcal{P} = \mathcal{P}(m)$  be the set of measure extension  $\tilde{\varphi}$  of  $\varphi$  in  $P(m)$ . Then we have the following.*

(1)  $\mathcal{P}$  is the nontrivial Gleason part of  $\tilde{m}$  in  $\mathcal{M}(H^\infty(m))$ .

(2) The map  $\hat{Z}|_{\mathcal{P}}$  is a one-one continuous map of the part  $\mathcal{P}$  (with the Gelfand topology) onto the open unit disk  $D$ , and thus the inverse map  $\tilde{\tau}$  of  $\hat{Z}|_{\mathcal{P}}$  is a homeomorphism of  $D$  onto  $\mathcal{P}$ .

*Proof.* If the Gelfand transform  $\hat{H}^\infty(m) = \hat{H}^\infty$  of  $H^\infty(m)$  is restricted to the maximal ideal space  $Y$  of  $L^\infty(m)$ , then  $\hat{H}^\infty$  is a logmodular algebra on  $Y$  (see Hoffman [3], Theorem 6.4, corollary), and therefore every complex homomorphism  $\varphi$  of  $H^\infty(m)$  has a unique representing measure on  $Y$  (see [3], Theorem 4.2). In particular,  $\tilde{m} \in \mathcal{M}(H^\infty(m))$  has a unique representing (normal) measure  $\tilde{m}$  on the hyperstonean space  $Y$ . Then for every  $f$  in  $L^\infty(m)$  we have

$$\int_x f dm = \int_Y \hat{f} d\tilde{m} .$$

And we can identify  $L^\infty(\tilde{m})$  with  $C(Y) = \hat{L}^\infty(m)$  (cf. Srinivasan-Wang [6], pp. 221-223).

Now let  $\Sigma$  be the Gleason part of  $\tilde{m}$ . For  $\tilde{\varphi}$  in  $\mathcal{P}$ , we have

$$(1.5) \quad \tilde{\varphi}(f) = \int_x f d\varphi = \int_x f h dm = \int_Y \hat{f} \hat{h} d\tilde{m} \quad (f \in H^\infty(m)) ,$$

where  $h$  is a function in  $L^\infty(m)$  with  $a < h < b$  for some positive constants  $a$  and  $b$ . From this we see that  $\tilde{\varphi}$  is in  $\Sigma$ .

Conversely if  $\lambda$  is an element of  $\Sigma$ , then  $\lambda$  has a unique representing measure  $\tilde{\lambda}$  on  $Y$ , and we have for every  $f \in H^\infty(m)$

$$\lambda(f) = \int_Y \hat{f} d\tilde{\lambda} = \int_Y \hat{f} \frac{d\tilde{\lambda}}{d\tilde{m}} d\tilde{m} .$$

Since  $d\tilde{\lambda}/d\tilde{m}$  is an element of  $L^\infty(\tilde{m})$ , there is a function  $h$  in  $L^\infty(m)$  such that  $d\tilde{\lambda}/d\tilde{m} = \hat{h}$  a.e. ( $d\tilde{m}$ ). Hence we have

$$\lambda(f) = \int_Y \hat{f} \hat{h} d\tilde{m} = \int_x f h dm$$

and thus  $\lambda \in \mathcal{P}$ . So we get  $\mathcal{P} = \Sigma$ . Then the rest part (2) of the proposition follows from Theorem 1.

## 2. Results.

**DEFINITION.** Let  $P(m)$  be the nontrivial Gleason part of  $m$  in

the maximal ideal space  $\mathcal{M}(A)$  of a uniform algebra  $A$ . A one-one continuous map  $\rho(t)$  of the open unit disk  $D$  onto  $P(m)$  (with the Gelfand topology) is called an *analytic map* if the composition  $\hat{f}(\rho(t))$  is analytic on  $D$ , for every  $f$  in  $A$ .

Now we are in a position to prove the following theorem.

**THEOREM 2.** *Let  $A$  be a uniform algebra on a compact space  $X$ . Suppose that  $m \in \mathcal{M}(A)$  has a unique representing measure  $m$  on  $X$ , and that the part  $P$  of  $m$  consists of more than one point. Let  $\tau(t)$  be an analytic map of the open unit disk  $D$  onto  $P$  which is obtained in Theorem 1. If  $\rho(t)$  is an analytic map of  $D$  onto  $P$  such that  $\rho(\alpha) = m$ , then we have*

$$(2.1) \quad \rho(t) = \tau\left(\beta \frac{t - \alpha}{1 - \bar{\alpha}t}\right),$$

where  $\beta$  is a constant of modulus 1. Furthermore,  $\tau(t)$  is a homeomorphism if and only if  $\rho(t)$  is a homeomorphism.

*Proof.* Let  $\mathcal{S}$ ,  $Z$ , and  $\tilde{\tau}$  be as in Theorem 1 and proposition. For any  $t \in D$ ,  $\rho(t)$  has a unique representing measure  $h_t dm$ , where  $h_t$  is an element of  $L^\infty(m)$ . Let  $\tilde{\rho}(t)$  be the measure extension of  $\rho(t)$  i.e.,  $\tilde{\rho}(t)(f) = \int f h_t dm$  for all  $f \in H^\infty(m)$ . For each  $f \in H^\infty(m)$  there exists a sequence  $\{f_n\}$  in  $A$  such that  $\|f_n\| \leq \|f\|$  for all  $n$  and  $f_n \rightarrow f$  a.e. ( $dm$ ) (Hoffman-Wermer theorem, see [1], Theorem 4.2.5). Then, by Lebesgue's dominant convergence theorem,  $\rho(t)(f_n) = \int f_n h_t dm \rightarrow \tilde{\rho}(t)(f)$  for every  $t$  in  $D$ . Since  $\rho(t)(f_n)$  ( $n = 1, 2, \dots$ ) are analytic in  $D$  and  $|\rho(t)(f_n)| \leq \|f_n\| \leq \|f\|$ , we see that, for every  $f$  in  $H^\infty(m)$ ,  $\tilde{\rho}(t)(f)$  is analytic in  $D$  (Vitali's theorem). Hence we see that  $\tilde{\rho}(t)$  is an analytic map of  $D$  onto  $\mathcal{S}$ . If we set  $g(t) = (\tilde{\tau}^{-1} \circ \tilde{\rho})(t) = \hat{Z}(\tilde{\rho}(t))$ , then  $g(t)$  is a one-one holomorphic map of  $D$  onto  $D$ , and  $g(\alpha) = 0$ . Hence we see

$$g(t) = \beta \frac{t - \alpha}{1 - \bar{\alpha}t},$$

where  $\beta$  is a constant of modulus 1, and thus we have

$$\tilde{\tau}\left(\beta \frac{t - \alpha}{1 - \bar{\alpha}t}\right) = \tilde{\rho}(t).$$

Since  $\tilde{\tau}(t)|_A = \tau(t)$  and  $\tilde{\rho}(t)|_A = \rho(t)$  we have

$$\tau\left(\beta \frac{t - \alpha}{1 - \bar{\alpha}t}\right) = \rho(t).$$

Next we prove that  $\tau(t)$  is a homeomorphism if and only if  $\rho(t) = \tau(\beta(t - \alpha)/(1 - \bar{\alpha}t))$  is a homeomorphism. We put  $L_\alpha(t) = (t + \alpha)/(1 + \bar{\alpha}t)$  and  $\beta = e^{i\theta}$ . Then  $\tau(t)$  is a homeomorphism of  $D$  onto  $P$  if and only if  $\hat{Z}(\varphi) (= \int Z d\varphi)$  is a continuous map of  $P$  onto  $D$  if and only if  $L_\alpha \circ e^{-i\theta} \circ \hat{Z}$  is a continuous map of  $P$  onto  $D$  if and only if

$$(L_\alpha \circ e^{-i\theta} \circ \hat{Z})^{-1}(t) = \tau(e^{i\theta} L_{-\alpha}(t)) = \tau\left( e^{i\theta} \frac{t - \alpha}{1 - \bar{\alpha}t} \right) = \rho(t)$$

is a homeomorphism, and the theorem is proved.

Next we shall prove the following theorem which generalizes a formula (6.12) in Hoffman [4], p. 105.

**THEOREM 3.** *Let  $A$  be a uniform algebra on  $X$ . Suppose that  $m \in \mathcal{M}(A)$  has a unique representing measure  $m$  on  $X$ , and that the part  $P$  of  $m$  consists of more than one point. If  $\rho(t)$  is an analytic map of the open unit disk  $D$  onto  $P(m)$ , then we have*

$$\begin{aligned} \sigma(\rho(t), \rho(s)) &= \sigma(t, s), \\ G(\rho(t), \rho(s)) &= G(t, s). \end{aligned}$$

For the definitions of  $\sigma, G$  see (1.1) ~ (1.4).

*Proof.* Let  $Z, \mathcal{P}, \tau$  and  $\tilde{\tau}$  be as Theorem 1 and proposition. Let  $\tilde{\tau}(t) = \tilde{\varphi}, \tilde{\tau}(s) = \tilde{\theta}, \tau(t) = \varphi$  and  $\tau(s) = \theta$ . From Lemma 4.4.4 in Browder [1], we see that

$$f \in H_{\tilde{\theta}}^\infty = \left\{ f : f \in H^\infty(m) = H^\infty(\theta), \tilde{\theta}(f) = \int f d\theta = 0 \right\}$$

if and only if  $f \in (Z - s)H^\infty(m)$ , and from this we easily get  $H_{\tilde{\theta}}^\infty = \{(Z - s)/(1 - \bar{s}Z)\}H^\infty(m)$ . So we have

$$\begin{aligned} \sigma(\tilde{\varphi}, \tilde{\theta}) &= \sup \{ |\tilde{\varphi}(f)| : f \in H^\infty(m), \|f\| \leq 1, \tilde{\theta}(f) = 0 \} \\ &= \sup \left\{ |\tilde{\varphi}(f)| : f \in \frac{Z - s}{1 - \bar{s}Z} H^\infty(m), \|f\| \leq 1 \right\} \\ &= \sup \left\{ \left| \tilde{\varphi}\left(\frac{Z - s}{1 - \bar{s}Z}\right) \tilde{\varphi}(g) \right| : g \in H^\infty(m), \|g\| \leq 1 \right\} \\ &= \left| \frac{t - s}{1 - \bar{s}t} \right| = \sigma(t, s). \end{aligned}$$

Since the closures of  $A_\theta$  and  $H_{\tilde{\theta}}^\infty$  in  $L^2(m)$  are the same  $H_\theta^2 = \left\{ f : f \in H^2(m) = H^2(\theta), \int f d\theta = 0 \right\}$ , we have the following equalities from the result which is stated as “the perhaps surprising equality” in

Browder [1], p. 134. (Note that  $\tilde{\varphi}(f) = \int_Y \hat{f} \hat{h} d\tilde{m} = \int_Y \hat{f} d\tilde{\varphi}$  (see (1.5)) and  $\int_X |f|^2 d\varphi = \int_Y |\hat{f}|^2 d\tilde{\varphi}$ , for any  $f \in H_\theta^\infty$ .)

$$\begin{aligned} \sigma(\varphi, \theta) &= \sup \{ |\varphi(f)| : f \in A_\theta, \|f\| \leq 1 \} \\ &= \sup \left\{ |\varphi(f)| : f \in A_\theta, \int |f|^2 d\varphi \leq 1 \right\} \\ &= \sup \left\{ |\varphi(f)| : f \in H_\theta^2, \int |f|^2 d\varphi \leq 1 \right\} \\ &= \sup \left\{ |\tilde{\varphi}(f)| : f \in H_\theta^\infty, \int |f|^2 d\varphi \leq 1 \right\} \\ &= \sup \{ |\tilde{\varphi}(f)| : f \in H_\theta^\infty, \|f\| \leq 1 \} \\ &= \sigma(\tilde{\varphi}, \tilde{\theta}) . \end{aligned}$$

Hence we have

$$\sigma(\tau(t), \tau(s)) = \sigma(\tilde{\tau}(t), \tilde{\tau}(s)) = \sigma(t, s) .$$

If  $\rho(t)$  is an analytic map of  $D$  onto  $P(m)$ , then by Theorem 2 we have  $\rho(t) = \tau(\beta(t - \alpha)/(1 - \bar{\alpha}t))$ , where  $\beta$  is a constant of modulus 1. Therefore we have

$$\sigma(\rho(t), \rho(s)) = \sigma\left(\beta \frac{t - \alpha}{1 - \bar{\alpha}t}, \beta \frac{s - \alpha}{1 - \bar{\alpha}s}\right) = \sigma(t, s) .$$

The following equality is proved by König [5], which holds for  $\varphi, \theta$  in the maximal ideal space  $\mathcal{M}(A)$  of any uniform algebra  $A$ .

$$2 \log \frac{2 + G(\varphi, \theta)}{2 - G(\varphi, \theta)} = \log \frac{1 + \sigma(\varphi, \theta)}{1 - \sigma(\varphi, \theta)} .$$

Using this we get

$$G(\rho(t), \rho(s)) = G(t, s) .$$

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