NON-HAUSDORFF MULTIFUNCTION GENERALIZATION OF
THE KELLEY-MORSE ASCOLI THEOREM

GEOFFREY FOX AND PEDRO MORALES
The paper generalizes the Kelley-Morse theorem to continuous point-compact multifunction context. The generalization, which is non-Hausdorff, contains the Ascoli theorem for continuous functions on a $k_3$-space by the authors and the known multifunction Ascoli theorems of Mancuso and of Smithson.

1. Introduction. The Kelley-Morse theorem [3, p. 236] is central among the topological Ascoli theorems for continuous functions on a $k$-space. It generalizes to the $k_3$-space theorem of [1], which contains all known Ascoli theorems for $k$-spaces or $k_3$-spaces.

Obviously a multifunction generalization depends on a multifunction extension of "even continuity". One such extension is that of Lin and Rose [5], but this was not applied in Kelley-Morse context. Another which was so applied [7, p. 24] is two-fold and leads to a two-fold multifunction Kelley-Morse theorem which, however, does not contain the Mancuso theorem [6, p. 470], nor the Smithson theorem [9, p. 259]. This paper gives a natural multifunction extension of the definition and leads to a multifunction theorem containing all the above-mentioned theorems.

2. Tychonoff sets. Let $X$ and $Y$ be nonempty sets. A multifunction is a point to set correspondence $f: X \to Y$ such that, for all $x \in X$, $fx$ is a nonempty subset of $Y$. For $A \subseteq X$, $B \subseteq Y$ it is customary to write $f(A) = \bigcup_{x \in A} fx$, $f^*(B) = \{x: x \in X$ and $fx \cap B \neq \emptyset\}$ and $f^+(B) = \{x: x \in X$ and $fx \subseteq B\}$. If $Y$ is a topological space, a multifunction $f: X \to Y$ is point-compact if $fx$ is compact for all $x \in X$.

Let $\{Y_x\}_{x \in X}$ be a family of nonempty sets. The $m$-product $P\{Y_x: x \in X\}$ of the $Y_x$ is the set of all multifunctions $f: X \to \bigcup_{x \in X} Y_x$ such that $fx \subseteq Y_x$ for all $x \in X$. In the case $Y_x = Y$ for all $x \in X$, the $m$-product of the $Y_x$, denoted $Y^{mx}$, is the set of all multifunctions on $X$ to $Y$. In particular, if $Y$ is a topological space, the symbol $(Y^{mx})$, will denote the set of all point-compact members of $Y^{mx}$. For $x \in X$, the $x$-projection $pr_x: P\{Y_x: x \in X\} \to Y_x$ is the multifunction defined by $pr_x f = fx$. If the $Y_x$ are topological spaces, the pointwise topology $\tau_p$ on $P\{Y_x: x \in X\}$ is defined to be the topology having as open subbase the sets of the forms $pr_i(U_i)$, $pr_i(U_i)$, where $U_i$ is open in $Y_x$, $x \in X$.

For $F \subseteq Y^{mx}$, $x \in X$, we write $F[x] = \bigcup_{f \in F} fx$. Let $Y$ be a topological space. A subset $F$ of $Y^{mx}$ is pointwise bounded if $F[x]$ has compact...
A subset $T$ of $Y^{mx}$ is Tychonoff if, for every pointwise bounded subset $F$ of $T$, $T \cap P\{F[x] : x \in X\}$ is $\tau_p$-compact. The following sets are Tychonoff:

1. $Y^X$, by the classical Tychonoff theorem.
2. $Y^{mx}$, by the theorem of Lin [4, p. 400].
3. The set of all point-closed members of $Y^{mx}$, by Corollary 7.5 of [7, p. 17].
4. $(Y^{mx})_b$, by Corollary 7.6 of [7, p. 17].

**Lemma 2.1.** If $F$ is a pointwise bounded subset of a Tychonoff set $T$, then the $\tau_p$-closure of $F$ in $T$ is compact.

**Proof.** Let $\bar{F}$ denote the $\tau_p$-closure of $F$ in $T$. Since $T \cap P\{F[x] : x \in X\}$ is a $\tau_p$-compact subset of $T$, it suffices to show that $\bar{F} \subseteq P\{F[x] : x \in X\}$. But this follows from Lemma 7.7 of [7, p. 17].

**3. Even continuity.** Let $X$ and $Y$ be topological spaces. A multifunction $f : X \to Y$ is lower semi-continuous (upper semi-continuous) if $f^-(U)(f^+(U))$ is open in $X$ whenever $U$ is open in $Y$. If $f$ is both lower semi-continuous and upper semi-continuous it is called continuous. Henceforth, the set of all continuous multifunctions on $X$ to $Y$ will be denoted $\mathcal{C}(X, Y)$. The multifunction $(f, x) \mapsto fx$ on $Y^{mx} \times X$ to $Y$, or any restriction, will be denoted by the symbol $\omega$. Let $F \subseteq Y^{mx}$. A topology $\tau$ on $F$ is said to be jointly continuous if $\omega : (F, \tau) \times X \to Y$ is continuous.

A subset $F$ of $Y^{mx}$ is evenly continuous if, whenever $x \in X$, $K$ is a compact subset of $Y$ and $V$ is a neighborhood of $K$, there exist neighborhoods $U$, $W$ of $x$, respectively, such that

(a) $f \in F$ and $fx \cap W \neq \emptyset$ imply $U \subseteq f^-(V)$, and

(b) $f \in F$ and $fx \subseteq W$ imply $U \subseteq f^+(V)$.

This extends the original Kelley-Morse definition [3, p. 235] by the substitution of compact subsets of $Y$ for points of $Y$. It is easily verified that every member of an evenly continuous subset of $Y^{mx}$ is lower semi-continuous. Moreover, every member of an evenly continuous subset of $(Y^{mx})_b$ is also upper semi-continuous, hence continuous.

**Lemma 3.1.** Let $Y$ be a regular space. If $F$ is an evenly continuous subset of $Y^{mx}$, then the $\tau_p$-closure of $F$ in $Y^{mx}$ is evenly continuous.

**Proof.** Let $\bar{F}$ denote the $\tau_p$-closure of $F$ in $Y^{mx}$. Let $x \in X$, let $K$ be a compact subset of $Y$ and let $V$ be a closed neighborhood of $K$. There exist open neighborhoods $U$, $W$ of $x$, $K$, respectively, such that, for all $f \in F$, $fx \cap W \neq \emptyset$ implies $U \subseteq f^-(V)$ and $fx \subseteq W$ implies $U \subseteq f^+(V)$. Let $g \in \bar{F}$ be such that $gx \cap W \neq \emptyset$. Let $\{g_\alpha\}$ be a net in $F$
which is $\tau_p$-convergent to $g$. Since $\{h: h \in Y^{mx} \text{ and } hx \cap W \neq \emptyset\}$ is a $\tau_p$-neighborhood of $g$, $g_x \cap W \neq \emptyset$ eventually, so $U \subseteq g^*_a(V)$ eventually. Suppose that $U \not\subseteq g^*(V)$. Then, for some $u \in U$, $gu \subseteq Y - V$, so $g_au \subseteq Y - V$ eventually, which is a contradiction.

Now let $g \in F$ be such that $gx \subseteq W$. Let $\{g_n\}$ be a net in $F$ which is $\tau_p$-convergent to $g$. Since $\{h: h \in Y^{mx} \text{ and } hx \cap W \neq \emptyset\}$ is a $\tau_p$-neighborhood of $g$, $g_{an} \subseteq W$ eventually, so $U \subseteq g^*_a(V)$ eventually. Suppose that $U \not\subseteq g^*(V)$. Then, for some $u \in U$, $gu \cap (Y - V) \neq \emptyset$, so $g_au \cap (Y - V) \neq \emptyset$ eventually, which is a contradiction.

**Lemma 3.2.** If $F$ is an evenly continuous subset of $(Y^{mx})_0$, then $\tau_p$ on $F$ is jointly continuous.

**Proof.** Let $\omega: (F, \tau_p) \times X \rightarrow Y$. Suppose that $(f, x) \in \omega^-(G)$, where $G$ is open in $Y$. Choose $y \in fx \cap G$. There are neighborhoods $U, W$ of $x, y$, respectively, such that $g \in F$ and $gx \cap W \neq \emptyset$ imply $U \subseteq g^*(G)$. Then $\{h: h \in F \text{ and } hx \cap W \neq \emptyset\} \times U$ is a neighborhood of $(f, x)$ which is contained in $\omega^-(G)$.

Now suppose that $(f, x) \in \omega^+(G)$, where $G$ is open in $Y$. There are neighborhoods $U, W$ of $x, fx$, respectively, such that $g \in F$ and $gx \subseteq W$ imply $U \subseteq g^*(G)$. Then $\{h: h \in F \text{ and } hx \subseteq W\} \times U$ is a neighborhood of $(f, x)$ which is contained in $\omega^+(G)$.

The following lemma generalizes an implicit lemma of Noble [8], stated explicitly as Lemma 1.4 in [7, p. 7]:

**Lemma 3.3.** Let $f \in \mathcal{C}(X \times Y, Z)$. If $X$ is compact and $Z$ is regular, then the set $F = \{f(x, \cdot): x \in X\}$ is evenly continuous.

**Proof.** Let $y \in Y$, let $K$ be a compact subset of $Z$ and let $V$ be an open neighborhood of $K$. Let $W$ be a closed neighborhood of $K$ which is contained in $V$. We construct a neighborhood $U$ of $y$ as follows: Since $f(\cdot, y)$ is continuous, $K_1 = f(\cdot, y)^-(W)$ and $K_2 = f(\cdot, y)^+(W)$ are closed in $X$, therefore compact. Thus the second projections $\text{pr}_2: K_1 \times Y \rightarrow Y$, $\text{pr}_2: K_2 \times Y \rightarrow Y$ are closed, so that

$$U_1 = Y - \text{pr}_2[(K_1 \times Y) - f^-(V)], \quad U_2 = Y - \text{pr}_2[(K_2 \times Y) - f^+(V)]$$

are open in $Y$. Because $K_1 \subseteq f(\cdot, y)^-(V)$, $K_2 \subseteq f(\cdot, y)^+(V)$, we have $K_1 \times \{y\} \subseteq f^-(V)$, $K_2 \times \{y\} \subseteq f^+(V)$. Hence $y \not\in \text{pr}_2[(K_1 \times Y) - f^-(V)]$, $y \not\in \text{pr}_2[(K_2 \times Y) - f^+(V)]$, that is, $y \in U_1 \cap U_2 = U$.

We show that the neighborhoods $U, W$ of $y, K$, respectively, satisfy the required implications: Let $g \in F$ be such that $gy \cap W \neq \emptyset$, so that $g = f(x, \cdot)$ for some $x \in K_1$. Let $u \in U$, so that $u \not\in \text{pr}_2[(K_1 \times Y) - f^-(V)]$. The proof then follows.
Then \((x, y) \in f^+(V)\), that is, \(g \cap V \neq \emptyset\). Now let \(g \in F\) be such that \(gy \subseteq W\), so that \(g = f(x, \cdot)\) for some \(x \in X\). Let \(u \in U\), so that \(u \notin \text{pr}_1((K \times Y) - f^+(V))\). Then \((x, u) \in f^+(V)\), that is, \(gu \subseteq V\).

Let \(X\) and \(Y\) be topological spaces. The **compact open topology** \(\tau_c\) on \(Y^mX\) is defined to be the topology having as open subbase the sets of the forms \(\{f : f(K) \subseteq U\}\), \(\{f : fx \cap U \neq \emptyset\text{ for all }x \in K\}\), where \(K\) is a compact subset of \(X\) and \(U\) is open in \(Y\). Obviously, \(\tau_c\) is larger than \(\tau_p\).

A subset \(F\) of \(Y^mX\) **satisfies the condition** \((G)\) if, for every \(\tau_c\)-closed subset \(F_0\) of \(F\), \(\bigcap_{f \in F_0} f^{-}(U)\) and \(\bigcap_{f \in F_0} f^+(U)\) are open in \(X\) whenever \(U\) is open in \(Y\). The following two lemmas relate this condition to even continuity:

**Lemma 3.4.** If \(Y\) is regular, then every subset of \(Y^mX\) satisfying the condition \((G)\) is evenly continuous.

**Proof.** Let \(F\) be a subset of \(Y^mX\) which satisfies the condition \((G)\). Let \(x \in X\), let \(K\) be a compact subset of \(Y\) and let \(V\) be an open neighborhood of \(K\). Let \(W\) be an open neighborhood of \(K\) such that \(K \subseteq W \subseteq \overline{W} \subseteq V\). Since \(F_1 = \{h : h \in F\text{ and }hx \cap \overline{W} \neq \emptyset\}\), \(F_2 = \{h : h \in F\text{ and }hx \subseteq \overline{W}\}\) are \(\tau_c\)-closed in \(F\), \(U_1 = \bigcap_{h \in F_1} h^{-}(V)\) and \(U_2 = \bigcap_{h \in F_2} h^+(V)\) are open in \(X\). Then \(U = U_1 \cap U_2\) is an open neighborhood of \(x\).

Let \(f \in F\) be such that \(fx \cap W \neq \emptyset\). Then \(f \in F_1\), so that \(U \subseteq U_1 \subseteq f^{-}(V)\). Now let \(f \in F\) be such that \(fx \subseteq W\). Then \(f \in F_2\), so that \(U \subseteq U_2 \subseteq f^+(V)\).

**Lemma 3.5.** Every \(\tau_c\)-compact evenly continuous subset of \((Y^mX)_0\) satisfies the condition \((G)\).

**Proof.** Let \(F\) be a \(\tau_c\)-compact evenly continuous subset of \((Y^mX)_0\). Since \(F\) is \(\tau_p\)-compact, it suffices, by Corollary 10.6 of [7, p. 23], to show that \(\tau_p\) on \(F\) is jointly continuous. For this we apply Lemma 3.2.

Let \(X\) be a topological space and let \(Y = (Y, \mathcal{U})\) be a uniform space. A subset \(F\) of \(Y^mX\) is **equicontinuous** if, for \((x, U) \in X \times \mathcal{U}\), there exists a neighborhood \(V\) of \(x\) such that, for all \(f \in F\), \(f(V) \subseteq U[f_x]\) and \(fz \cap U[y] \neq \emptyset\) whenever \((z, y) \in V \times fx\). The following two lemmas relate equicontinuity to even continuity:

**Lemma 3.6.** If \(Y = (Y, \mathcal{U})\) is a uniform space, then every equicontinuous subset of \(Y^mX\) is evenly continuous.

**Proof.** Let \(F\) be an equicontinuous subset of \(Y^mX\). Let \(x \in X\), let \(K\) be a compact subset of \(Y\) and let \(U\) be a symmetric member of \(\mathcal{U}\). There is a neighborhood \(V\) of \(x\) such that, for all \(f \in F\), \(f(V) \subseteq f_x \cap W \neq \emptyset\) whenever \((z, y) \in V \times fx\).
LEMMA 3.7. If $Y$ is a uniform space, then every evenly continuous pointwise bounded subset of $(Y^m)^0_0$ is equicontinuous.

Proof. Let $F$ be an evenly continuous pointwise bounded subset of $(Y^m)^0_0$. Let $\bar{F}$ denote the $\tau_p$-closure of $F$ in $(Y^m)^0_0$. By the Lemmas 3.1, 3.2, $\tau_p$ on $\bar{F}$ is jointly continuous. Since $(Y^m)^0_0$ is a Tychonoff set, by Lemma 2.1, $\bar{F}$ is $\tau_p$-compact. Then, by the Lemma 8 of Smithson [9, p. 258], $\bar{F}$ is equicontinuous.

4. Ascoli theorem. Let $X = (X, \tau)$ be a topological space. The $k$-extension of $\tau$ is the family $k(\tau)$ of all subsets $U$ of $X$ such that $U \cap K$ is open in $K$ for every compact subset $K$ of $X$. It is clear that $k(\tau)$ is a topology on $X$ which is larger than $\tau$. The topological space $kX = (X, k(\tau))$ is called the $k$-extension of $X$. A topological space $X$ is called a $k$-space if $kX = X$. For an arbitrary topological space $X$, $kkX = kX$, so $kX$ is a $k$-space. Familiar examples of $k$-spaces are the locally compact spaces and the spaces satisfying the first countability axiom.

Let $X$ and $Y$ be topological spaces. A function $f: X \to Y$ is called $k$-continuous if its restriction to each compact subset of $X$ is continuous. Henceforth, the set of all continuous ($k$-continuous) functions on $X$ to $Y$ will be denoted $C(X, Y)$ ($C_k(X, Y)$). It can be shown that a topological space $X$ is a $k$-space if and only if $C_k(X, Y) = C(X, Y)$ for every topological space $Y$ [7, p. 9]. A topological space $X$ is a $k_3$-space if $C_k(X, Y) = C(X, Y)$ for every regular space $Y$. Thus a $k$-space is a $k_3$-space but not conversely. In fact, the product of uncountably many copies of the real line, which is not a $k$-space, is a $k_3$-space. We write $C_0(X, Y) = (Y^m)^0_0 \cap C(X, Y)$.

We note that if $Y$ is regular, then $(C_0(X, Y), \tau_c)$ is a regular space for every topological space $X$.

In a regular space there was introduced in [7, p. 11] the following equivalence relation $R$: $xRy$ if every open neighborhood of $x$ contains $y$. For a subset $F$ of such a space, $F^*$ denotes its $R$-saturation, that is, the smallest $R$-saturated set containing $F$.

THEOREM 4.1. Let $X$, $Y$ be topological spaces, let $T$ be a Tychonoff subset of $(Y^m)^0_0$ and let $F \subseteq (T \cap C(X, Y), \tau_c)$. If $Y$ is regular, the following conditions are sufficient for the compactness of $F$: $U[fx]$ and $fx \subseteq U[fz]$ for all $z \in V$. Write $W = U[K]$. Let $f \in F$ be such that $fx \cap W \neq \emptyset$. If $z \in V$ then $fx \subseteq U[fz]$, so that $U[fz] \cap W \neq \emptyset$, therefore $V \subseteq f(U^2[K])$. Now let $f \in F$ be such that $fx \subseteq W$. Then $f(V) \subseteq U[fx] \subseteq U^2[K]$, that is, $V \subseteq f(U^2[K])$. 

(a) $F^*$ is closed in $T \cap \mathcal{C}(X, Y)$.
(b) $F$ is pointwise bounded, and
(c) $F$ is evenly continuous.

If $X$ is a $k$-space and $Y$ is regular, then the conditions (a), (b) and (c) are necessary for the compactness of $F$.

**Proof.** Sufficiency. Let $\bar{F}$ denote the $\tau_p$-closure of $F$ in $T$. Since $T \subseteq (Y^{=X})_p$, (c) implies, by Lemmas 3.1 and 3.2, that $\omega: (\bar{F}, \tau_p) \times X \rightarrow Y$ is continuous, and, in particular, that $\bar{F} \subseteq \mathcal{C}(X, Y)$. By Lemma 8.1 of [7, p. 18], $\omega: (\bar{F}, \tau_p) \rightarrow (\mathcal{C}(X, Y), \tau_c)$ is continuous. Since $T$ is a Tychonoff set, (b) implies, by Lemma 2.1, that $\bar{F}$ is $\tau_p$-compact, so $\bar{\omega}(\bar{F}) = \bar{F}$ is a $\tau_c$-compact subset of $T \cap \mathcal{C}(X, Y)$. But (a) implies $F \subseteq \bar{F} \subseteq F^*$, so, by Theorem 4.1 (b) of [7, p. 11], $F$ is $\tau_c$-compact.

**Necessity.** By Theorem 4.1 (c) of [7, p. 11], $F^*$ is closed in $(T \cap \mathcal{C}(X, Y), \tau_c)$. It is clear that $F$ is pointwise bounded. Since $X$ is a $k$-space, by Theorem 9.4 of [7, p. 21], $\omega: (F, \tau_c) \times X \rightarrow Y$ is continuous. So by Lemma 3.3, $F = \{\omega(f, \cdot): f \in F\}$ is evenly continuous.

**Corollary 1.** Let $F \subseteq (\mathcal{C}_0(X, Y), \tau_c)$. If $Y$ is regular, the following conditions are sufficient for the compactness of $F$:
(a) $F^*$ is closed in $\mathcal{C}_0(X, Y)$,
(b) $F$ is pointwise bounded, and
(c) $F$ is evenly continuous.

If $X$ is a $k$-space and $Y$ is regular, then the conditions (a), (b) and (c) are necessary for the compactness of $F$.

**Corollary 2.** Let $F \subseteq (C(X, Y), \tau_c)$. If $Y$ is regular, the following conditions are sufficient for the compactness of $F$:
(a) $F^*$ is closed in $C(X, Y)$,
(b) $F$ is pointwise bounded, and
(c) $F$ is evenly continuous.

If $X$ is a $k$-space and $Y$ is regular, then the conditions (a), (b) and (c) are necessary for the compactness of $F$.

**Corollary 3.** If $Y$ is regular, a subset $F$ of $(C_1(X, Y), \tau_c)$ is compact if and only if
(a) $F^*$ is closed in $C_1(X, Y)$,
(b) $F$ is pointwise bounded, and
(c) $F$ is evenly continuous on compacta.

**Proof.** For the sufficiency, we note that $C_1(X, Y) = C(kX, Y)$ and apply the Lemma 3.4 of [7, p. 11]. For the necessity, we consider $F$ as a subset of $(C(kX, Y), \tau_c)$ and deduce from Corollary 2 the conditions (a),
(b) and the even continuity of $F$. Then it is clear that $F$, considered as a subset of $(C_k(X, Y), \tau_c)$, is evenly continuous on compacta.

**Corollary 4.** ([1, p. 635]). Let $F \subseteq (C(X, Y), \tau_c)$. If $Y$ is regular, the following conditions are sufficient for the compactness of $F$:

(a) $F$ is closed in $C(X, Y)$,
(b) $F$ is pointwise bounded, and
(c) $F$ is evenly continuous.

If $X$ is a $k_\gamma$-space and $Y$ is regular, then the conditions (a), (b) and (c) are necessary for the compactness of $F$.

**Proof.** For the necessity, we note that, since $Y$ is regular, $C_k(X, Y) = C(X, Y)$; then we apply Corollary 3 and Lemma 3.4 of [7, p. 11].

**Remarks.** (1) By Lemmas 3.4, 3.5, the Corollary 1 is equivalent to the Theorem 10.10 of [7, pp. 23–24], which contains the Ascoli theorem of Gale [2, p. 304] and the multifunction Ascoli theorem of Mancuso [6, p. 470].

(2) Let $Y$ be a uniform space. By Lemmas 3.6, 3.7 and Theorem 12.2 of [7, p. 28], the Corollary 1 in this context is equivalent to the Theorem 12.8 of [7, p. 31], which contains the multifunction Ascoli theorem of Smithson [9, p. 259].

**References**


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**Université de Montréal**
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