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**ON THE CONSTRUCTION OF ONE-PARAMETER
SEMIGROUPS IN TOPOLOGICAL SEMIGROUPS**

JOHN YUAN

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Let S be a topological Hausdorff semigroup and $s \in S$ be a strongly root compact element. Then there are an algebraic morphism $f: Q_+ \cup \{0\} \rightarrow S$ with $f(0) = e$, $f(1) = s$, and a one-parameter semigroup $\phi: H \rightarrow S$ which satisfy the following properties: If $K = \cap \{f(]0, \varepsilon[_Q): 0 < \varepsilon < 1\}$, then K is a compact connected abelian subgroup of $\mathcal{H}(e)$, $\phi(0) = e$, $\phi(H)$ is in the centralizer $Z = \{x \in eSe: xk = kx \text{ for all } k \in K\}$ of K in eSe , and $\phi(t) \in f(t)K$ for each $t \in Q_+$. Furthermore, if \mathcal{U} is any neighborhood of s in S , then ϕ may be chosen so that $\phi(1) \in \mathcal{U}$: and, in fact, if K is arcwise connected, then ϕ may be chosen so that $\phi(1) = s$. The above statements also hold for strongly p th root compact elements almost everywhere.

1. Introduction. We are concerned with the question of when a divisible element in a topological semigroup can be embedded in a one-parameter semigroup which has many applications in Probability theory (cf. [4], [8]).

The first result about the existence of one-parameter semigroups in a compact semigroup which we call the One-Parameter Semigroup Theorem is due to Mostert and Shields [7], 1957. In 1960, an independent proof based on the local nature of the compact semigroup was given by Hoffmann (cf. [5], [6]). In 1970, a global proof was presented by Carruth and Lawson [1]. The first result of a generalized one-parameter semigroup theorem dealing with the embedding problems which we will call the Embedding and Density Theorem is indicated by Hofmann in [4] and later proved by Siebert [8]. Siebert's proof is based on the notion of a local semigroup called nucleus (cf. [6]). We will present in this paper a global proof of this theorem by applying the One-Parameter Semigroup Theorem.

Throughout this paper, we maintain that R_+ , Q_+ and Z_+ are the totalities of strictly positive real numbers, rational numbers and integers, respectively, $H = R_+ \cup \{0\}$ and $Q_+^p = \{n/p^m: n \in Z_+, m \in Z_+ \cup \{0\}\}$ for a prime p . For convenience, we will use $]a, b[_Q$ (resp. $]a, b[_Q$, etc.) and $]a, b[_{Q^p}$ (resp. $]a, b[_{Q^p}$) to denote $]a, b[\cap Q_+$ (resp. $]a, b[\cap Q_+$, etc.) and $]a, b[\cap Q_+^p$ (resp. $]a, b[\cap Q_+^p$) respectively. We also maintain that S is a topological (Hausdorff) semigroup and $\mathcal{H}(e)$ is the maximal group of units in the closed subsemigroup eSe for an idempotent $e \in S$.

2. On the existence of a one-parameter semigroup in $\overline{f(A)}$ where $f: A \rightarrow S$ is an algebraic morphism with $A = Q_+, Q_+^p$. Throughout this section, we will always assume that $f: Q_+$ (resp. Q_+^p) $\rightarrow S$ is an algebraic morphism so that $\overline{f([0, d]_Q)}$ (resp. $\overline{f([0, d]_{Q^p})}$) is compact for some $d > 0$ unless mentioned otherwise. As the discussions for Q_+ and for Q_+^p would be almost the same, we will concentrate on Q_+ only.

DEFINITION. For each $s \in S$ and each $n \geq 1$, let $W_n(s) = \{t \in S: t^n = s\}$, $W(n; s) = \{t^m: 1 \leq m \leq n, t^n = s\}$. s is said to be divisible (resp. p -divisible) if $W_n(s) \neq \emptyset$ (resp. $W_{p^n}(s) \neq \emptyset$) for all $n \geq 1$; root compact (resp. p th root compact) if $W_n(s)$ (resp. $W_{p^n}(s)$) is in addition compact for each $n \geq 1$; strongly root compact (resp. strongly p th root compact) if $W_\infty(s) = \cup \{W(n; s): n \geq 1\}$ (resp. $W_{p^\infty}(s) = \cup \{W(p^n; s): n \geq 1\}$) is in addition relatively compact.

PROPOSITION 2.1. Let s be a root compact (resp. p th root compact) element in S . Then there is an algebraic morphism $f: Q_+$ (resp. Q_+^p) $\rightarrow S$ so that $f(1) = s$. If s is strongly root compact (resp. strongly p th root compact), then f may be chosen so that $\overline{f([0, 1]_Q)}$ (resp. $\overline{f([0, 1]_{Q^p})}$) is compact.

Proof. For each $n \geq 1$ and $i \geq 0$, pick an $s_{n+i} \in W_{(n+i)!}(s)$ (resp. $s_{n+i} \in W_{p^{(n+i)}}(s)$) and let

$$a_n = (s_n^{n_1}, s_n^{n_1/2^1}, \dots, s_n, s_{n+1}, \dots)$$

(resp. $a_n = (s_n^{p^n}, s_n^{p^{n-1}}, \dots, s_n, s_{n+1}, \dots)$).

Then $\{a_n\}$ is a sequence in the compact set $\prod_{n \geq 1} W_{n!}(s)$ (resp. $\prod_{n \geq 1} W_{p^n}(s)$). Hence there is a convergent subnet $\{a_{n(k)}\}$ converging to $a = (t_1, t_2, \dots) \in \prod_{n \geq 1} W_{n!}(s)$ (resp. $\prod_{n \geq 1} W_{p^n}(s)$).

Then

$$t_{q+1}^{q+1} = (\lim s_n^{n(k)!/(q+1)!})^{q+1}$$

$$= \lim s_n^{n(k)!/q!} = t_q$$

(resp. $t_{q+1}^p = (\lim s_n^{p^n(k)-q})^p$)

$$= \lim s_n^{p^n(k)-q+1} = t_q$$

for all $q \geq 1$, and $t_1 = s$. If $n/m! = b/a!$ (resp. $n/p^m = b/p^a$), then

$$t_m^n = (t_m^{m!/a!})^b = t_b^a$$

(resp. $t_m^n = (t_m^{p^m-a})^b = t_b^a$).

Hence $f: Q_+$ (resp. Q_+^p) $\rightarrow S$ given by $f(n/m!) = t_m^n$ (resp. $f(n/p^m) = t_m^n$)

is well-defined. If $n/m!, b/a! \in Q_+$ (resp. $n/p^m, b/p^a \in Q_+^p$), assuming $a \geq m$, then

$$\begin{aligned}
 f(n/m! + b/a!) &= f\left(\frac{n(a!/m!) + b}{a!}\right) \\
 &= t_a^{n(a!/m!)} t_a^b = t_m^n t_a^b \\
 \text{resp. } f(n/p^m + b/p^a) &= f\left(\frac{np^{a-m} + b}{p^a}\right) \\
 &= t_a^{np^{a-m}} t_a^b = t_m^n t_a^b,
 \end{aligned}$$

whence f is an algebraic morphism so that $f(1) = s$. The rest is simple.

LEMMA 2.2. *for each $x > 0$, let $S(x) = \overline{f(]0, x[_\mathbb{Q})}$. Then*

(1) $S(x + y) = S(x)S(y)$ for all $x, y > 0$. In particular, $S(x)$ is compact for each $x > 0$

(2) $\overline{f(Q_+)}$ has the identity e so that $K = \cap \{S(x) : x \in Q_+\}$ is a divisible compact abelian subgroup of $\mathcal{H}(e)$. In particular, we may extend f to $Q_+ \cup \{0\}$ so that $f(0) = e$

(3) $\overline{Kf(]x, y[_\mathbb{Q})} = \overline{f(]x, y[_\mathbb{Q})}$ for all $x < y \in Q_+$.

Proof. Straightforward (cf. § 3, Chapter B, [6]).

LEMMA 2.3. *The following statements are equivalent:*

- (1) $K = \{f(0)\}$
- (2) f is continuous at 0
- (3) f is continuous.

Proof. (cf. 3.9, p. 102, [6].)

LEMMA 2.4. *If f is continuous, then there is a unique one-parameter semigroup ϕ so that $\phi|(Q_+ \cup \{0\}) = f$.*

Proof. Given a $d > 0$, there is a net $\{x_\alpha\}$ in $]0, d + 1[_\mathbb{Q}$ with $\lim x_\alpha = d$. Since $\{(f(x_\alpha))\}$ is a net in $S(d + 1)$, there is a convergent subnet $\{f(x_\beta)\}$. Define $F(d) = \lim f(x_\beta)$. It is straightforward to check that $F: H \rightarrow S$ is a well defined morphism so that $\cup \{F(]0, x[_\mathbb{Q}) : x > 0\} = \{f(0)\}$, whence F is continuous (cf. 3.9, p. 102, [6]).

LEMMA 2.5. *Let $\phi: H \rightarrow S$ be a nontrivial one-parameter semigroup. Then there is a $d \in]0, 1]$ so that $\phi|[0, d]$ is injective. Moreover, if $c > 0$, one may reparameterize ϕ so that $\phi|[0, c]$ is injective (cf. 3.9, p. 102, [6]).*

Since K acts on $\overline{f(Q_+)}$ and $\overline{f([x, y]_\rho)}$, one has the orbit spaces $\overline{f(Q_+)}/K$ and $\overline{f([x, y]_\rho)}/K$. We will use the same letter π to denote the orbit maps.

LEMMA 2.6. $\overline{f(Q_+)}/K$ is a topological monoid under the multiplication $xK \cdot yK = xyK$.

LEMMA 2.7. If $f(Q_+) \not\subset K$, then $\pi \circ f: Q_+ \cup \{0\} \rightarrow \overline{f(Q_+)}/K$ is non-trivial continuous morphism so that $\pi(\overline{f([x, y]_\rho)}) = \overline{f([x, y]_\rho)}/K$ for all $x < y \in Q_+ \cup \{0\}$.

Proof. The continuity of $\pi \circ f$ follows from 2.3. The rest follows from the closedness of π .

In the remainder of this section, we maintain that $f(1) \notin K$ and so $\pi \circ f$ extends to a unique one-parameter semigroup $g: \mathbf{H} \rightarrow \overline{f(Q_+)}/K$ that $g|_{[0, 2]}$ is injective by a suitable reparameterization of g or f , i.e. the following diagram commutes:

$$\begin{array}{ccc}]0, 2[_\rho & \xrightarrow{f} & S(2) \\ \downarrow & & \downarrow \pi \\ [0, 2] & \xrightarrow{g} & S(2)/K. \end{array}$$

Let $\rho = g^{-1} \circ \pi: S(2) \rightarrow [0, 2]$. Then ρ is a continuous map such that

$$\rho(f(r)) = (g^{-1} \circ \pi)(f(r)) = r \quad \text{for all } r \in [0, 2]_\rho$$

and that the following condition is satisfies:

$$\rho(xy) = \rho(x) + \rho(y) \quad \text{for all } x, y \in S(1).$$

LEMMA 2.8. The following statements hold:

- (1) $x \in Kf(r)$ iff $x \in \pi^{-1}(g(r))$ for each $r \in Q_+ \cup \{0\}$
- (2) $x \in S(2)$ iff there is a unique $t \in [0, 2]$ so that $x \in \pi^{-1}(g(t))$
- (3) $\pi^{-1}(g([x, y])) = Kf([x, y]_\rho) = \overline{f([x, y]_\rho)}$ for all $x, y \in Q_+ \cup \{0\}$
- (4) $S(1)Kf(1) \subset Kf([1, 2]_\rho)$
- (5) $S(1) \setminus Kf(1) = S(2) \setminus \overline{Kf([1, 2]_\rho)}$.

Proof. Straightforward.

Define a multiplication on the space X obtained from $S(1)$ by collapsing $Kf(1)$ to a point as follows:

$$m_X(x, y) = \begin{cases} xy & \text{if } x, y, xy \in S(1) \setminus Kf(1) \\ Kf(1) & \text{otherwise.} \end{cases}$$

Let $\pi': S(2) \rightarrow X$ be defined via

$$\begin{aligned} \pi' \mid S(1) \setminus Kf(1) &= \pi \mid S(2) \setminus \overline{Kf([1, 2]_q)} \quad \text{and} \\ \pi'(\overline{Kf([1, 2]_q)}) &= \{Kf(1)\}; \end{aligned}$$

then

$$\begin{array}{ccc} S(1) \times S(1) & \xrightarrow{m} & S(2) \\ \pi' \times \pi' \downarrow & & \downarrow \pi' \\ X \times X & \xrightarrow{m_R} & X \end{array}$$

commutes, hence m_R is a global multiplication on X .

LEMMA 2.9. *X is a compact abelian monoid in the quotient topology.*

Proof. Since π' is a closed map, m_R is continuous.

Let $[0, 1]_*$ denote the space $[0, 1]$ equipped with the multiplication $x + y = \min\{1, x + y\}$. Then $[0, 1]_*$ is a compact monoid in the usual topology. In particular, we have the following factorization:

$$\begin{array}{ccc} S(2) & \xrightarrow{\rho} & [0, 2] \\ \pi' \downarrow & & \downarrow \tau \\ X & \xrightarrow{\rho_R} & [0, 1]_* = H/[1, \infty], \end{array}$$

where $\tau: H \rightarrow [0, 1]_*$ is the canonical map and $\rho_R: X \rightarrow [0, 1]_*$ is the unique continuous morphism making the diagram commute.

LEMMA 2.10. *The following statements hold:*

- (1) *X has exactly two idempotents e and $0 \equiv Kf(1)$*
- (2) *K is the maximal group of units in X*
- (3) *K is not open in X*
- (4) *$X \setminus \{0\}$ is isomorphic to $S(1) \setminus Kf(1)$.*

Proof. (1) and (4) are clear. (2): We have $X \setminus K = \rho_R^{-1}([0, 1])$ which is an ideal. Thus K is maximal. (3): If K were open, then $X \setminus K$ would be closed, hence compact, and thus $\rho_R(X \setminus K) =]0, 1[$ would be compact which is not the case.

PROPOSITION 2.11. *There is a continuous morphism $\phi_*: [0, 1]_* \rightarrow X$ so that $\phi_*(0) = e$ and $\phi_*^{-1}(\{0\}) = \{1\}$.*

Proof. By 2.10 we can apply the One-Parameter Semigroup Theorem (Thm. 1, p. 510, [7]; [1]) to obtain ϕ_* .

PROPOSITION 2.12. $\rho_R \circ \phi_*$ is the identity map on $[0, 1]_*$.

Proof. We observe first that $\rho_R \circ \phi_*$ is an endomorphism α of $[0, 1]_*$ with $\alpha^{-1}(\{1\}) = \{1\}$ and is therefore the identity.

PROPOSITION 2.13. There is a one-parameter semigroup $\phi: \mathbf{H} \rightarrow S$ such that $\phi(r) \in Kf(r)$ for all $r \in Q_+$.

Proof. For all $r \in [0, 1]_{[e]}$, $r = \rho_R \circ \phi_*(r) = \rho \circ \phi_*(r)$ and so $\phi_*(r) \in \rho^{-1}(r) = Kf(r)$. Let ϕ be the unique lifting of ϕ_* to \mathbf{H} . Then $\phi(r) \in Kf(r)$ for all $r \in Q_+$.

3. On the Embedding and Density Theorem.

PROPOSITION 3.1. Let G be a locally compact abelian group and $LG = \text{Hom}(R, G)$ the totality of one-parameter subgroups in G . If $\text{exp}: LG \rightarrow G$ denotes the map $\text{exp}(f) = f(1)$, then

- (1) $\overline{\text{exp}(GL)} = G_0$, where G_0 is the identity component of G
- (2) $\text{exp}(LG) = G_0$ iff G_0 is arcwise connected.

Proof. (1) (25.20, p. 410, [3]). (2) (Thm. 1, p. 40, [2]).

EMBEDDING AND DENSITY THEOREM 3.2. Let s be strongly root compact in S . Then there are an algebraic morphism $f: Q_+ \cup \{0\} \rightarrow S$ with $f(0) = e$, $f(1) = s$, and a one-parameter semigroup $\phi: \mathbf{H} \rightarrow S$ which satisfy the following properties: If $K = \bigcap \{f([0, \varepsilon]_{[e]}): 0 < \varepsilon < 1\}$, then K is a compact connected abelian subgroup of $\mathcal{H}(e)$, $\phi(0) = e$, $\phi(\mathbf{H})$ is in the centralizer $Z = \{x \in eSe: xk = kx \text{ for all } k \in K\}$ of K in eSe , and $\phi(t) \in Kf(t)$ for each $t \in Q_+$.

Furthermore, if \mathcal{U} is any neighborhood of s in S , then ϕ may be chosen so that $\phi(1) \in \mathcal{U}$; and, in fact, if K is arcwise connected, then ϕ may be chosen so that $\phi(1) = s$.

Proof. By 2.1, there is an algebraic morphism $f: Q_+ \cup \{0\} \rightarrow S$ such that $f(0) = e$, $f(1) = s$, $\overline{f([0, 1]_{[e]})}$ is compact, $K \subset \mathcal{H}(e)$ is a compact connected abelian subgroup and $\overline{f(Q_+)} \subset eSe$.

If $s \in K$, then by 3.1 the assertion is true. If $s \notin K$, then by 2.13 there is a one-parameter semigroup $\phi: \mathbf{H} \rightarrow S$ so that $\phi(\mathbf{H}) \subset \overline{f(Q_+)} \subset eSe$ and $\phi(r) \in Kf(r)$ for all $r \in Q_+ \cup \{0\}$. In particular, $\phi(\mathbf{H})$ is in the centralizer of K in eSe . Let \mathcal{U} be a neighborhood of s in S ; then there is a neighborhood U of e in K so that $sU \subset \mathcal{U}$. Pick

a $k \in K$ so that $\phi(1) = sk$, by the fact that $\overline{\exp(LK)} = K$, there is an $\psi \in LK$ so that $\psi(1) \in Uk^{-1}$. Let $\phi_1: \mathbf{H} \rightarrow S$ be defined via $\phi_1(r) = \phi(r)\psi(r)$. As $\phi(\mathbf{H})$ is in the centralizer of K in eSe , then ϕ_1 is a well-defined one-parameter semigroup so that

$$\phi_1(1) = \phi(1)\psi(1) \in skUk^{-1} = sU.$$

It is easy to check that ϕ_1 also satisfies the same properties as stated above. If K is arcwise connected, by 3.1 ψ may be chosen so that $\psi(1) = k^{-1}$ and so $\phi_1(1) = s$.

COROLLARY 3.3. *If K is a Lie group, then there is a one-parameter semigroup ϕ so that $\phi(1) = s$ (cf. Thm. 7, p. 141, [9]).*

THEOREM 3.4. *Let s be a strongly p th root compact element in S . Then there are an algebraic morphism $f: \mathbb{Q}_+^p \cup \{0\} \rightarrow S$ with $f(0) = e$, $f(1) = s$, and a one-parameter semigroup $\phi: \mathbf{H} \rightarrow S$ which satisfy the following properties: If $K_p = \cap \{f([0, \varepsilon]_{\mathbb{Q}_+^p}): 0 < \varepsilon < 1\}$, then K_p is a p -divisible compact abelian subgroup of $\mathcal{H}(e)$, $\phi(0) = e$, $\phi(\mathbf{H})$ is in the centralizer Z of K_p in eSe , and $\phi(r) \in K_p f(r)$ for all $r \in \mathbb{Q}_+^p$.*

REMARK. K_p is in general not divisible (cf. p. 265, [5]; p. 117, [6]).

PROPOSITION 3.5. *Let s be a strongly root compact (resp. strongly p th root compact) element in S and f and ϕ be as stated in 3.2 (resp. 3.4). Then there is an algebraic morphic morphism $h: \mathbb{Q}_+ \rightarrow K$ (resp. $h: \mathbb{Q}_+^p \rightarrow K_p$) so that $\phi(r) = f(r)h(r)$ for all $r \in \mathbb{Q}_+$ (resp. \mathbb{Q}_+^p).*

Proof. For each $n \geq 1$, let $A_{n!} = \{x \in K: f(1/n!)x = \phi(1/n!)\}$ (resp. $B(p; n) = \{x \in K_p: f(1/p^n)x = \phi(1/p^n)\}$). Clearly, $A_{n!}$ (resp. $B(p; n)$) is a nonempty compact subset for each $n \geq 1$. The construction of h then follows as in 2.1.

The following example shows that there are elements which are not strongly root compact but which are nevertheless embeddable in one-parameter semigroups:

EXAMPLE 3.5. Let $S = SL(2; R)$ and $s = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$: then s is divisible and $W_2(s) \supset \left\{ \begin{pmatrix} 0 & y \\ z & 0 \end{pmatrix}: yz = -1 \right\}$ is not compact, whence s is not even 2th root compact. But the map $f: R \rightarrow S$ defined via

$$f(t) = \begin{pmatrix} \cos \pi t & \sin \pi t \\ -\sin \pi t & \cos \pi t \end{pmatrix}$$

is a one-parameter subgroup so that $f(1) = s$.

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Vol. 65, No. 1

September, 1976

David Lee Armacost, <i>Compactly cogenerated LCA groups</i>	1
Sun Man Chang, <i>On continuous image averaging of probability measures</i>	13
J. Chidambaraswamy, <i>Generalized Dedekind ψ-functions with respect to a polynomial. II</i>	19
Freddy Delbaen, <i>The Dunford-Pettis property for certain uniform algebras</i>	29
Robert Benjamin Feinberg, <i>Faithful distributive modules over incidence algebras</i>	35
Paul Froeschl, <i>Chained rings</i>	47
John Brady Garnett and Anthony G. O'Farrell, <i>Sobolev approximation by a sum of subalgebras on the circle</i>	55
Hugh M. Hilden, José M. Montesinos and Thomas Lusk Thickstun, <i>Closed oriented 3-manifolds as 3-fold branched coverings of S^3 of special type</i>	65
Atsushi Inoue, <i>On a class of unbounded operator algebras</i>	77
Peter Kleinschmidt, <i>On facets with non-arbitrary shapes</i>	97
Narendrakumar Ramanlal Ladhawala, <i>Absolute summability of Walsh-Fourier series</i>	103
Howard Wilson Lambert, <i>Links which are unknottable by maps</i>	109
Kyung Bai Lee, <i>On certain g-first countable spaces</i>	113
Richard Ira Loebel, <i>A Hahn decomposition for linear maps</i>	119
Moshe Marcus and Victor Julius Mizel, <i>A characterization of non-linear functionals on W_1^p possessing autonomous kernels. I</i>	135
James Miller, <i>Subordinating factor sequences and convex functions of several variables</i>	159
Keith Pierce, <i>Amalgamated sums of abelian l-groups</i>	167
Jonathan Rosenberg, <i>The C^*-algebras of some real and p-adic solvable groups</i>	175
Hugo Rossi and Michele Vergne, <i>Group representations on Hilbert spaces defined in terms of ∂_b-cohomology on the Silov boundary of a Siegel domain</i>	193
Mary Elizabeth Schaps, <i>Nonsingular deformations of a determinantal scheme</i>	209
S. R. Singh, <i>Some convergence properties of the Bubnov-Galerkin method</i>	217
Peggy Strait, <i>Level crossing probabilities for a multi-parameter Brownian process</i>	223
Robert M. Tardiff, <i>Topologies for probabilistic metric spaces</i>	233
Benjamin Baxter Wells, Jr., <i>Rearrangements of functions on the ring of integers of a p-series field</i>	253
Robert Francis Wheeler, <i>Well-behaved and totally bounded approximate identities for $C_0(X)$</i>	261
Delores Arletta Williams, <i>Gauss sums and integral quadratic forms over local fields of characteristic 2</i>	271
John Yuan, <i>On the construction of one-parameter semigroups in topological semigroups</i>	285