ON THE CONSTRUCTION OF ONE-PARAMETER SEMIGROUPS IN TOPOLOGICAL SEMIGROUPS

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Let $S$ be a topological Hausdorff semigroup and $s \in S$ be a strongly root compact element. Then there are an algebraic morphism $f : \mathbb{Q}_+ \cup \{0\} \to S$ with $f(0) = e$, $f(1) = s$, and a one-parameter semigroup $\phi : H \to S$ which satisfy the following properties: If $K = \cap \{f(]0, \varepsilon[_\mathbb{Q}) : 0 < \varepsilon < 1\}$, then $K$ is a compact connected abelian subgroup of $\mathbb{H}(e)$, $\phi(0) = e$, $\phi(H)$ is in the centralizer $Z = \{x \in eSe : xk = kx \text{ for all } k \in K\}$ of $K$ in $eSe$, and $\phi(t) \in f(t)K$ for each $t \in \mathbb{Q}_+$. Furthermore, if $\mathbb{H}$ is any neighborhood of $s$ in $S$, then $\phi$ may be chosen so that $\phi(1) \in \mathbb{H}$; and, in fact, if $K$ is arcwise connected, then $\phi$ may be chosen so that $\phi(1) = s$. The above statements also hold for strongly $p$th root compact elements almost everywhere.

1. Introduction. We are concerned with the question of when a divisible element in a topological semigroup can be embedded in a one-parameter semigroup which has many applications in Probability theory (cf. [4], [8]).

The first result about the existence of one-parameter semigroups in a compact semigroup which we call the One-Parameter Semigroup Theorem is due to Mostert and Shields [7], 1957. In 1960, an independent proof based on the local nature of the compact semigroup was given by Hoffmann (cf. [5], [6]). In 1970, a global proof was presented by Carruth and Lawson [1]. The first result of a generalized one-parameter semigroup theorem dealing with the embedding problems which we will call the Embedding and Density Theorem is indicated by Hofmann in [4] and later proved by Siebert [8]. Siebert’s proof is based on the notion of a local semigroup called ducleus (cf. [6]). We will present in this paper a global proof of this theorem by applying the One-Parameter Semigroup Theorem.

Throughout this paper, we maintain that $\mathbb{R}_+$, $\mathbb{Q}_+$ and $\mathbb{Z}_+$ are the totalities of strictly positive real numbers, rational numbers and integers, respectively, $H = \mathbb{R}_+ \cup \{0\}$ and $Q_p^n = \{n/p^m : n \in \mathbb{Z}_+, m \in \mathbb{Z}_+ \cup \{0\}\}$ for a prime $p$. For convenience, we will use $]a, b[_\mathbb{Q}$ (resp. $]a, b[_\mathbb{Q}_+$, etc.) and $]a, b[Q_p$ (resp. $]a, b[Q_p^n$ (resp. $]a, b[Q_p^n \cap \mathbb{Q}_+$, etc.) and $]a, b[ \cap Q_p^n$ (resp. $]a, b[ \cap Q_p^n$) respectively. We also maintain that $S$ is a topological (Hausdorff) semigroup and $\mathbb{H}(e)$ is the maximal group of units in the closed subsemigroup $eSe$ for an idempotent $e \in S$. 

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2. On the existence of a one-parameter semigroup in \( f(A) \) where \( f: A \to S \) is an algebraic morphism with \( A = Q_+, Q_\varepsilon^* \). Throughout this section, we will always assume that \( f: Q_+ (\text{resp. } Q_\varepsilon^*) \to S \) is an algebraic morphism so that \( f([0, d]_Q) (\text{resp. } f([0, d]_Q^)) \) is compact for some \( d > 0 \) unless mentioned otherwise. As the discussions for \( Q_+ \) and for \( Q_\varepsilon^* \) would be almost the same, we will concentrate on \( Q_+ \) only.

**Definition.** For each \( s \in S \) and each \( n \geq 1 \), let \( W_n(s) = \{ t \in S: t^n = s \}, \ W(n; s) = \{ t^n: 1 \leq m \leq n, t^n = s \} \). \( s \) is said to be divisible (resp. \( p \)-divisible) if \( W_n(s) \neq \emptyset \) (resp. \( W_p^n(s) \neq \emptyset \) ) for all \( n \geq 1 \); root compact (resp. \( p \)th root compact) if \( W_n(s) \) (resp. \( W_p^n(s) \) ) is in addition compact for each \( n \geq 1 \); strongly root compact (resp. strongly \( p \)th root compact) if \( W_\infty(s) = \bigcup \{ W(n; s): n \geq 1 \} \) (resp. \( W_p^\infty(s) = \bigcup \{ W(p^n; s): n \geq 1 \} \) ) in addition relatively compact.

**Proposition 2.1.** Let \( s \) be a root compact (resp. \( p \)th root compact) element in \( S \). Then there is an algebraic morphism \( f: Q_+ (\text{resp. } Q_\varepsilon^*) \to S \) so that \( f(1) = s \). If \( s \) is strongly root compact (resp. strongly \( p \)th root compact), then \( f \) may be chosen so that \( f([0, 1]_Q) \) (resp. \( f([0, 1]_Q^) \) ) is compact.

**Proof.** For each \( n \geq 1 \) and \( i \geq 0 \), pick an \( s_{n+i} \in W_{(n+i)}(s) \) (resp. \( s_{n+i} \in W_{p(n+i)}(s) \) ) and let

\[
\alpha_n = (s_n^1, s_n^{1/2}, \ldots, s_{n}, s_{n+1}, \ldots)
\]

(resp. \( \alpha_n = (s_p^n, s_p^{n-1}, \ldots, s_n, s_{n+1}, \ldots) \)).

Then \( \{ \alpha_n \} \) is a sequence in the compact set \( \prod_{n \geq 1} W_n(s) \) (resp. \( \prod_{n \geq 1} W_p^n(s) \) ). Hence there is a convergent subnet \( \{ \alpha_{n(k)} \} \) converging to \( \alpha = (t_1, t_2, \cdots) \in \prod_{n \geq 1} W_n(s) \) (resp. \( \prod_{n \geq 1} W_p^n(s) \) ).

Then

\[
t_{q+1}^\frac{1}{q+1} = \lim s_{n(k)}^\frac{1}{q+1} = t_q
\]

(resp. \( t_{q+1}^p = \lim s_p^{n(k)} - q = t_q \))

for all \( q \geq 1 \), and \( t_1 = s \). If \( n/m! = b/a! \) (resp. \( n/p^m = b/p^n \) ), then

\[
t_m^a = (t_m^a)^b = t_b^a
\]

(resp. \( t_m^a = (t_m^{p-m-a})^b = t_b^b \)).

Hence \( f: Q_+ (\text{resp. } Q_\varepsilon^*) \to S \) given by \( f(n/m!) = t_m^n \) (resp. \( f(n/p^m) = t_m^n \) )
is well-defined. If \( n/m!, \ b/a! \in \mathbb{Q}_+ \) (resp. \( n/p^m, \ b/p^a \in \mathbb{Q}_+ \)), assuming \( a \geq m \), then

\[
\begin{align*}
    f(n/m! + b/a!) &= f\left(\frac{n(a!/m!)}{a!} + b\right) \\
    &= t^{n(a!/m!)}_a t^b_a = t^{n/a}_m t^b_a
\end{align*}
\]

resp.

\[
\begin{align*}
    f(n/p^m + b/p^a) &= f\left(\frac{np^{a-m}}{p^a} + b\right) \\
    &= t^{n/p^{a-m}}_a t^b_a = t^{n/a}_m t^b_a
\end{align*}
\]

whence \( f \) is an algebraic morphism so that \( f(1) = s \). The rest is simple.

**Lemma 2.2.** for each \( x > 0 \), let \( S(x) = f(x) \). Then

1. \( S(x + y) = S(x)S(y) \) for all \( x, y > 0 \). In particular, \( S(x) \)
   is compact for each \( x > 0 \)

2. \( f(\mathbb{Q}_+) \) has the identity \( e \) so that \( K = \cap \{ S(x) \mid x \in \mathbb{Q}_+ \} \) is a
   divisible compact abelian subgroup of \( H(e) \). In particular, we may
   extend \( f \) to \( Q_+ \cup \{0\} \) so that \( f(0) = e \)

3. \( Kf([x, y]) = F([x, y]) \) for all \( x < y \in \mathbb{Q}_+ \).

*Proof.* Straightforward (cf. § 3, Chapter B, [6]).

**Lemma 2.3.** The following statements are equivalent:

1. \( K = \{ f(0) \} \)

2. \( f \) is continuous at 0

3. \( f \) is continuous.

*Proof.* (cf. 3.9, p. 102, [6].)

**Lemma 2.4.** If \( f \) is continuous, then there is a unique one-
parameter semigroup \( \phi \) so that \( \phi | (Q_+ \cup \{0\}) = f \).

*Proof.* Given a \( d > 0 \), there is a net \( \{ x_\alpha \} \) in \( ]0, d + 1[_Q \) with
\( \lim x_\alpha = d \). Since \( \{ f(x_\alpha) \} \) is a net in \( S(d + 1) \), there is a convergent
subnet \( \{ f(x_\beta) \} \). Define \( F(d) = \lim f(x_\beta) \). It is straightforward to
check that \( F: H \to S \) is a well defined morphism so that \( \cup \{ F([0, x]) : x > 0 \} = \{ f(0) \} \), whence \( F \) is continuous (cf. 3.9, p. 102, [6]).

**Lemma 2.5.** Let \( \phi: H \to S \) be a nontrivial one-parameter semi-
group. Then there is a \( d \in [0, 1] \) so that \( \phi | [0, d] \) is injective.
Moreover, if \( c > 0 \), one may reparameterize \( \phi \) so that \( \phi | [0, c] \) is
injective (cf. 3.9, p. 102, [6]).
Since $K$ acts on $\overline{f(Q_+)}$ and $\overline{f([x, y[Q_+])}$, one has the orbit spaces $\overline{f(Q_+)/K}$ and $\overline{f([v, y[Q_+)/K}$. We will use the same letter $\pi$ to denote the orbit maps.

**Lemma 2.6.** $\overline{f(Q_+)/K}$ is a topological monoid under the multiplication $xK \cdot yK = xyK$.

**Lemma 2.7.** If $f(Q_+) \not\subset K$, then $\pi \circ f: Q_+ \cup \{0\} \to \overline{f(Q_+)/K}$ is non-trivial continuous morphism so that $\pi(\overline{f([x, y[Q_+])} = \overline{f([x, y[Q_+)/K}$ for all $x < y \in Q_+ \cup \{0\}$.

*Proof.* The continuity of $\pi \circ f$ follows from 2.3. The rest follows from the closedness of $\pi$.

In the remainder of this section, we maintain that $f(1) \not\subset K$ and so $\pi \circ f$ extends to a unique one-parameter semigroup $g: H \to \overline{f(Q_+)/K}$ that $g|[0, 2]$ is injective by a suitable reparameterization of $g$ or $f$, i.e. the following diagram commutes:

$$
\begin{array}{ccc}
0, 2[0 & \xrightarrow{f} & S(2) \\
\downarrow & & \downarrow \pi \\
[0, 2] & \xrightarrow{g} & S(2)/K.
\end{array}
$$

Let $\rho = g^{-1} \circ \pi: S(2) \to [0, 2]$. Then $\rho$ is a continuous map such that

$$
\rho(f(r)) = (g^{-1} \circ \pi)(f(r)) = r \quad \text{for all} \quad r \in [0, 2],
$$

and that the following condition is satisfies:

$$
\rho(xy) = \rho(x) + \rho(y) \quad \text{for all} \quad x, y \in S(1).
$$

**Lemma 2.8.** The following statements hold:

1. $x \in Kf(r)$ if and only if $x \in \pi^{-1}(g(r))$ for each $r \in Q_+ \cup \{0\}$
2. $x \in S(2)$ if and only if there is a unique $t \in [0, 2]$ so that $x \in \pi^{-1}(g(t))$
3. $\pi^{-1}(g([x, y[)] = Kf([x, y[Q_+]) = \overline{f([x, y[Q_+])}$ for all $x, y \in Q_+ \cup \{0\}$
4. $S(1) \setminus Kf(1) \subset Kf([1, 2[Q_+]$
5. $S(1) \setminus Kf(1) = S(2) \setminus Kf([1, 2[Q_+)$.

*Proof.* Straightforward.

Define a multiplication on the space $X$ obtained from $S(1)$ by collapsing $Kf(1)$ to a point as follows:

$$
m_x(x, y) = \begin{cases} 
xy & \text{if} \quad x, y, xy \in S(1) \setminus Kf(1) \\
Kf(1) & \text{otherwise}.
\end{cases}
$$
Let $\pi': S(2) \to X$ be defined via

$$\pi' \mid S(1) \setminus Kf(1) = \pi \mid S(2) \setminus K\overline{f([1, 2[Q])} \quad \text{and} \quad \pi'(K\overline{f([1, 2[Q)) = \{Kf(1)\};$$

then

$$
\begin{array}{ccc}
S(1) \times S(1) & \xrightarrow{m} & S(2) \\
\pi' \times \pi' & \downarrow & \pi' \\
X \times X & \xrightarrow{m_B} & X
\end{array}
$$

commutes, hence $m_B$ is a global multiplication on $X$.

**Lemma 2.9.** $X$ is a compact abelian monoid in the quotient topology.

**Proof.** Since $\pi'$ is a closed map, $m_B$ is continuous.

Let $[0, 1]_*$ denote the space $[0, 1]$ equipped with the multiplication $x + y = \min \{1, x + y\}$. Then $[0, 1]_*$ is a compact monoid in the usual topology. In particular, we have the following factorization:

$$
\begin{array}{ccc}
S(2) & \xrightarrow{\nu} & [0, 2] \\
\pi' & \downarrow & \tau \\
X & \xrightarrow{\rho_b} & [0, 1]_* = H/[1, \infty],
\end{array}
$$

where $\tau: H \to [0, 1]_*$ is the canonical map and $\rho_b: X \to [0, 1]_*$ is the unique continuous morphism making the diagram commute.

**Lemma 2.10.** The following statements hold:

1. $X$ has exactly two idempotents $e$ and $0 = Kf(1)$
2. $K$ is the maximal group of units in $X$
3. $K$ is not open in $X$
4. $X\setminus\{0\}$ is isomorphic to $S(1) \setminus Kf(1)$.

**Proof.** (1) and (4) are clear. (2): We have $X\setminus K = \rho_b^{-1}(]0, 1])$ which is an ideal. Thus $K$ is maximal. (3): If $K$ were open, then $X\setminus K$ would be closed, hence compact, and thus $\rho_b(X\setminus K) = ]0, 1]$ would be compact which is not the case.

**Proposition 2.11.** There is a continuous morphism $\phi_*: [0, 1]_* \to X$ so that $\phi_*(0) = e$ and $\phi_*([0]) = \{1\}$. 
Proof. By 2.10 we can apply the One-Parameter Semigroup Theorem (Thm. 1, p. 510, [7]; [1]) to obtain $\phi_*$. 

PROPOSITION 2.12. $\rho_{\mathbb{R}} \circ \phi_*$ is the identity map on $[0, 1]_*$. 

Proof. We observe first that $\rho_{\mathbb{R}} \circ \phi_*$ is an endomorphism $\alpha$ of $[0, 1]_*$ with $\alpha^{-1}(\{1\}) = \{1\}$ and is therefore the identity.

PROPOSITION 2.13. There is a one-parameter semigroup $\phi: H \to S$ such that $\phi(r) \in Kf(r)$ for all $r \in Q_+$. 

Proof. For all $r \in [0, 1]_\mathbb{Q}$, $r = \rho_{\mathbb{R}} \circ \phi_*(r) = \rho \circ \phi_*(r)$ and so $\phi_*(r) \in \rho^{-1}(r) = Kf(r)$. Let $\phi$ be the unique lifting of $\phi_*$ to $H$. Then $\phi(r) \in Kf(r)$ for all $r \in Q_+$.

3. On the Embedding and Density Theorem.

PROPOSITION 3.1. Let $G$ be a locally compact abelian group and $LG = \text{Hom}(R, G)$ the totality of one-parameter subgroups in $G$. If $\exp: LG \to G$ denotes the map $\exp(f) = f(1)$, then

(1) $\exp(GL) = G_o$, where $G_o$ is the identity component of $G$ 
(2) $\exp(LG) = G_o$ iff $G_o$ is arcwise connected.

Proof. (1) (25.20, p. 410, [3]). (2) (Thm. 1, p. 40, [2]).

EMBEDDING AND DENSITY THEOREM 3.2. Let $s$ be strongly root compact in $S$. Then there are an algebraic morphism $f: Q_+ \cup \{0\} \to S$ with $f(0) = e$, $f(1) = s$, and a one-parameter semigroup $\phi: H \to S$ which satisfy the following properties: If $K = \cap \{f([0, \varepsilon]): 0 < \varepsilon < 1\}$, then $K$ is a compact connected abelian subgroup of $\mathcal{H}(e)$, $\phi(0) = e$, $\phi(H)$ is in the centralizer $Z = \{x \in eSe: xk = kx \text{ for all } k \in K\}$ of $K$ in $eSe$, and $\phi(t) \in Kf(t)$ for each $t \in Q_+$.

Furthermore, if $\mathcal{U}$ is any neighborhood of $s$ in $S$, then $\phi$ may be chosen so that $\phi(1) \in \mathcal{U}$; and, in fact, if $K$ is arcwise connected, then $\phi$ may be chosen so that $\phi(1) = s$.

Proof. By 2.1, there is an algebraic morphism $f: Q_+ \cup \{0\} \to S$ such that $f(0) = e$, $f(1) = s$, $f([0, 1]_\mathbb{Q})$ is compact, $K \subset \mathcal{H}(e)$ is a compact connected abelian subgroup and $f(Q_+) \subset eSe$.

If $s \in K$, then by 3.1 the assertion is true. If $s \not\in K$, then by 2.13 there is a one-parameter semigroup $\phi: H \to S$ so that $\phi(H) \subset f(Q_+) \subset eSe$ and $\phi(r) \in Kf(r)$ for all $r \in Q_+ \cup \{0\}$. In particular, $\phi(H)$ is in the centralizer of $K$ in $eSe$. Let $\mathcal{U}$ be a neighborhood of $s$ in $S$; then there is a neighborhood $U$ of $e$ in $K$ so that $sU \subset \mathcal{U}$. Pick
a \ k \in \ K \text{ so that } \phi(1) = sk, \text{ by the fact that } \exp(\mathbf{L}K) = K, \text{ there is an } \psi \in \mathbf{L}K \text{ so that } \psi(1) \in \mathbf{U}k^{-1}. \text{ Let } \phi_i: \mathbf{H} \to S \text{ be defined via } \phi_i(r) = \phi(r)\psi(r). \text{ As } \phi(H) \text{ is in the centralizer of } K \text{ in } \mathbf{e}S, \text{ then } \phi_i \text{ is a well-defined one-parameter semigroup so that }
\phi_i(1) = \phi(1)\psi(1) \in sk\mathbf{U}k^{-1} = s\mathbf{U}.

It is easy to check that \phi_i \text{ also satisfies the same properties as stated above. If } K \text{ is arcwise connected, by 3.1 } \psi \text{ may be chosen so that } \psi(1) = k^{-1} \text{ and so } \phi_i(1) = s.

**Corollary 3.3.** If \( K \) is a Lie group, then there is a one-parameter semigroup \( \phi \) so that \( \phi(1) = s \) (cf. Thm. 7, p. 141, [9]).

**Theorem 3.4.** Let \( s \) be a strongly pth root compact element in \( S \). Then there are an algebraic morphism \( f: Q_+^p \cup \{0\} \to S \) with \( f(0) = e, f(1) = s \), and a one-parameter semigroup \( \phi: \mathbf{H} \to S \) which satisfy the following properties: If \( K_p = \cap \{ f([0, \varepsilon[\mathbf{Q}_p]): 0 < \varepsilon < 1 \} \), then \( K_p \) is a \( p \)-divisible compact abelian subgroup of \( \mathbb{R}^e(e), \phi(0) = e, \phi(H) \) is in the centralizer \( Z \) of \( K_p \) in \( eS \), and \( \phi(r) \in K_pf(r) \) for all \( r \in Q_+^p \).

**Remark.** \( K_p \) is in general not divisible (cf. p. 265, [5]; p. 117, [6]).

**Proposition 3.5.** Let \( s \) be a strongly root compact (resp. strongly pth root compact) element in \( S \) and \( f \) and \( \phi \) be as stated in 3.2 (resp. 3.4). Then there is an algebraic morphic morphism \( h: Q_+ \to K \) (resp. \( h: Q_+^p \to K_p \)) so that \( \phi(r) = f(r)h(r) \) for all \( r \in Q_+ \) (resp. \( Q_+^p \)).

**Proof.** For each \( n \geq 1 \), let \( A_n = \{ x \in K: f(1/n!)x = \phi(1/n!) \} \) (resp. \( B(p; n) = \{ x \in K_p: f(1/p^n)x = \phi(1/p^n) \} \)). Clearly, \( A_n \) (resp. \( B(p; n) \)) is a nonempty compact subset for each \( n \geq 1 \). The construction of \( h \) then follows as in 2.1.

The following example shows that there are elements which are not strongly root compact but which are nevertheless embeddable in one-parameter semigroups:

**Example 3.5.** Let \( S = SL(2; R) \) and \( s = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \); then \( s \) is divisible and \( W(s) \supset \{ \begin{pmatrix} 0 & y \\ z & 0 \end{pmatrix}: yz = -1 \} \) is not compact, whence \( s \) is not even 2th root compact. But the map \( f: R \to S \) defined via
\[
\begin{pmatrix} \cos \pi t & \sin \pi t \\ -\sin \pi t & \cos \pi t \end{pmatrix}
\]
is a one-parameter subgroup so that \( f(1) = s \).

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REFERENCES


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