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**DELOOPING THE CONTINUOUS K -THEORY OF A
VALUATION RING**

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DELOOPING THE CONTINUOUS K -THEORY OF A VALUATION RING

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In this note the continuous algebraic K -theory groups of a complete discrete valuation ring are described as the inverse limit of the ordinary algebraic K -theory of its finite quotient rings.

In [4] we defined continuous algebraic K -theory groups K_i^{top} , $i \geq 2$, both for a complete discrete valuation ring \mathcal{O} with finite residue field of positive characteristic p and for its fraction field and proved that K_2^{top} agrees with the fundamental group of the special linear group as defined in [2] by means of universal topological central extensions. The definition of K_i^{top} in [4] is in terms of BN -pairs and is similar to the theory K_i^{BN} of [5] which is known [6] to deloop to ordinary algebraic K -theory. The purpose of this note is to deloop $K_i^{\text{top}}(\mathcal{O})$ in the sense of the following result: Let $\mathcal{P} \cap \mathcal{O}$ be the maximal ideal and let K_i be the algebraic K -theory groups of Quillen [3].

THEOREM. *For $i \geq 2$ there is a natural isomorphism*

$$K_i^{\text{top}}(\mathcal{O}) \cong \varprojlim_n K_i(\mathcal{O}/\mathcal{P}^n).$$

In a forthcoming paper of the author and R. J. Milgram, this equation allows us to use the continuous cohomology of $SL(l, \mathcal{O})$ to compute the rank of the free part of $K_i^{\text{top}}(\mathcal{O})$ as a module over the p -adic completion of the rational integers.

In §2 a step in the proof of this theorem is used to describe the homotopy fiber of $BE(A)^+ \rightarrow BE(A/J)^+$ where J is an ideal in a commutative ring A such that $1 + J \subset A^*$. At least, we construct a space $B\{U_{\bar{x}}(A, J)\}^+$ whose homotopy groups fit into the appropriate exact sequence.

Actually, in this paper we shall let

$$K_i^{\text{top}}(\mathcal{O}) = \varprojlim_{m|n} [\varinjlim_l \pi_{i-1} SL_n^{\text{top}}(l, \mathcal{O})]$$

whereas in [4] the order of the inverse and direct limits is reversed. The above definition is perhaps better as it still gives the main results of [4]. To see the two are the same one would have to prove that

$$\longrightarrow \pi_{i-1} SL_n^{\text{top}}(l, \mathcal{O}) \longrightarrow \pi_{i-1} SL_n^{\text{top}}(l+1, \mathcal{O}) \longrightarrow \dots$$

eventually stabilizes to an isomorphism.

The theorem makes it clear that the natural map $K_i(\mathcal{O}) \rightarrow K_i^{\text{top}}(\mathcal{O})$ comes from the ring maps $\mathcal{O} \rightarrow \mathcal{O}/\mathcal{P}^n$.

1. **Delooping.** Let n and l be fixed. The main step is to prove

PROPOSITION 1.1. *There is a natural homotopy equivalence*

$$\text{SL}^{ab}(l, \mathcal{O}/\mathcal{P}^n) \cong \text{SL}_n^{\text{top}}(l, \mathcal{O})$$

such that if $m|n$ there is a homotopy commutative diagram

$$\begin{array}{ccc}
 \text{SL}^{ab}(l, \mathcal{O}/\mathcal{P}^n) & \cong & \text{SL}_n^{\text{top}}(l, \mathcal{O}) \\
 \downarrow & & \downarrow \\
 \text{SL}^{ab}(l, \mathcal{O}/\mathcal{P}^m) & \cong & \text{SL}_m^{\text{top}}(l, \mathcal{O}).
 \end{array}$$

(*)

See [4] for notation. From this result and [6] we see that for $i \geq 2$

$$\begin{aligned}
 \lim_{\substack{\longrightarrow \\ i}} \pi_{i-1} \text{SL}_n^{\text{top}}(l, \mathcal{O}) &= \lim_{\substack{\longrightarrow \\ i}} \pi_{i-1} \text{SL}^{ab}(l, \mathcal{O}/\mathcal{P}^n) \\
 &= \pi_{i-1} \text{SL}^{ab}(\mathcal{O}/\mathcal{P}^n) \\
 &= K_i(\mathcal{O}/\mathcal{P}^n).
 \end{aligned}$$

Here $\text{SL}^{ab}(A)$ of [4] is the same as $E^{BN}(A)$ of [5]. The main theorem now follows from commutativity of (*).

For simplicity of notation let $S_i = \text{SL}^{ab}(l, \mathcal{O}/\mathcal{P}^n)$ and $T_i = \text{SL}_n^{\text{top}}(l, \mathcal{O})$. Let P^i (resp. Q^i) be the complex whose k -simplices are $(k+1)$ -tuples $(F_0 < F_1 < \dots < F_k)$ where F_i is a linear (resp. affine) facet or R^i . $P^i \subset S_i$ by the imbedding $F \rightarrow U_F$ and $Q^i \subset T_i$ via $F \rightarrow U_F^n$. Let $\text{st}_i(\Delta) \subset Q^i$ be the star of Δ consisting of all affine facettes F such that $\Delta < F$. Let $K_i \subset T_i$ be the subcomplex whose k -simplices $(\alpha_0 \cdot U_{F_0}^n < \dots < \alpha_k \cdot U_{F_k}^n)$ have $F_i \in \text{st}_i(\Delta)$.

Now for each affine facet $F \in \text{st}_i(\Delta)$ there is a unique linear facet F' which contains F such that $F < G$ implies $F' < G'$. The map $\text{st}_i(\Delta) \rightarrow P^i$ sending F to F' is an isomorphism of partially ordered sets. Let $\pi: \text{SL}(l, \mathcal{O}) \rightarrow \text{SL}(l, \mathcal{O}/\mathcal{P}^n)$ be reduction modulo \mathcal{P}^n . We claim that

$$(1.2) \quad \pi(U_F^n) = U_{F'}$$

for $F \in \text{st}_i(\Delta)$. This is clear for the fundamental chamber $C = \{x_i + 1 > x_1 > \dots > x_i\}$ and also for any $F < C$. For an arbitrary $F \in \text{st}_i(\Delta)$ choose an element w of the linear Weyl group W_0 so that $w \cdot F < C$. Thus by [4, Lemma 3]

$$\begin{aligned} \pi(U_F^n) &= \pi(w^{-1}(wU_F^n w^{-1})w) \\ &= w^{-1} \cdot \pi(U_{w \cdot F}^n) \cdot w \\ &= w^{-1} \cdot U_{w \cdot F'} \cdot w \\ &= U_{F'} . \end{aligned}$$

Moreover for each $F \in \text{st}_i(\Delta)$ we have

$$(1.2') \quad \pi^{-1}(U_{F'}) = U_F^n$$

These two equations imply the correspondence

$$\alpha \cdot U_F^n \longrightarrow \pi(\alpha) \cdot U_{F'}$$

preserves order and defines a simplicial isomorphism $K_i \rightarrow S_i$. Hence to prove (1.1) it suffices to show K_i is a deformation retract to T_i .

Let $f; g: Q^i \rightarrow Q^i$ be two simplicial maps arising from order preserving maps of vertices.

LEMMA 1.3. *There is a triangulation $(Q_i \times I)'$ of $Q_i \times I$ as a partially ordered set which refines the standard triangulation leaving $Q^i \times 0$ and $Q^i \times 1$ fixed and there is a simplicial map $w: (Q^i \times I)' \rightarrow Q^i$ such that*

- (a) $w|_{Q^i \times 0} = f$ and $w|_{Q^i \times 1} = g$
- (b) if $\sigma = (v_0 < \dots < v_n)$ is a simplex of the standard triangulation $Q \times I$, $v \in \sigma$ is a vertex in the new triangulation, and $e_{ij}(\lambda)$ is in $U_{w(v_s)}^n$ for $0 \leq s \leq k$, then

$$e_{ij}(\lambda) \in U_{w(v)}^n .$$

This is the affine analogue of Lemma 3.3 of [6] and the proof is similar. For (b) compare (B) of Lemma 4 of [4].

Now let $r: Q^i \rightarrow \text{st}_i(\Delta) \subset Q^i$ be defined by

$$r(F) = \begin{cases} \text{the unique affine facette of } \text{st}_i(\Delta) \text{ which is} \\ \text{contained in the same linear facette as } F. \end{cases}$$

This is an order preserving map which is the identity on $\text{st}_i(\Delta)$.

LEMMA 1.4. *For each affine facette F we have $U_F^n \subset U_{r(F)}^n$.*

Proof. If $w \in W_0$, then $w \cdot r(F) = r(w \cdot F)$ and $w \cdot F_F^n \cdot w^{-1} = U_{w \cdot F}^n$; so by choosing a w such that $w \cdot F$ is contained in the closure \bar{C}_0 of the fundamental linear chamber $C_0 = \{x_1 > \dots > x_i\}$ we can assume $F \subset \bar{C}_0$. In this case $r(F) = C$. When $i > j$, $e_i - e_j \geq 0$ on F ; so for the generator $e_{ij}(\lambda)$ of U_F^n the element $\lambda \in \mathcal{O}$ can be arbitrary and $e_{ij}(\lambda) \in U_C^n$. When $i < j$, $e_i - e_j \leq 0$ on F so $k(F, e_i - e_j)_n \geq n = k(C, e_i - e_j)_n$; hence any generator $e_{ij}(\lambda)$ of U_F^n also belongs to U_C^n .

We can now complete the proof of (1.1). Apply Lemma 1.3 in the case $f = \text{id}$ and $g = r$ to get $w: (Q^l \times I)' \rightarrow Q^l$ satisfying (a) and (b). The map $\rho: T_l \rightarrow Q^l$ taking $\alpha \cdot U_F^n$ to F is nondegenerate on simplices and so is $\rho \times 1: T_l \times I \rightarrow Q^l \times I$. Therefore the triangulation $(Q^l \times I)'$ induces a subdivision $(T_l \times I)'$ of $T_l \times I$. Let $\sigma = (\alpha_0 \cdot U_{F_0}^n < \dots < \alpha_k \cdot U_{F_k}^n)$ be a simplex of T_l and let v be a vertex of $\sigma \times I$. Let $u = (\rho \times 1)(v)$. By (1.4) we have $U_{F_0}^n \subset U_{w(v)}^n$. Hence by (b) of (1.3) we still have

$$(1.5) \quad U_{F_0}^n \subset U_{w(v)}^n$$

if v is any vertex of $(\sigma \times I)'$.

Let $R: T_l \rightarrow T_l$ be defined by $R(\alpha \cdot U_F^n) = \alpha \cdot U_{r(F)}^n$. This retracts T_l onto K_l . Define a homotopy $H: (T_l \times I)' \rightarrow T_l$ from the identity to R as follows: Let v be a vertex $(\sigma \times I)'$ and let $u = (\rho \times 1)(v)$. Let

$$H(v) = \alpha_0 \cdot U_{w(v)}^n .$$

Then (1.5) shows this is independent of the choice $\alpha_0 \in U_{F_0}^n$ so we get a well defined map.

2. A fibration in K -theory. Let A be a commutative ring and $J \subset A$ be an ideal such that $1 + J \subset A^*$. Then $K_i(A) \rightarrow K_i(A/J)$ is surjective for $i = 1, 2$. In this section we build a space $B\{U_F(A, J)\}^+$ such that for $i \geq 2$ there is a natural exact sequence

$$(2.1) \quad \begin{aligned} \dots &\longrightarrow K_{i+1}(A/J) \longrightarrow \pi_i B\{U_F(A, J)\}^+ \\ &\longrightarrow K_i(A) \longrightarrow K_i(A/J) \longrightarrow \dots \end{aligned}$$

Let P^l denote the set of linear facettes in R^l and identify P^l as a subset of P^{l+1} by the map

$$(x_1, \dots, x_l) \longrightarrow (x_1, \dots, x_l, x_l) .$$

Let $P^\infty = \cup_l P^l$. If $F \in P^\infty$ define the subgroup $U_F(A, J)$ of the group $E(A)$ of elementary matrices to be the one generated by

- (a) $e_{ij}(\lambda)$ where $\lambda \in A$ for $e_i - e_j > 0$ on F
- (b) $e_{ij}(\lambda)$ where $\lambda \in J$ for $e_i - e_j < 0$ on F
- (c) diagonal matrices $\text{diag} \{1 + \lambda_1, \dots, 1 + \lambda_r\}$ of determinant one where $\lambda_i \in J$.

If $F < G$, then $U_F(A, J) < U_G(A, J)$. When $J = 0$, we just get the groups U_F of [4] and [5]. In this case we write $U_F(A, J) = U_F(A)$. Let $\pi: E(A) \rightarrow E(A/J)$ be reduction mod J . Then as in (1.2) and (1.2)' we have

$$(2.2) \quad \pi[U_F(A, J)] = U_F(A/J) \quad \text{and} \quad \pi^{-1}[U_F(A/J)] = U_F(A, J) .$$

Let $B\{U_F(A, J)\}$ be the realization of the simplicial space which in dimension $k \geq 0$ is the disjoint union of the spaces

$$(F_0 < \dots < F_k) \times BU_{F_0}(A, J)$$

where $F_i \in P^\infty$. Let $E\{\alpha \cdot U_F(A, J)\}$ be defined as the pullback

$$\begin{array}{ccc} E\{\alpha \cdot U_F(A, J)\} & \longrightarrow & EG \\ \downarrow & & \downarrow \\ B\{U_F(A, J)\} & \longrightarrow & BG \end{array}$$

where $G = E(A)$. When $J = 0$ we recover $E\{\alpha \cdot U_F\}$ as in [1]. Moreover just as in [1] the space $E\{\alpha \cdot U_F(A, J)\}$ has the homotopy type of the space $E^{BN}(A, J)$ whose k -simplices are $(k + 1)$ -tuples

$$\sigma_0 \cdot U_{F_0}(A, J) < \dots < \alpha_k \cdot U_{F_k}(A, J)$$

where $\alpha \cdot U_F(A, J) < \beta \cdot U_G(A, J)$ iff $F < G$ and $\alpha \cdot U_F(A, J) \subset \beta \cdot U_G(A, J)$. As in [1] we have a homotopy fibration

$$E\{\alpha \cdot U_F(A, J)\} \longrightarrow B\{U_F(A, J)\} \longrightarrow BE(A).$$

Suppose for the moment we have

LEMMA 2.3. $\pi_1 B\{U_F(A, J)\}$ is perfect.

Then essentially the same argument as in [1] shows that

$$(**) \quad E\{\alpha \cdot U_F(A, J)\} \longrightarrow B\{U_F(A, J)\}^+ \longrightarrow BE(A)^+$$

is also a homotopy fibration. It follows from (2.2) that the map

$$E^{BN}(A, J) \longrightarrow E^{BN}(A/J)$$

given by $\alpha \cdot U_F(A, J) \rightarrow \pi(\alpha) \cdot U_F(A/J)$ is an isomorphism. By [6] we therefore have $\pi_{i-1} E^{BN}(A, J) = K_i(A/J)$ and the homotopy sequence of the fibration (**) gives (2.1).

To prove the lemma, it is enough to show the generators are products of commutators and the formula $w \cdot U_F \cdot w^{-1} = U_{u \cdot F}$ reduces the argument to the case where $F = C_0 = \{x_1 > x_2 > \dots > x_l\}$ considered as lying in P^l . Here $l \geq 3$. For generators $e_{ij}(\lambda)$ of $\pi_1(BU_{C_0})$ the third Steinberg relation $e_{ij}(\alpha\beta) = [e_{ij}(\alpha), e_{jk}(\beta)]$ shows $e_{ij}(\lambda)$ is a commutator: for example, if $\lambda \in J$ we have $e_{21}(\lambda) = [e_{23}(1), e_{31}(\lambda)]$. Now consider the generators $\begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}$, $\lambda \in 1 + J$, where λ is in the i th row and i th column and λ^{-1} is in the j th row and j th column. For simplicity take $i = 1$ and $j = 2$. Recall that if $M, N \in U_F$ are considered as generators of $\pi_1 BU_F$ their composition as loops is homotopic to

MN. Let $\lambda = 1 + \sigma$ and $\lambda^{-1} = 1 + \tau$ where $\tau, \sigma \in J$. We have the following matrix identity valid in $E(A)$:

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix} = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\tau & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & \sigma \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}.$$

Thus modulo the commutator subgroup

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & \sigma \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+\sigma & \sigma \\ -\sigma & 1-\sigma \end{pmatrix}.$$

Now let $D = \{x_1 = x_2 > \dots > x_i\}$ and $C'_0 = \{x_2 > x_1 > \dots > x_i\}$. We have $U_{C'_0} \supset U_D \subset U_{C'_0}$ and the matrix $\begin{pmatrix} 1+\sigma & \sigma \\ -\sigma & 1-\sigma \end{pmatrix}$ lies in U_D . Each of $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & \sigma \\ 0 & 1 \end{pmatrix}$, and $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ belong to $U_{C'_0}$ and therefore by the above argument lie in the commutator subgroup. Therefore so does $\begin{pmatrix} 1+\sigma & \sigma \\ -\sigma & 1-\sigma \end{pmatrix}$, and we conclude that the loop $\begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}$ lies in the commutator subgroup.

It is probably true that

$$B\{U_F(A, J)\}^+ \longrightarrow BE(A)^+ \longrightarrow BE(A/J)^+$$

is a homotopy fibration.

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