

# Pacific Journal of Mathematics

## **NOETHERIAN FIXED RINGS**

DANIEL REUVEN FARKAS AND ROBERT L. SNIDER

## NOETHERIAN FIXED RINGS

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One of the basic questions of noncommutative Galois theory is the relation between a ring  $R$  and the ring  $S$  fixed by a group of automorphisms of  $R$ . This paper explores what happens when the group is finite and the fixed ring  $S$  is assumed to be Noetherian. Easy examples show that  $R$  may not be Noetherian; however, in this paper it is shown that  $R$  is Noetherian with some rather natural assumptions. More precisely we prove the Theorem 2: Let  $S$  be a semi-prime ring. Assume that  $G$  is a finite group of automorphisms of  $S$  and that  $S$  has no  $|G|$ -torsion. If  $S^G$  is left noetherian then  $S$  is left noetherian.

Theorem 2 answers a question raised by Fisher and Osterburg [4].

This result rests on calculations which can best be described as belonging to noncommutative Galois theory. The basic theorem here may be of independent interest.

**THEOREM 1.** *Let  $R$  be a semisimple artinian ring. If  $G$  is a finite group of automorphisms of  $R$  and  $|G|$  is invertible in  $R$  then  $R$  is a finitely generated ring  $R^G$ -module.*

The proof of Theorem 1 follows the spirit of Karchenko's work on polynomial identity rings ([6]).

1. A proof of Theorem 1. We will repeatedly need Levitzki's fixed ring theorem ([8]): Suppose  $R$  is a semisimple artinian ring. If  $G$  is a finite group of automorphisms of  $R$  with  $|G|$  invertible in  $R$  then  $R^G$  is semisimple artinian.

**LEMMA 1.** *If Theorem 1 is true when  $G$  is a simple group then it is true for an arbitrary finite  $G$ .*

*Proof.* By induction on the length of a composition series for  $G$ .

If  $G$  is not already simple choose  $H \triangleleft G$  with  $1 \neq H \neq G$ . By Levitzki's theorem  $R^H$  is semisimple artinian.  $G/H$  acts on  $R^H$  and  $R^H$  has no  $|G/H|$ -torsion; by induction  $R^H$  is a finitely generated right  $R^G$ -module. Again, induction shows that  $R$  is a finitely generated right  $R^H$ -module. The lemma follows.

We eventually assume that  $G$  is simple. In that case either  $G$  consists entirely of outer automorphisms or entirely of inner automorphisms.

LEMMA 2. *Let  $B$  be a simple artinian ring and let  $G$  be a finite group of outer automorphisms of  $B$ . Then  $B$  is a finitely generated right  $B^G$ -module.*

*Proof.* By [1],  $B^G$  is a simple ring and  $B$  is a free module over  $B^G$  of rank  $|G|$ . (Cf. [5] for the case of a division ring.)

LEMMA 3. *Let  $B$  be a simple artinian ring and let  $G$  be a finite group of inner automorphisms of  $B$ . Assume  $|G|$  is invertible in  $B$ . Then  $B$  is a finitely generated right  $B^G$ -module.*

*Proof.* Let  $F$  be the center of  $B$ .

For each  $g \in G$  pick one  $x \in B$  such that  ${}^g b = xbx^{-1}$  for all  $b \in B$ . Call the finite set so chosen,  $\bar{G}$ . Then collection of sums,  $F\bar{G}$ , is a finite dimensional algebra over  $F$ . Since  $1/|G| \in F$ , Maschke's theorem for twisted group algebras ([9]) states that  $F\bar{G}$  is a separable algebra. Thus there is a finite extension field  $K$  of  $F$  such that  $K$  is a splitting field for each simple constituent of  $F\bar{G}$ .

$K \otimes_F B$  is a simple artinian ring with center  $K$ .  $G$  acts on  $K \otimes_F B$  by

$${}^g(k \otimes b) = k \otimes {}^g b.$$

Obviously this action, too, is induced by inner automorphisms. A straight-forward calculation shows that  $(K \otimes B)^G = K \otimes B^G$ . Similarly, if  $K \otimes B$  is a finitely generated right  $(K \otimes B)^G$ -module then  $B$  is a finitely generated  $B^G$ -module.

Thus we replace  $B$  with  $K \otimes_F B$  and assume each simple constituent of  $F\bar{G}$  is a total matrix ring with entries in  $F$ . Let  $\mathcal{E}$  be the set of centrally primitive idempotents in  $F\bar{G}$ .

The crux of this lemma is to show that if  $e \in \mathcal{E}$  then  $eBe$  is a finitely generated right  $B^G$ -module. An element of  $B^G$  commutes with elements of  $F\bar{G}$  so it certainly commutes with  $e$ ; hence  $eBe$  is a right  $B^G$ -module. Let  $\varepsilon_{ij}$  be a set of matrix units for  $eF\bar{G}$ . If  $x$  is in  $eBe$ , set

$$\pi_{ij}(x) = \sum_k \varepsilon_{ki} x \varepsilon_{jk}$$

$\pi_{ij}(x)$  commutes with each of the matrix units. Since  $F$  is the center of  $B$ , it commutes with  $eF\bar{G}$ . Thus it commutes with  $F\bar{G}$ . In other words,  $\pi_{ij}(x)$  is in  $B^G$ . The map  $\pi_{ij}: eBe \rightarrow B^G$  is a right  $B^G$ -module map by the argument at the beginning of this paragraph. We claim that the map

$$\sum_{i,j} \pi_{ij}: eBe \longrightarrow \bigoplus_{i,j} B^G$$

is injective. For if  $\sum_k \varepsilon_{ki} x \varepsilon_{jk} = 0$  for all  $i$  and  $j$ , multiple on the right by  $\varepsilon_{ij}$ :

$$\varepsilon_{ii} x \varepsilon_{jj} = 0 \text{ for all } i \text{ and } j.$$

Hence  $exe = 0$ . But  $x \in eBe$  implies  $exe = x$ . We finish this paragraph by noticing that Levitzki's theorem says that  $B^G$  is right noetherian. Since  $eBe$  is isomorphic to a submodule of a finitely generated  $B^G$ -module,  $eBe$  is finitely generated.

Next we show that if  $e$  and  $f$  are different elements of  $\mathcal{E}$  then  $fBe$  is a finitely generated right  $B^G$ -module. (Of course it is a  $B^G$ -module as above.) Since  $B$  is simple,  $BeB = B$ . Thus we can choose  $v_i \in fBe$  and  $u_i \in eBf$  so that

$$f = \sum_i v_i u_i .$$

Define  $\varphi: fBe \rightarrow \bigoplus \sum_i eBe$  by  $\varphi(y) = (u_i y)$ , a right  $B^G$ -module map.  $\varphi(y) = 0 \Rightarrow u_i y = 0$  for each  $i \Rightarrow (\sum v_i u_i) y = 0 \Rightarrow f y = 0$ . But  $f y = y$ . Hence  $\varphi$  is injective. Finish the argument as before.

Because  $B = \sum_{e, f \in \mathcal{E}} fBe$ ,  $B$  is a finitely generated right  $B^G$ -module.

*Proof of Theorem 1.* Induct on the order of  $G$ . Assume  $G$  is simple.

Let  $e$  be a centrally primitive idempotent in  $R$ .  $eR$  is a simple artinian ring. Moreover the stabilizer  $H = \text{Stab}_G(e)$  acts on  $eR$  and  $1/|H|e \in eR$ . By Lemmas 2 and 3,  $eR$  is a finitely generated right  $(eR)^H$ -module.

*Claim.*  $(eR)^H = e(R^G)$ .

Certainly  $e(R^G) \subseteq (eR)^H$ . Let  $G = \bigcup_{\gamma \in \Gamma} \gamma H$  be a coset decomposition of  $G$  with  $1 \in \Gamma$ .  $G$  permutes the centrally primitive idempotents of  $R$  and for  $\alpha \neq \beta$  in  $\Gamma$ ,  ${}^\alpha e \neq {}^\beta e$ . Equivalently, if  $\gamma \neq 1$  is in  $\Gamma$ ,  $e({}^\gamma e) = 0$ . If  $x \in (eR)^H$  define  $t_r(x) = \sum_{\gamma \in \Gamma} ({}^\gamma x)$ . If  $g \in G$ ,  $\{g\gamma \mid \gamma \in \Gamma\}$  are also coset representatives for  $H$ . Thus  ${}^g t_r(x) = t_r(x)$ . That is,  $t_r(x) \in R^G$ . But  $e t_r(x) = x$  by the remarks above about multiplying idempotents. Thus  $(eR)^H \subseteq e(R^G)$ .

We now know that  $eR$  is a finitely generated right  $e(R^G)$ -module. That means  $eR$  is a finitely generated  $R^G$ -module. Since  $R = \sum_e eR$ , we are done.

## 2. Theorem 2 and its relatives.

**LEMMA 4.** *Let  $A$  be a semiprime ring. Assume  $G$  is a finite group of automorphisms of  $A$  and  $A$  has no  $|G|$ -torsion. Then  $tr_G$  does not vanish on any nonzero right ideal of  $A$ .*

$$\text{(Here } tr_G(a) = \sum_{g \in G} ({}^g a)\text{.)}$$

*Proof.* Suppose  $I$  is a right ideal of  $A$  with  $tr_G(I) = 0$ . If  $J = \sum_{g \in G} {}^g I$  then  $J$  is a  $G$ -invariant right ideal of  $A$  with  $tr_G(J) = 0$ . By [2],  $J$  is nilpotent. But the only nilpotent right ideal in a semi-prime ring is 0.

*Proof of Theorem 2.*  $S^G$  is left Goldie, so according to [6],  $S$  is (semiprime) left Goldie. Let  $R$  be the left quotient ring for  $S$ ;  $R$  is semisimple artinian. By Theorem 1 we can find a finite set of generators  $x_1, \dots, x_n$  for  $R$  as a right  $R^G$ -module. Choose a regular  $t$  and  $s_i$  both in  $S$  such that  $x_i = t^{-1}s_i$ .

$R = \sum_{i=1}^n t^{-1}s_i R^G \Rightarrow tR = \sum_i s_i R^G$ . But  $tR = R$  since  $t$  is invertible. Thus we assume  $x_i \in S$ .

Define  $T: S \rightarrow \bigoplus \sum_{i=1}^n S^G$  by  $T(a) = [tr_G(ax_i)]_{i=1}^n$ .  $T$  is clearly a left  $S^G$ -module map. We will be done once we prove that  $T$  is injective.

$T(a) = 0$  implies  $tr_G(ax_i) = 0$  for all  $i$ . But  $tr_G$  is a right  $R^G$ -module map. Thus  $tr_G(aR) = 0$ . By the previous lemma,  $a = 0$ .

We have actually proved that  $S$  is a finitely generated  $S^G$ -module!

One might well ask whether the requirement that  $S$  have no  $|G|$ -torsion can be dropped. Consider the following counterexample. Let  $F$  be a field of characteristic  $p > 2$  and let  $\Phi$  be the free group on  $x$  and  $y$ . If  $S$  denotes the ring of two-by-two matrices over the group algebra  $F[\Phi]$  then  $S$  is semiprime but not noetherian. Let  $G$  be the multiplicative subgroup of  $S$  generated by

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix}.$$

$G$  is isomorphic to the semidirect product of  $Z/p \oplus Z/p \oplus Z/p$  with  $Z/2$ . Since  $\text{char } F \neq 2$ ,  $S^{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}$  is the collection of diagonal matrices. The only diagonal matrices fixed by  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  are the scalar matrices. Now a simple calculation shows that  $S^G$  consists of those scalars in the center of  $F[\Phi]$ . But it is well known that the center is  $F$ , a patently noetherian ring.

However, the  $|G|$ -torsion restriction is not needed when  $S$  is (semiprime) commutative or, more generally, when  $S$  has no nilpotent elements. There are several difficulties in proving the last statement along the lines of Theorem 2. First, there are division rings on which  $tr_G$  vanishes. Even if this objection is met, our induction and restriction techniques all ignore the question of fidelity of action. Reconsider, for instance, Lemma 4. The Bergman-Isaacs theorem states that if  $H$  is a group of automorphisms of  $J$  and  $tr_H(J) = 0$

then  $J = 0$ . Thus implicit in our argument is the proposition that  $tr_G(J) = 0 \Rightarrow tr_{G/K}(J) = 0$  where  $K$  is the kernel of the action of  $G$  on  $J$ . The implication is true because  $J$  has no  $|K|$ -torsion.

We avoid these complications (and, of course, replace them with other complications) by refining the notion of trace. Let  $G$  be a finite group acting on a ring  $R$ . If  $\wedge$  is a subset of  $G$  define  $t_\wedge: R \rightarrow R$  by

$$t_\wedge(r) = \sum_{\lambda \in \wedge} (\lambda r).$$

$t_\wedge$  is an  $R^G$ -bimodule map. Notice that  $tr_G \equiv t_G$ .

**LEMMA 5.** *Let  $G$  be a finite group acting on the division ring  $D$ . Then there is a subset  $\wedge \subseteq G$  such that  $t_\wedge$  is a mapping from  $D$  onto  $D^G$ .*

*Proof.* Suppose we can find  $\wedge$  such that  $t_\wedge$  is a nonzero function from  $D$  into  $D^G$ . Say  $d \in D$  such that  $t_\wedge(d) = w \neq 0$ . If  $x \in D^G$ ,  $t_\wedge(dw^{-1}x) = t_\wedge(d)w^{-1}x = x$ . Thus  $t_\wedge$  is surjective.

We argue by induction on the length of a composition series for  $G$ . If  $G$  is simple and does not act faithfully then  $G$  acts trivially; choose  $\wedge = \{1\}$ . If  $G$  is simple group of automorphisms, a result of Faith ([3]) shows that  $t_G$  is not identically zero.

When  $G$  is not simple choose  $H \triangleleft G$  with  $H \neq 1$  and  $H \neq G$ . By induction there is a subset  $A \subseteq H$  such that  $t_A: D \rightarrow D^H$  is surjective.  $G/H$  acts on  $D^H$ , so we can find  $C \subseteq G/H$  such that  $t_C: D^H \rightarrow D^G$  is surjective. If  $B$  consists of representatives in  $G$  for elements of  $C$  then  $t_C = t_B$ . Now  $t_{B \cdot A} = t_B \cdot t_A$  is the desired map.

Let  $S$  be a ring without nilpotent elements. Suppose  $G$  is a finite group of automorphisms of  $S$  such that  $S^G$  is left noetherian. By [7]  $S$  is a semiprime left Goldie ring. By the Faith-Utumi theorem the quotient ring,  $R$ , of  $S$  has no nilpotent elements. Let  $e$  be a centrally primitive idempotent of  $R$ .

**LEMMA 6.**  *$S \cap eR$  is a finitely generated left  $S^G$ -module.*

*Proof.* We first observe that the left quotient ring of  $S \cap eR$  in  $eR$  is the entire division ring  $eR$ . Choose  $z$  and  $s$  in  $S$  with  $z$  regular such that  $e = z^{-1}s$ . Then  $s = ze \in S \cap eR$ . If  $x \in eR$  choose  $q$  and  $w$  in  $S$  with  $q$  regular such that  $qx = w$ . Then  $(sq)x = sw$ . But  $sq$  and  $sw$  are in  $S \cap eR$  with  $sq$  regular when considered as an element in  $eR$ .

$H = \text{Stab}_G(e)$  is a group which acts on  $S \cap eR$ . Pick a transversal,  $G = \Gamma \cdot H$ . As in Theorem 1, if  $a \in S^H \cap eR$  then

$$t_r(a) \in S^G \quad \text{and} \quad e \cdot t_r(a) = a .$$

Thus  $t_r$  is an injective left  $S^G$ -module map from  $S^H \cap eR$  into  $S^G$ .

The Galois theory for division rings ([5]) as applied to  $eR$  implies that  $eR$  is a finite dimensional right  $(eR)^H$ -vector space. As in the proof of Theorem 2 we can choose a basis  $x_1, \dots, x_n$  in  $S \cap eR$ . Use Lemma 5 to find  $\wedge \subseteq H$  so that  $t_\wedge$  is nondegenerate on  $eR$ . Define  $T: S \cap eR \rightarrow \bigoplus \sum_{i=1}^n S^G$  by

$$T(a) = [t_{r \cdot \wedge}(ax_i)]_{i=1}^n .$$

It is easy to check that  $T$  is a well defined left  $S^G$ -module map. The lemma is completed by showing that  $T$  is injective. Suppose  $a \neq 0$  and  $T(a) = 0$ . Then  $t_r \cdot t_\wedge(ax_i) = 0$  for each  $i$ . Since  $t_r$  is injective,  $t_\wedge(ax_i) = 0$  for each  $i$ . That is,  $t_\wedge(a \cdot eR) = 0$ . But  $eR$  is a division ring:  $a \cdot eR = eR$ . We have contradicted the nonvanishing of  $t_\wedge$ .

**THEOREM 3.** *Let  $S$  be a ring without nilpotent elements. If  $G$  is a finite group of automorphisms of  $S$  and  $S^G$  is left noetherian then  $S$  is left noetherian (in fact, is finitely generated as an  $S^G$ -module).*

*Proof.* So far we have proved that  $\sum_e (S \cap eR)$  is a finitely generated left  $S^G$ -module, where the sum is taken over the centrally primitive idempotents of  $R$ .

As observed in the first paragraph of Lemma 6,  $S \cap eR$  contains an element invertible in  $eR$ . Consequently there is an element  $d \in \Sigma(S \cap eR)$  which is invertible in  $R$ . Define  $f: S \rightarrow \Sigma(S \cap eR)$  by  $f(s) = sd$ . Since  $f$  is an injective left  $S^G$ -module map,  $S$  is a finitely generated left  $S^G$ -module.

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