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**A FORMULA FOR THE NORMAL PART OF THE
LAPLACE-BELTRAMI OPERATOR ON THE FOLIATED
MANIFOLD**

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A FORMULA FOR THE NORMAL PART OF THE LAPLACE-BELTRAMI OPERATOR ON THE FOLIATED MANIFOLD

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In this paper, we give a formula for the normal part of the Laplace-Beltrami operator with respect to the second connection on a foliated manifold with a bundle-like metric. This formula is analogous to the formula obtained by S. Helgason.

1. Introduction. We shall be in C^∞ -category and manifolds are supposed to be paracompact, connected Hausdorff spaces.

Let M be a complete $(p + q)$ -dimensional Riemannian manifold and H a compact subgroup of the Lie group of all isometries of M . We suppose that all orbits of H have the same dimension p . Then H defines a p -dimensional foliation F whose leaves are orbits of H , and the Riemannian metric is a bundle-like metric with respect to the foliation F . A quotient space $B = M/F$ is a Riemannian V -manifold [5]. Let L_D be the Laplace-Beltrami operator on M with respect to the second connection D [8], and let $\Delta(L_D)$ denote the operator defined by (*) in §4. Our goal in this paper is the following theorem:

THEOREM. *Let L_D be the Laplace-Beltrami operator on M with respect to the second connection D and L_B the Laplace-Beltrami operator on B with respect to the Levi-Civita connection associated with the Riemannian metric defined by the normal component of the metric on M . Then*

$$\Delta(L_D) = \delta^{-1/2} L_B \circ \delta^{1/2} - \delta^{-1/2} L_B (\delta^{1/2})$$

where δ is the function given by (**) below.

This theorem is analogous to the following result obtained by S. Helgason [2]: *Suppose V is a Riemannian manifold, H a closed unimodular subgroup of the Lie group of all isometries of V (with the compact open topology). Let $W \subset V$ be a submanifold satisfying the condition: For each $w \in W$,*

$$(H \cdot w) \cap W = \{w\}, \quad V_w = (H \cdot w)_w \oplus W_w,$$

where \oplus denotes orthogonal direct sum. Let L_V and L_W denote the Laplace-Beltrami operators on V and W , respectively. Then

$$\Delta(L_V) = \delta^{-1/2} L_W \circ \delta^{1/2} - \delta^{-1/2} L_W (\delta^{1/2})$$

where $\Delta(L_V)$ denotes the operator called the radial part of L_V and δ is the function given by $d\sigma_w = \delta(w)dh$ ($d\sigma_w$ is the Riemannian volume element on the orbit $H \cdot w$ and dh is an H -invariant measure on each orbit $H \cdot w = H/(\text{the isotropy subgroup of } H \text{ at } w)$).

2. Definition of V -manifold [1, 6, 7]. The concept of V -manifold is defined by I. Satake. Let M be a Hausdorff space. A C^∞ -local uniformizing system $\{\tilde{U}, G, \varphi\}$ for an open set U in M is a collection of the following objects:

\tilde{U} : a connected open set in the m -dimensional Euclidean space (or C^∞ -manifold).

G : a finite group of C^∞ -transformations of \tilde{U} .

φ : a continuous map from \tilde{U} onto U such that $\varphi \circ \sigma = \varphi$ for all $\sigma \in G$, inducing a homeomorphism from the quotient space \tilde{U}/G onto U .

Let $\{\tilde{U}, G, \varphi\}$, $\{\tilde{U}', G', \varphi'\}$ be local uniformizing systems for U , U' respectively, and let $U \subset U'$. By a C^∞ -injection $\lambda: \{\tilde{U}, G, \varphi\} \rightarrow \{\tilde{U}', G', \varphi'\}$ we mean a C^∞ -isomorphism from \tilde{U} onto an open subset of \tilde{U}' such that for any $\sigma \in G$ there exists $\sigma' \in G'$ satisfying relations $\varphi = \varphi' \circ \lambda$ and $\lambda \circ \sigma = \sigma' \circ \lambda$.

A C^∞ - V -manifold consists of a connected Hausdorff space M and a family \mathcal{F} of C^∞ -local uniformizing systems for open subsets in M satisfying the following conditions:

(I) If $\{\tilde{U}, G, \varphi\}$, $\{\tilde{U}', G', \varphi'\} \in \mathcal{F}$ and $U \subset U'$, then there exists a C^∞ -injection $\lambda: \{\tilde{U}, G, \varphi\} \rightarrow \{\tilde{U}', G', \varphi'\}$.

(II) The open sets U , for which there exists a local uniformizing system $\{\tilde{U}, G, \varphi\} \in \mathcal{F}$, form a basis of open sets in M .

The set R of all real numbers is regarded as a V -manifold defined by a single local uniformizing system $\{R, \{1\}, 1\}$, then a C^∞ -function on a V -manifold (M, \mathcal{F}) is defined as a C^∞ -map $M \rightarrow R$ defined by a C^∞ - V -manifold map $(M, \mathcal{F}) \rightarrow (R, \{R, \{1\}, 1\})$.

A C^∞ - V -bundle over C^∞ - V -manifold is also defined, and in particular the tangent bundle (TM, \mathcal{F}^*) of a C^∞ - V -manifold (M, \mathcal{F}) is defined. Let (M, \mathcal{F}) be a C^∞ - V -manifold, then an h -form ω on (M, \mathcal{F}) is a collection of h -forms $\{\omega_{\tilde{v}}\}$, where $\omega_{\tilde{v}}$ is a G -invariant h -form on \tilde{U} such that $\omega_{\tilde{v}} = \omega_{\tilde{v} \circ \lambda}$ for any injection $\lambda: \{\tilde{U}, G, \varphi\} \rightarrow \{\tilde{U}', G', \varphi'\}$ ($\{\tilde{U}, G, \varphi\}, \{\tilde{U}', G', \varphi'\} \in \mathcal{F}$), and if the support of ω is contained in $U = \varphi(\tilde{U})$,

$$\int_M \omega = \frac{1}{N_G} \int_{\tilde{v}} \omega_{\tilde{v}},$$

where N_G denotes the order of G . A Riemannian metric g on (M, \mathcal{F}) is a collection of Riemannian metrics $\{g_{\tilde{v}}\}$, where $g_{\tilde{v}}$ is a G -invariant Riemannian metric on \tilde{U} satisfying some condition with

any injection $\lambda: \{\tilde{U}, G, \varphi\} \rightarrow \{\tilde{U}', G', \varphi'\}$.

3. Review of the results from [4, 5]. Let M be a complete $(p + q)$ -dimensional manifold with a "bundle-like metric" with respect to a p -dimensional foliation F . We suppose that each leaf of the foliation F is closed.

The quotient space $B = M/F$ is the space formed from M by identifying each leaf to a point, and let $\pi: M \rightarrow B$ denote the identification map. $H(S)$ denotes the holonomy group of a leaf S . Since M has the bundle-like metric with respect to F and all leaves are closed, $H(S)$ is a finite group for any S and B is a metric space defining the distance between two points of B to be the minimum distance between them considered as leaves in M . B is a connected Hausdorff space, since it is metric space and is the continuous image of M under π . Given any point $b \in B$, let $S = \pi^{-1}(b)$. Let U be a flat coordinate neighborhood of some point of S . Since $H(S)$ may be considered as a group of isometries of the sphere of unit vectors orthogonal to the leaf S at some arbitrary point of S , $H(S)$ operates the q -ball orthogonal to S . Thus we may consider that $H(S)$ operates on U such a manner that $\{U, H(S), \pi\}$ is a local uniformizing system for the neighborhood $\pi(U)$ in B . The natural injection map of two such local uniformizing systems are of C^∞ . Thus B is a C^∞ - V -manifold. Since $H(S)$ is an isometry on the normal vectors at a point of S , the normal component of the metric of M defines a Riemannian structure on B . Thus B is a Riemannian V -manifold.

4. Laplace-Beltrami operator with respect to the second connection. Let M be a $(p + q)$ -dimensional manifold with a Riemannian metric $\langle \cdot, \cdot \rangle$ and a p -dimensional foliation F . Let $(U, (x^1, \dots, x^p, y^1, \dots, y^q))$ be a flat coordinate neighborhood system, that is, in U , the foliation F is defined by $dy^\alpha = 0$ for $1 \leq \alpha \leq q$. Hereafter we will agree on the following ranges of indices: $1 \leq i, j, k \leq p, 1 \leq \alpha, \beta, \gamma, \delta \leq q$.

We may choose in each flat coordinate neighborhood system $(U, (x^1, \dots, x^p, y^1, \dots, y^q))$ 1-forms w^1, \dots, w^p such that $\{w^1, \dots, w^p, dy^1, \dots, dy^q\}$ is a basis for the cotangent space, and vectors v_1, \dots, v_q such that $\{\partial/\partial x^i, \dots, \partial/\partial x^p, v_1, \dots, v_q\}$ is the dual base for the tangent space. Then we may get

$$w^i = dx^i + A_\alpha^i dy^\alpha, \quad v_\alpha = \frac{\partial}{\partial y^\alpha} - A_\alpha^i \frac{\partial}{\partial x^i}.$$

We may choose A_α^i such that $\langle \partial/\partial x^i, v_\alpha \rangle = 0$, then the metric has the local expression

$$ds^2 = g_{ij}(x, y)w^i w^j + g_{\alpha\beta}(x, y)dy^\alpha dy^\beta$$

where

$$g_{ij} = \left\langle \frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j} \right\rangle, \quad g_{\alpha\beta} = \langle v_\alpha, v_\beta \rangle$$

and $x = (x^1, \dots, x^p)$, $y = (y^1, \dots, y^q)$.

We may uniquely define the "second connection" D on M as follows (cf. [8]):

$$(a) \quad D_{\partial/\partial x^i} \frac{\partial}{\partial x^j} = \Gamma_{ij}^k \frac{\partial}{\partial x^k}, \quad D_{v_\alpha} \frac{\partial}{\partial x^j} = \Gamma_{\alpha j}^k \frac{\partial}{\partial x^k},$$

$$D_{\partial/\partial x^i} v_\beta = \Gamma_{i\beta}^\gamma v_\gamma, \quad D_{v_\alpha} v_\beta = \Gamma_{\alpha\beta}^\gamma v_\gamma,$$

$$(b) \quad \frac{\partial}{\partial x^i} \left\langle \frac{\partial}{\partial x^j}, \frac{\partial}{\partial x^k} \right\rangle = \left\langle D_{\partial/\partial x^i} \frac{\partial}{\partial x^j}, \frac{\partial}{\partial x^k} \right\rangle + \left\langle \frac{\partial}{\partial x^j}, D_{\partial/\partial x^i} \frac{\partial}{\partial x^k} \right\rangle,$$

$$v_\alpha \langle v_\beta, v_\gamma \rangle = \langle D_{v_\alpha} v_\beta, v_\gamma \rangle + \langle v_\beta, D_{v_\alpha} v_\gamma \rangle,$$

$$(c) \quad T\left(\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j}\right) = T_{ij}^\gamma v_\gamma, \quad T\left(\frac{\partial}{\partial x^i}, v_\beta\right) = 0,$$

$$T\left(v_\alpha, \frac{\partial}{\partial x^j}\right) = 0, \quad T(v_\alpha, v_\beta) = T_{\alpha\beta}^k \frac{\partial}{\partial x^k},$$

where T denotes the torsion of D , that is, for any vector fields X, Y on M , $T(X, Y) := D_X Y - D_Y X - [X, Y]$ ($[,]$ denotes the usual bracket operator). Note that, in general, the torsion of D doesn't vanish. If the metric has the local expression

$$ds^2 = g_{ij}(x, y)w^i w^j + g_{\alpha\beta}(y)dy^\alpha dy^\beta,$$

the metric is called a "bundle-like metric" with respect to the foliation F . Hereafter we suppose that M has a bundle-like metric with respect to F . Then we get

$$\frac{\partial}{\partial x^i} \langle v_\alpha, v_\beta \rangle = \langle D_{\partial/\partial x^i} v_\alpha, v_\beta \rangle + \langle v_\alpha, D_{\partial/\partial x^i} v_\beta \rangle.$$

For a vector field X on M , $\text{div}_D X$ is defined by

$$\text{div}_D X := \text{Trace} (Y \longrightarrow D_Y X),$$

for any vector field Y on M . For a function f on M , $\text{grad}_D f$ is defined by

$$\begin{aligned} \text{grad}_D f &:= (\tilde{g}^{ij} D_{\partial/\partial x^j} f) \frac{\partial}{\partial x^i} + (\tilde{g}^{\alpha\beta} D_{v_\beta} f) v_\alpha \\ &= \left(\tilde{g}^{ij} \frac{\partial}{\partial x^j} (f) \right) \frac{\partial}{\partial x^i} + (\tilde{g}^{\alpha\beta} v_\beta (f)) v_\alpha \end{aligned}$$

where (\tilde{g}^{ij}) and $(\tilde{g}^{\alpha\beta})$ are inverse matrices of (g_{ij}) and $(g_{\alpha\beta})$ respectively. We define the Laplace-Beltrami operator L_D with respect to the second connection D by

$$L_D(f) := \operatorname{div}_D \operatorname{grad}_D f,$$

that is,

$$\begin{aligned} L_D(f) &= \tilde{g}^{ij} \frac{\partial}{\partial x^i} \left(\frac{\partial}{\partial x_j} (f) \right) - \tilde{g}^{ij} \Gamma_{ij}^k \frac{\partial}{\partial x^k} (f) \\ &\quad + \tilde{g}^{\alpha\beta} v_\alpha (v_\beta (f)) - \tilde{g}^{\alpha\beta} \Gamma_{\alpha\beta}^\gamma v_\gamma (f). \end{aligned}$$

Let B be the C^∞ - V -manifold M/F . Let $\mathcal{E}(B)$ (resp. $\mathcal{D}(B)$) be the space of C^∞ -functions (resp. C^∞ -functions of compact support) on B , and let $\mathcal{E}_s(M)$ be the space of C^∞ -functions on M which are constants on leaves. We may define a map $\Phi: \mathcal{E}_s(M) \rightarrow \mathcal{E}(B)$ by $\Phi(f)(\pi(m)) := f(m)$ where $f \in \mathcal{E}_s(M)$, $m \in M$ and $\pi: M \rightarrow B$, then Φ is of one-to-one. Let $\mathcal{E}_s^0(M) := \Phi^{-1}(\mathcal{D}(B))$.

It is clear that $f \in \mathcal{E}_s(M)$ if and only if $\partial/\partial x^i(f) = 0$ for $1 \leq i \leq p$.

LEMMA. *If $f \in \mathcal{E}_s(M)$, then $L_D(f) \in \mathcal{E}_s(M)$.*

Proof. For $f \in \mathcal{E}_s(M)$, we get

$$L_D(f) = \tilde{g}^{\alpha\beta} v_\alpha (v_\beta (f)) - \tilde{g}^{\alpha\beta} \Gamma_{\alpha\beta}^\gamma v_\gamma (f).$$

Since $g_{\alpha\beta} = g_{\alpha\beta}(y)$ and $\Gamma_{\alpha\beta}^\gamma = (1/2)\tilde{g}^{\gamma\delta}\{v_\alpha(g_{\delta\beta}) + v_\beta(g_{\alpha\delta}) - v_\delta(g_{\alpha\beta})\}$, we get $\tilde{g}^{\alpha\beta} = \tilde{g}^{\alpha\beta}(y)$ and so $\partial/\partial x^i(L_D(f)) = 0$. Thus we get $L_D(f) \in \mathcal{E}_s(M)$.

REMARK. Let L be the Laplace-Beltrami operator with respect to the Levi-Civita connection associated with the bundle-like metric. In general $L(f) \notin \mathcal{E}_s(M)$ for $f \in \mathcal{E}_s(M)$.

For L_D and $f \in \mathcal{E}(B)$, we define $\Delta(L_D)$ by

$$(*) \quad \Delta(L_D)(\underline{f})(b) := L_D(\Phi^{-1}(\underline{f}))(\pi^{-1}(b)), \quad b \in B.$$

This is well-defined by lemma. Roughly speaking, $\Delta(L_D)$ seems to be an operator projected on B of the normal part of L_D .

5. Proof of theorem. Using the same notations as above sections, we give a proof of our theorem.

The isotropy subgroup H_m at each point $m \in M$ is compact and the orbit $H \cdot m$ is compact. We fix a Haar measure on H and a Haar measure on H_m , we get an H -invariant measure $d\hat{h}$ on each orbit $H \cdot m = H/H_m$. Since M has the bundle-like metric, $ds^2 = g_{ij}(x, y)w^i w^j + g_{\alpha\beta}(y)dy^\alpha dy^\beta$, the volume element dM of M is given by

$$\begin{aligned} dM &= G(x, y) dx^1 \wedge \cdots \wedge dx^p \wedge dy^1 \wedge \cdots \wedge dy^q \\ &= G(x, y) w^1 \wedge \cdots \wedge w^p \wedge dy^1 \wedge \cdots \wedge dy^q \end{aligned}$$

where

$$G(x, y) := \sqrt{\det \begin{pmatrix} g_{\alpha\beta} & j\mathbf{0} \\ 0 & g_{\alpha\beta} \end{pmatrix}}.$$

For a flat coordinate system $(U, (x^1, \dots, x^p, y^1, \dots, y^q))$ and the projection $\pi: M \rightarrow B$,

$$d\sigma = G'(y) dy^1 \wedge \cdots \wedge dy^q,$$

where $G'(y) := \sqrt{|\det(g_{\alpha\beta})|}$, is regarded as the volume element dB of B , since $\{U, H(S), \pi\}$ is a local uniformizing system for $\pi(U)$ in B . Also we get

$$G(x, y) = \sqrt{|\det(g_{ij}(x, y))|} \cdot G'(y).$$

However

$$\sqrt{|\det(g_{ij}(x, y))|} w^1 \wedge \cdots \wedge w^p$$

is the volume element dS_m on the leaf S_m through a point $m = (x, y)$ (that is, on the orbit $H \cdot m$). Thus, if $f \in \mathcal{E}_s^0(M)$ we get from the Fubini's theorem that

$$\int_M f dM = \int_B \left[\int_{H \cdot m} f dS_m \right] dB(\pi(m))$$

where “ $\underline{\quad}$ ” denotes the image under Φ . dS_m is invariant under H , so it must be a scalar multiple of $d\dot{h}$,

$$dS_m = \bar{\delta}(m) d\dot{h}.$$

Then the function $\bar{\delta}$ belongs to $\mathcal{E}_s(M)$. We put

$$(**) \quad \delta := \Phi(\bar{\delta}).$$

Thus we get

$$\int_K f dM = \int_B \left[\int_{H \cdot m} f(h \cdot m) d\dot{h} \right] \delta(\pi(m)) dB(\pi(m)).$$

The normal component of the bundle-like metric $ds^2 = g_{ij}(x, y) w^i w^j + g_{\alpha\beta}(y) dy^\alpha dy^\beta$ is $ds_N^2 = g_{\alpha\beta}(y) dy^\alpha dy^\beta$, thus L_B is defined by the Levi-Civita connection associated with the metric defined from ds_N^2 . Thus we observe that

$$\Delta(L_D) = L_B + \text{lower order terms}.$$

The operator L_D restricted to $\mathcal{E}_s^0(M)$ is symmetric with respect to dM (cf. [8]), that is,

$$(***) \quad \int_M L_D(f_1)f_2dM = \int_M f_1L_D(f_2)dM$$

for $f_1, f_2 \in \mathcal{E}_s^0(M)$.

For $f \in \mathcal{E}_s(M)$ and $m \in M$, we get

$$\int_{H \cdot m} f d\dot{h} = \underline{f}(\pi(m))c$$

where c denotes a nonzero constant $\int_{H \cdot m} d\dot{h}$. Putting $\underline{f}_1 = \Phi(f_1)$, $\underline{f}_2 = \Phi(f_2)$ for $f_1, f_2 \in \mathcal{E}_s^0(M)$, we get

$$\begin{aligned} \int_M L_D(f_1)f_2dM &= \int_B \left[\int_{H \cdot m} L_D(f_1)f_2d\dot{h} \right] \delta dB \\ &= \int_B \left[\int_{H \cdot m} L_D(f_1)d\dot{h} \right] c \delta \underline{f}_2 dB \\ &= c^2 \int_B \underline{L_D(f_1)} \underline{f}_2 \delta dB. \end{aligned}$$

Thus we get from (***)

$$\int_B \underline{L_D(f_1)} \underline{f}_2 \delta dB = \int_B \underline{f_1} \underline{L_D(f_2)} \delta dB$$

for $f_1, f_2 \in \mathcal{E}_s^0(M)$. By the definition of $\Delta(L_D)$ we get $\underline{L_D(f)} = \Delta(L_D)(\underline{f})$ for $f \in \mathcal{E}_s(M)$, so

$$\int_B \Delta(L_D)(\underline{f_1}) \underline{f_2} \delta dB = \int_B \underline{f_1} \Delta(L_D)(\underline{f_2}) \delta dB.$$

This expression implies that $\Delta(L_D)$ is symmetric with respect to δdB . Since L_B is symmetric with respect to dB , $\delta^{-1/2}L_B \circ \delta^{1/2}$ is symmetric with respect to δdB and it clearly agrees with L_B up to lower order terms. The symmetric operators $\Delta(L_D)$ and $\delta^{-1/2}L_B \circ \delta^{1/2}$ agree up to an operator of order ≤ 1 , thus this operator, being symmetric, must be a function. By applying the operators to the constant function 1, we get

$$\Delta(L_D)(1) - \delta^{-1/2}L_B \circ \delta^{1/2}(1) = -\delta^{-1/2}L_B(\delta^{1/2}).$$

Thus

$$\Delta(L_D) = \delta^{-1/2}L_B \circ \delta^{1/2} - \delta^{-1/2}L_B(\delta^{1/2}).$$

This completes the proof of our theorem.

REMARK. The example of “RS-manifold of almost fibered type”

given by S. Kashiwabara (Appendix 5 in [3]) is a foliated manifold with a 1-dimensional foliation and bundle-like metric. Each leaf of the foliation is a "S-geodesic." This example is constructed from the space D which consists of all points $x_1e_1 + x_2e_2 + x_3e_3 + te_4$ such that $|x_i| \leq 1 (i = 1, 2, 3)$, $0 \leq t \leq 1$, where (e_1, e_2, e_3, e_4) denotes an orthonormal frame with origin o in Euclidean 4-space. If S-geodesics are of direction of e_4 , a leaf through the origin o has nontrivial holonomy group. Then $\delta = 1$.

REMARK. The semi-reducible Riemannian space are a special class of foliated manifolds with bundle-like metrics. The metric of such a space has the local expression

$$d s^2 = \sigma(y)q_{ij}(x)dx^i dx^j + g_{\alpha\beta}(y)dy^\alpha dy^\beta$$

(cf. [4]). Then δ is defined from σ .

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