Pacific Journal of Mathematics

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Vol. 70, No. 2 October 1977

PACIFIC JOURNAL OF MATHEMATICS
Vol. 70, No. 2, 1977

ON PRESERVATION OF E-COMPACTNESS

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In this paper we study preservation of E-compactness under taking finite unions (the finite additivity theorems of E-compactness) and under taking quotient images.

Throughout this paper spaces are assumed to be Hausdorff, and maps are continuous onto functions. Given a space E, we shall call a space X *E-completely regular* (*E-compact*) provided that X is homeomorphic to a subspace (respectively, closed subspace) of a product E^m for some cardinal m.

As far as additivity theorems are concerned, the first author has shown in **[1]** that *if a space X is normal and if it can be expressed as the union of a countable collection of closed R-compact spaces* (*R denotes the space of all real numbers*), *then X is R-compact.* The assumption that X is normal in the above theorem is essential. In fact, in **[2]**, **[4]** the first author has constructed an example of a completely regular, non-R-compact space X which can be expressed as the union of two closed R-compact subspaces. This example shows that finite additivity relative to closed subspaces fails for R-compactness. It can be shown that the same example satisfies the above statement with "R-compact" replaced by "N-compact". (N denotes the space of all nonnegative integers.) Using the same example it was shown that the image of an R-compact (N-compact) space under a perfect map need not be R-compact (respectively, N-compact). In **[4]**, some positive results in this direction have been obtained. The purpose of this paper is to generalize some of the results in **[4]** to a certain class of E-compact spaces which contains both the class of R-compact spaces and the class of N-compact spaces. Many theorems concerning the preservation of E-compactness can be stated in a more comprehensive form as rules concerning "E-defect" of spaces (for definition of E-defect, see next paragraph). In §2 we shall state the additivity theorems of E-compactness both in words and as rules concerning E-defects of spaces.

The reader is referred to **[3]** for basic results of E-completely regular spaces and E-compact spaces. For convenience we review the notations and terminology. Given two spaces X and E, $C(X, E)$ denotes the set of all continuous functions from X into E. A class $\mathscr{F} \subseteq C(X, E)$ is called an *E-non-extendable class* for X provided that there is no proper extension ϵX of X such that every $f \in \mathscr{F}$ admits a continuous extension $f^*: \epsilon X \rightarrow E$. The *$E$-defect* of a space X (in symbols, $\mathrm{def}_E X$) is the smallest (finite or infinite) cardinal p such that there exists an E-nonextendable class for X of cardinal p. A subspace X_0 of a space X is said to be *complementatively E-compact* in X provided that every closed subspace of X disjoint from X_0 is E-compact. X_0 is said to be *E-embedded* in X provided that every continuous function $f\colon X_0 \to E$ admits a continuous extension $f^*\colon X \to E$. For two subsets A, B of a space X, B is said to be *E-functionally contained* in A (in symbols, $B \subset_f A$) provided that there exists a map $g\colon X \to E$ such that

$$\mathrm{cl}(g(X-A)) \cap \mathrm{cl}(g(B)) = \varnothing.$$

It should be noted that in §§2 and 3, E is assumed to satisfy a set of rather complex conditions; a way of avoiding these conditions is indicated in §4.

2. Additivity theorems of E-compactness. In §§2 and 3 we assume that E is a space with a continuous binary operation θ and two fixed distinct points e_0 and e_1 satisfying the following properties:

(α) *$e\theta e_0 = e_0$, $e\theta e_1 = e$ for every $e \in E$.*

(β) *for every closed subset A of E^n ($n \in N$) and for every $p \in E^n - A$, there exists an $f \in C(E^n, E)$ such that $f(A) = e_0$ and $f(p) = e_1$.*

(γ) *for every two disjoint closed subsets A, B of E, there exists a $g \in C(E, E)$ such that $g(A) = e_0$ and $g(B) = e_1$.*

We first observe the following results.

2.1. *If E satisfies (β), then it is regular and if it satisfies (γ), then it is normal.*

2.2. *Let E be a space satisfying (β). Then X is E-completely regular iff for every closed subset F of X and every point $x \in X - F$, there exists an $f \in C(X, E)$ such that $f(X) = e_1$, $f(F) = e_0$.*

2.3. *Let E be a space satisfying (β) and (γ). Then X is E-completely regular iff for every closed subset F of X and every point $x \in X - F$, there exist two disjoint neighborhoods U and V of x and F, respectively, and a map $g \in C(X, E)$ such that $g(U) = e_1$, $g(V) = e_0$.*

2.4. *Let E be a space satisfying (γ). Then for two subsets A and B of X, $B \subset_f A$ iff there exists a map $g \in C(X, E)$ such that $g(X-A) = e_0$ and $g(B) = e_1$.*

2.5. *Let E be a space satisfying (α), (β) and (γ). If A, B are two closed subsets of X with $B \subset_f A$, then for each $f \in C(A, E)$, there is an $f' \in C(A, E)$ such that f' admits a continuous extension $f^* \in C(X, E)$ such that $f^* | B = f | B$.*

Proof. 2.1–2.4 are straightforward. We now prove 2.5. By 2.4, there exists a map $g \in C(X, E)$ such that $g(X - A) = e_0$ and $g(B) = e_1$. Let $f \in C(A, E)$ be given. We define f': $A \to E$ as follows $f'(x) = f(x)\theta g(x)$ for every $x \in A$. Clearly $f' \in C(A, E)$. Then f^* can be defined by letting $f^*(x) = f'(x)$ for $x \in A$ and $f^*(x) = e_0$ for $x \in X - A$.

From now on all spaces will be assumed to be E-completely regular. We first prove two lemmas which are needed for the proof of our main theorems.

2.6. LEMMA. *An E-compact, E-embedded subspace X_0 of an E-completely regular space X is closed in $\beta_E X$.*

Proof. Since X_0 is E-compact, $\beta_E X_0 = X_0$. Hence it suffices to show that $\mathrm{cl}_{\beta_E X} X_0 = \beta_E X_0$. First, $\mathrm{cl}_{\beta_E X} X_0$ is obviously E-compact. Also, since X_0 is E-embedded in X, it is also E-embedded in $\beta_E X$, so it is E-embedded in $\mathrm{cl}_{\beta_E X} X_0$. Thus by 4.14 (a), (b) of **[3]**, $\mathrm{cl}_{\beta_E X} X_0 = \beta_E X_0$.

2.7. LEMMA. *If a space X contains a complementatively E-compact subspace X_0 which is closed in $\beta_E X$, then X is E-compact.*

Proof. Assume that X is not E-compact. Choose a point p_0 in $\beta_E X - X$ and let $\epsilon X = X \cup \{p_0\}$. Then ϵX is a proper extension of X and X is E-embedded in ϵX. Clearly, $p_0 \notin X_0$ and X_0 is closed in ϵX. By 2.3, there exist a map $g \in C(\epsilon X, E)$ and two disjoint neighborhoods U and V in ϵX of p_0 and X_0, respectively, such that $g(U) = e_1$ and $g(V) = e_0$. We claim that $X - V$ is not E-compact. First note that $p_0 \in \mathrm{cl}_{\epsilon X}(X - V)$. Now given $f \in C(X - V, E)$, we define a map $h: X \to E$ as follows: $h(x) = f(x)\theta g(x)$ for $x \in X - V$ and $h(x) = e_0$ for $x \in V$. One easily verifies that $h \in C(X, E)$ and consequently h admits a continuous extension $h^* \in C(\epsilon X, E)$. Now for any $x \in U \cap X$, we have $h^*(x) = h(x) = f(x)\theta g(x) = f(x)\theta e_1 = f(x)$, i.e., f agrees with h^* on a deleted neighborhood of p_0, hence f can be extended likewise. Therefore, $X - V$ is not E-compact and this contradicts the fact that X_0 is complementatively E-compact.

We are now ready to prove the main theorems. In the following for a space X and a subspace X_0 of X we shall use $D(X_0)$ and $FC(X_0)$ to denote the class of all closed subsets of X which are disjoint from X_0 and which are E-functionally contained in X_0, respectively.

2.8. THEOREM. *If X contains a compact and complementatively E-compact subspace X_0, then X is E-compact.*

More precisely, we have the following formula for E-defect of X:

(a) $\operatorname{def}_E X \leqq \Sigma\{\operatorname{card}(FC(A)) \cdot \operatorname{def}_E A : A \in D_0(X_0)\}$
where $D_0(X_0)$ *is a cofinal subset of* $D(X_0)$.

Proof. The first part follows immediately from 2.7. We now prove formula (a). For each $A \in D(X_0)$, let $\mathscr{F}_A$ be an E-nonextendable class for A with $\operatorname{card} \mathscr{F}_A = \operatorname{def}_E A$. Let B be an arbitrary set of $FC(A)$. Then by 2.5, for each $f \in \mathscr{F}_A$, there are two maps $f'_B \in C(A, E)$, $f^*_B \in C(X, E)$ such that $f^*_B | B = f | B$. Let $\mathscr{F}_{(A,B)}$ be the class of such f^*_B. Then $\operatorname{card} \mathscr{F}_{(A,B)} \leqq \operatorname{def}_E A$ for each $B \in FC(A)$. Let $\mathscr{F}^*_A = \bigcup\{\mathscr{F}_{(A,B)}: B \in FC(A)\}$. Then $\operatorname{card} \mathscr{F}^*_A \leqq \Sigma\{\operatorname{card} \mathscr{F}_{(A,B)}: B \in FC(A)\} \leqq \operatorname{card} FC(A) \cdot \operatorname{def}_E A$. Finally, let $\mathscr{F} = \bigcup\{\mathscr{F}^*_A: A \in D_0(X_0)\}$. Then

$$\operatorname{card} \mathscr{F} \leqq \sum \{\operatorname{card} \mathscr{F}^*_A: A \in D_0(X_0)\}$$

$$\leqq \sum \{\operatorname{card} FC(A) \cdot \operatorname{def}_E A : A \in D_0(X_0)\}.$$

It is easy to show that $\mathscr{F}$ is an E-nonextendable class for X.

2.9. THEOREM. *If $X_1, \cdots, X_n$ are E-compact, E-embedded subspaces of X such that $\bigcup_{i=1}^n X_i$ is complementatively E-compact, then X is E-compact.*

More precisely, we have the following formula for the E-defect of X:

(b) $\operatorname{def}_E X \leqq \Sigma_{i=1}^n \operatorname{def}_E X_i + \Sigma\{\operatorname{card} FC(A) \cdot \operatorname{def}_E A :$
$A \in D_0(\bigcup_{i=1}^n X_i)\}$
where $D_0(\bigcup_{i=1}^n X_i)$ *is a cofinal subset of* $D(\bigcup_{i=1}^n X_i)$.

Proof. The first part follows from 2.6 and 2.7. We now prove formula (b). For each $i = 1, \cdots, n$, let $\mathscr{F}_i$ be an E-nonextendable class for X_i with $\operatorname{card} \mathscr{F}_i = \operatorname{def}_E X_i$. Since X_i is E-embedded in X, for each $f \in \mathscr{F}_i$, we choose an extension $f^* \in C(X, E)$ of f and denote by $\mathscr{F}^*_i$ the class of all such extensions. Clearly, $\operatorname{card} \mathscr{F}^*_i \leqq \operatorname{def}_E X_i$ for $i = 1, \cdots, n$. Let $\mathscr{F}_I = \bigcup_{i=1}^n \mathscr{F}^*_i$. Then $\operatorname{card} \mathscr{F}_I \leqq \Sigma_{i=1}^n \operatorname{def}_E X_i$. For each $A \in D(\bigcup_{i=1}^n X_i)$, let $\mathscr{F}_A$ be an E-nonextendable class for A with $\operatorname{card} \mathscr{F}_A = \operatorname{def}_E A$. Let B be an arbitrary set of $FC(A)$. Then for each $f \in \mathscr{F}_A$, by 2.5, there exist two maps $f'_B \in C(A, E)$, $f^*_B \in C(X, E)$ with $f^*_B | B = f | B$. Let $\mathscr{F}_{(A,B)}$ be the class of all such f^*_B. Then $\operatorname{card} \mathscr{F}_{(A,B)} = \operatorname{def}_E A$ for each $B \in FC(A)$. Let $\mathscr{F}^*_A = \bigcup\{\mathscr{F}_{(A,B)}: B \in FC(A)\}$. Then $\operatorname{card} \mathscr{F}^*_A \leqq \Sigma\{\operatorname{card} \mathscr{F}_{(A,B)}: B \in FC(A)\} \leqq \operatorname{card} FC(A) \cdot \operatorname{def}_E A$. Finally, let $\mathscr{F}_{II} = \bigcup\{\mathscr{F}^*_A: A \in D_0(\bigcup_{i=1}^n X_i)\}$. Then

$$\operatorname{card} \mathscr{F}_{II} \leqq \sum \{\operatorname{card} \mathscr{F}^*_A: A \in D_0(\bigcup_{i=1}^n X_i)\}$$

$$\leqq \sum \{\operatorname{card} FC(A) \cdot \operatorname{def}_E A : A \in D_0(\bigcup_{i=1}^n X_i)\}.$$

It is easy to see that the class $\mathscr{F} = \mathscr{F}_I \cup \mathscr{F}_{II}$ is an E-nonextendable class for X.

The following corollaries follow from 2.7, 2.8 and 2.9.

2.10. COROLLARY. *If $X = X_1 \cup X_2$ where X_1 is E-compact and X_2 is closed in $\beta_E X$, then X is E-compact.*

2.11. COROLLARY. *If $X = X_1 \cup X_2$ where X_1 is E-compact and X_2 is compact, then X is E-compact.*

2.12. COROLLARY. *If X is the union of finitely many E-compact subspaces, each of which is E-embedded in X, except at most one, then X is E-compact.*

2.13. REMARK. Unlike 2.9, considering more than one subspace in 2.7 and 2.8 will not generalize the theorems. In fact, if $X_1, \cdots, X_n$ are subspaces of X which are closed in $\beta_E X$ (compact) such that $\bigcup_{i=1}^n X_i$ is complementatively E-compact, then we could simply let $X_0 = \bigcup_{i=1}^n X_i$ which is closed in $\beta_E X$ (respectively, compact) and is complementatively E-compact.

2.14. REMARK. We shall now show that formulas (a) and (b) of 2.8 and 2.9 are the best estimations for the E-defects of X.

For each ordinal α, let $S(\alpha) = \{\lambda : \lambda < \alpha\}$ and let Ω be the first uncountable ordinal. Let $X = (R \times S(\Omega)) \cup \{p_0\}$ where $p_0 \notin R \times S(\Omega)$. Topologize X as follows: every open set in $R \times S(\Omega)$ is open in X: a base of neighborhoods of p_0 consists of sets of the form $(R \times B) \cup \{p_0\}$ where $B \subseteq S(\Omega)$ and $S(\Omega) - B$ is countable. It follows from 2.8 that X is R-compact and $\operatorname{def}_R X \leqq \aleph_1$. Also, it is easy to show that $\operatorname{def}_R X \geqq \aleph_0$. In order to show that formula (a) in 2.8 is the best estimation for $\operatorname{def}_R X$, we must show that $\operatorname{def}_R X \neq \aleph_0$. Assume the contrary, i.e., assume that $\operatorname{def}_R X = \aleph_0$. Let $\mathscr{F}$ be an R-nonextendable class for X with card $\mathscr{F} = \aleph_0$. For an arbitrary rational number r and for each $f \in \mathscr{F}$, there is an ordinal $\alpha_f \in S(\Omega)$ such that f is constant on $\{r\} \times (S(\Omega) - S(\alpha_f))$. Obviously, the set $\{\alpha_f : f \in \mathscr{F}\}$ has an upper bound, say α_r, in $S(\Omega)$ and every $f \in \mathscr{F}$ is constant on $\{r\} \times (S(\Omega) - S(\alpha_r))$. It is also clear that the set $\{\alpha_r : r \in P\}$, where P denotes the set of all rational numbers, has an upper bound, say α, in $S(\Omega)$ and every $f \in \mathscr{F}$ is constant on $P \times (S(\Omega) - S(\alpha))$. Since P is dense in R, every $f \in \mathscr{F}$ is then constant on $R \times (S(\Omega) - S(\alpha))$. Now choose a point $p_1 \in \beta X - X$ such that $p_1 \in \operatorname{cl}_{\beta X}(R \times \{\alpha\})$. Then $X \cup \{p_1\}$ is a proper extension of X with the property that every $f \in \mathscr{F}$ admits a continuous extension $f^* : X \cup \{p_1\} \to R$. This contradicts the fact that $\mathscr{F}$ is an R-nonextendable class for X.

2.15. REMARK. Recall that for $E = R$ and $E = N$ we have the following countable theorem for E-compactness: If $X = \bigcup_{i=1}^{\infty} X_i$ *where* X_i *is E-compact, E-embedded in X for each i, then X is E-compact.* We shall now show that, however, for the infinite additivity theorems of E-compactness, it is impossible to find formulas for the E-defects analogous for formulas (a) and (b) of 2.8 and 2.9.

Let $X = \bigcup_{n=1}^{\infty} [0, n]^m$ where m is an infinite cardinal. Then X, being σ-comapct, is R-compact. We shall prove our claim by showing that $\text{def}_R X \geqq m$.

CASE 1. $m = \aleph_0$. If $\text{def}_R X < m$, then by Theorem 5.9 of **[3]** X is Lindelöf and locally compact which is a contradiction (since X is not locally compact).

CASE 2. $m > \aleph_0$. If $\text{def}_R X = p < m$. Let $\mathcal{F}$ be an R-nonextendable class for X with card $\mathcal{F} = p$. It is well known that for each $f \in \mathcal{F}$, there exists a countable subset $\Xi_f \subset \Xi$ such that if $x_1, x_2 \in X$ and $x_1 | \Xi_f = x_2 | \Xi_f$, then $f(x_1) = f(x_2)$. Let $\Xi_{\mathcal{F}} = \bigcup \{\Xi_f : f \in \mathcal{F}\}$. Then card $\Xi_{\mathcal{F}} < m$. Hence there exists $\xi_0 \in \Xi - \Xi_{\mathcal{F}}$. Let $X_0 = \{x \in X : \pi_\xi(x) = 0$ for every $\xi \neq \xi_0\}$. Then every $f \in \mathcal{F}$ is constant on X_0. Now choose a point p_1 in $\beta X - X$ such that $p_1 \in \text{cl}_{\beta X} X_0$. Then every $f \in \mathcal{F}$ admits a continuous extension $f^* : X \cup \{p_1\} \to R$. Hence $\mathcal{F}$ is not an R-nonextendable class for X which is a contradiction.

3. Quotient images of E-compact spaces. We now turn to the preservation of E-compactness under quotient maps. Given a map $\varphi : X \to Y$ and a point y in Y, we shall call card $\varphi^{-1}(y)$ the *multiplicity of y* (with respect to φ). A point of Y is called a *multiple point of* φ provided that its multiplicity is greater than one.

3.1. THEOREM. *Given a quotient map $\varphi : S \to X$. If S is an E-compact space and if the set M of all multiple points of φ satisfies one of the following conditions, then X is E-compact.*

(i) *M is closed in $\beta_E X$.*

(ii) *M is compact.*

(iii) *M can be expressed as the union of finitely many E-compact E-embedded subspaces of X.*

Proof. It is obvious that if M satisfies any of the three conditions then it is closed in X. Hence $S - \varphi^{-1}(M)$ is open in S and φ restricted to $S - \varphi^{-1}(M)$ is a homeomorphism. If F is a closed subset of X disjoint from M, then F is homeomorphic to $\varphi^{-1}(F)$; consequently, F is E-compact, i.e., M is complementatively E-compact in X. By 2.7, 2.8 and 2.9, X is E-compact.

4. Applicability of the theorems. In §§2 and 3, E was assumed to satisfy rather complex conditions (α), (β) and (γ). However, sometimes the results can be applied to an E which does not satisfy these conditions. The procedure is to find another representative E' of $\mathfrak{R}(E)$ which satisfies the assumptions of the theorems. As an example of this procedure we shall show that *all theorems of* §§2 *and* 3 *are true when* E *is an arbitrary* 0-*dimensional linearly ordered space.* (Obviously, these theorems are true for $E = R$ and for $E = N$.) The statements which lead to this result are as follows:

4.1. *Every linearly ordered space which has first and last elements satisfies* (α).

4.2. *Every* 0-*dimensional space satisfies* (β).

4.3. *Every strongly* 0-*dimensional normal space satisfies* (γ).

4.4. *Every* 0-*dimensional linearly ordered space is strongly* 0-*dimensional.*

4.5. *Every* 0-*dimensional linearly ordered space with first and last element satisfies* (α), (β) *and* (γ).

4.6. *Let* X_0 *be an* E-*embedded subspace of* X, $E' \subset_{\text{top}} E^m$ *for some cardinal* m. *If* E' *is a retract of* E^m, *then* X_0 *is also* E'-*embedded in* X.

Proof. Let $f \in C(X_0, E')$. Then f can be considered as a continuous map from X_0 into E^m. Hence f admits a continuous extension $f^*: X \to E^m$. Thus, $r \circ f^*$, where r is the retraction of E^m onto E', is a continuous extension of f over X.

4.7. *For every* 0-*dimensional linearly ordered space* E, *there exists a* 0-*dimensional linearly ordered space* E' *which has first and last elements and satisfies the following conditions.*

(1) $E \subset_{\text{cl}} E'^2$, $E' \subset E^2$, (*hence* $\mathfrak{R}(E) = \mathfrak{R}(E')$).

(2) E' *is a retract of* E^2 (*hence any* E-*embedded subspace* X_0 *of* X *is also* E'-*embedded*).

Proof. If E itself has both first and last element, then by letting $E' = E$, we are done. Otherwise we consider two cases.

CASE 1. E has exactly one of the first and the last elements. Without loss of generality, we assume that E has first element (say a) but has no last element. Let E^* be the linearly ordered set formed by all elements of E with the reverse order of E. Let $E' = E \oplus E^*$, i.e., $E' = E \cup E^*$ with the order be defined by letting $x < x^*$ for every $x \in E$ and $x^* \in E^*$. Then E' has first and last elements. Let $b \in E$ with $b \neq a$. Clearly, $E \subset_{\text{cl}} E'$ and $E' \subset_{\text{top}} \{(x, a): x \in E\} \cup \{(x, b): x \in E\} \subset_{\text{cl}} E^2$. To show that E' is a retract of E^2, we let c be a cut between a and b, and define a map $p: E^2 \to E^2$ as follows: $p(x, y) = (x, b)$ for each $x \in E$ and $c < y$; $p(x, y) = (x, a)$ for each $x \in E$ and $y < c$. Then the map $h^{-1} \circ p$ is a retraction from E^2 onto E' where h is the homeomorphism from E' into E^2.

CASE 2. E has neither first nor last element. Choose an arbitrary point $a \in E$. Let $E_1 = \{x \in E: x \geqq a\}$, $E_2 = \{x \in E: x \leqq a\}$ and $E' = E_1 \oplus E_2$. Then E' is a linearly ordered set with first and last elements (say a_1 and a_2, respectively). Let b be an element of E with $b \neq a$. Without loss of generality, we assume that $a < b$. Clearly, $E \subset_{\text{cl}} E'^2$ and $E' \subset_{\text{top}} \{(x, a): x \in E, x \geqq a\} \cup \{(x, b): x \in E, a \leqq x\} \subset_{\text{cl}} E^2$. To show that E' is a retract of E^2, we let c be a cut between a and b, and define two maps s and t: $E^2 \to E^2$ as follows $s(x, y) = (x, b)$ for each $x \in E, c < y$; $s(x, y) = (x, a)$ for each $x \in E$, $y < c$ and $t(x, y) = (x, y)$ for $x < a$, $y = b$ or $a < x$, $y = a$; $t(x, y) = a_2$ for $a \leqq x$, $y = b$; $t(x, y) = a_1$ for $x \leqq a$, $y = b$. Then $k^{-1} \circ s \circ t$ is a retraction from E^2 into E' where k is the homeomorphism from E' into E^2.

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Received March 26, 1974 and in revised form January 23, 1976. The second author wishes to express his appreciation to State University of New York College at Geneseo for providing a Faculty Released-time Research Grant during Spring of 1973 for the preparation of this paper.

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Printed at Jerusalem Academic Press, POB 2390, Jerusalem, Israel.

Pacific Journal of Mathematics

Vol. 70, No. 2 October, 1977