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**WEAK AND NORM APPROXIMATE IDENTITIES ARE  
DIFFERENT**

CHARLES ALLEN JONES AND CHARLES DWIGHT LAHR

## WEAK AND NORM APPROXIMATE IDENTITIES ARE DIFFERENT

CHARLES A. JONES AND CHARLES D. LAHR

**An example is given of a convolution measure algebra which has a bounded weak approximate identity, but no norm approximate identity.**

1. Introduction. Let  $A$  be a commutative Banach algebra,  $A'$  the dual space of  $A$ , and  $\Delta A$  the maximal ideal space of  $A$ . A weak approximate identity for  $A$  is a net  $\{e(\lambda): \lambda \in \Lambda\}$  in  $A$  such that

$$\chi(e(\lambda)a) \longrightarrow \chi(a)$$

for all  $a \in A$ ,  $\chi \in \Delta A$ . A norm approximate identity for  $A$  is a net  $\{e(\lambda): \lambda \in \Lambda\}$  in  $A$  such that

$$\|e(\lambda)a - a\| \longrightarrow 0$$

for all  $a \in A$ . A net  $\{e(\lambda): \lambda \in \Lambda\}$  in  $A$  is bounded and of norm  $M$  if there exists a positive number  $M$  such that  $\|e(\lambda)\| \leq M$  for all  $\lambda \in \Lambda$ .

It is well known that if  $A$  has a bounded weak approximate identity for which  $f(e(\lambda)a) \rightarrow f(a)$  for all  $f \in A'$  and  $a \in A$ , then  $A$  has a bounded norm approximate identity [1, Proposition 4, page 58]. However, the situation is different if weak convergence is with respect to  $\Delta A$  and not  $A'$ . An example is given in § 2 of a Banach algebra  $A$  which has a weak approximate identity, but does not have a norm approximate identity. This algebra provides a counterexample to a theorem of J. L. Taylor [4, Theorem 3.1], because it is proved in [3, Corollary 3.2] that the structure space of a convolution measure algebra  $A$  has an identity if and only if  $A$  has a bounded weak approximate identity of norm one.

2. The example. Throughout this paper the set of complex numbers is denoted  $\mathbf{C}$  and the set of real numbers  $\mathbf{R}$ .

Let  $S$  be a commutative semigroup, and  $\mathcal{L}_1(S)$  the Banach space of all complex functions  $\alpha: S \rightarrow \mathbf{C}$  such that  $\|\alpha\| = \sum_{x \in S} |\alpha(x)|$  is finite, made into a convolution algebra under the product

$$\alpha * \beta = \sum_{x \in S} \sum_{\substack{u, v \\ uv=x}} \alpha(u)\beta(v)\delta_x,$$

where  $\delta_x$  represents the point mass at  $x \in S$ ,  $\alpha = \sum_{x \in S} \alpha(x)\delta_x$  and  $\beta = \sum_{x \in S} \beta(x)\delta_x$ . A semicharacter on  $S$  is a bounded nonzero function  $\chi: S \rightarrow \mathbf{C}$  such that  $\chi(xy) = \chi(x)\chi(y)$  for all  $x, y \in S$ . The set of

all semicharacters is denoted  $\hat{S}$ .

It has been shown in a previous paper [3] that if  $\mathcal{L}_1(S)$  is semisimple, then the existence of a bounded weak approximate identity of norm one in  $\mathcal{L}_1(S)$  is equivalent to the existence of a net  $\{u_d\}$  in  $S$  such that  $\chi(u_d) \rightarrow 1$  for all  $\chi \in \hat{S}$ . It has also been shown that the existence of a norm approximate identity bounded by 1 is equivalent to the existence of a net  $\{u_d\}$  in  $S$  with the following property: for each  $x \in S$ , there exists  $d_x$  such that  $xu_d = x$  for all  $d \geq d_x$ . For the particular semigroup  $S$  to follow, it will be shown that  $\mathcal{L}_1(S)$  does indeed have a bounded weak approximate identity, but does not have a norm approximate identity.

Let the set of integers be denoted by  $\mathbf{Z}$  and the set of positive integers by  $\mathbf{Z}^+$ . Further, let  $S = \{m/n: m, n \in \mathbf{Z}^+\}$  under addition. Then  $S$  is a cancellative semigroup and so  $\mathcal{L}_1(S)$  is semisimple [2]. If  $\chi \in \hat{S}$ , then  $\chi$  is uniquely determined by its values on  $\{1/n: n \in \mathbf{Z}^+\}$ . For if  $m$  is any positive integer, then for all  $n \in \mathbf{Z}^+$ ,  $\chi(m/n) = \chi(1/n)^m$ . In fact  $\chi(1) = \chi(n/n) = \chi(1/n)^n$  for all  $n \in \mathbf{Z}^+$ , and so  $\chi(1/n)$  is an  $n$ th root of  $\chi(1)$ . Now, each pair  $(k, z)$ , where  $k \in \mathbf{Z}$  and  $z = re^{i\theta}$  with  $|z| \leq 1$  and  $r, \theta \in \mathbf{R}$ , determines a semicharacter  $\chi_{k,z}$  of  $S$  by defining

$$\chi_{k,z}(m/n) = r^{m/n} e^{im(\theta + 2k\pi)/n}$$

for all  $m/n$  in  $S$ . It is clear that  $\chi_{k,z}(1/n) \rightarrow 1$  for each  $\chi_{k,z} \in \hat{S}$ . However, not all semicharacters have such a nice form. In constructing an arbitrary semicharacter  $\chi$ , there are very few restrictions imposed upon how the  $n$ th root of  $\chi(1)$  is to be chosen. Thus, a more elaborate argument is required to obtain a weak approximate identity for  $\mathcal{L}_1(S)$ .

**LEMMA 2.1.** *Let  $G$  be an infinite discrete group with identity  $e$ . Then there exists a net  $\{g_\lambda\} \subset G$ ,  $g_\lambda \neq e$  for all  $\lambda$ , such that  $\chi(g_\lambda) \rightarrow 1$  for each  $\chi \in \hat{G}$ .*

*Proof.* Let  $\bar{G}$  be the Bohr compactification of  $G$ . Then there is an algebra isomorphism  $i$  of  $G$  onto a dense subset of  $\bar{G}$ . Specifically, for each  $g \in \bar{G}$ , there exists a net  $\{i(g_\lambda): g_\lambda \in G\}$  such that  $i(g_\lambda) \rightarrow g$ ; equivalently,  $\bar{\chi}(i(g_\lambda)) \rightarrow \bar{\chi}(g)$  for each  $\chi \in \hat{G}$ , where  $\bar{\chi}$  is the unique extension of  $\chi \in \hat{G}$  to  $\bar{\chi} \in \hat{\bar{G}}$  [3]. Since  $\bar{G}$  is infinite and compact, the identity  $i(e)$  of  $\bar{G}$  is not isolated in  $\bar{G}$ . Hence, there is a net  $\{i(g_\lambda): g_\lambda \in G\}$ ,  $g_\lambda \neq e$  for all  $\lambda$ , such that  $i(g_\lambda) \rightarrow i(e)$ . Therefore,

$$\chi(g_\lambda) = \bar{\chi}(i(g_\lambda)) \longrightarrow \bar{\chi}(i(e)) = 1 \text{ for each } \chi \in \hat{G}.$$

Let  $T = \{z \in \mathbf{C}: |z| = 1\}$  and  $D = \{z \in \mathbf{C}: |z| \leq 1\}$ . Then the previous lemma yields the following number-theoretic result.

**THEOREM 2.2.** *Let  $\{z_1, z_2, \dots, z_p\} \subset T$ ,  $p \in \mathbf{Z}^+$ . Then for each  $\varepsilon > 0$  there exists  $m \in \mathbf{Z}^+$  such that  $|(z_i)^m - 1| < \varepsilon$  for all  $i$ ,  $1 \leq i \leq p$ .*

*Proof.* Consider the group  $G = \mathbf{Z}$  under addition. Then  $\hat{G} = \{\chi_z: z \in T\}$ , where  $\chi_z(n) = z^n$ ,  $n \in G$ . Now, let  $\varepsilon > 0$  be given. By Lemma 2.1, there exists a net  $\{n_\lambda: \lambda \in A\} \subset G$ ,  $n_\lambda \neq 0$  for all  $\lambda$ , such that  $z^{n_\lambda} = \chi_z(n_\lambda) \rightarrow 1$  for each  $\chi_z \in \hat{G}$ . Without loss of generality, assume that  $n_\lambda \in \mathbf{Z}^+$  for all  $\lambda$ . Hence, given  $\{z_1, z_2, \dots, z_p\} \subset T$ , there exist  $\lambda_1, \lambda_2, \dots, \lambda_p$  in  $A$  such that  $|z_i^{n_\lambda} - 1| < \varepsilon$  for all  $\lambda \geq \lambda_i$ ,  $1 \leq i \leq p$ . Thus, if  $\lambda_0 \in A$  is such that  $\lambda_0 \geq \lambda_i$ ,  $1 \leq i \leq p$ , then with  $m = n_{\lambda_0}$ ,

$$|(z_i)^m - 1| < \varepsilon \quad \text{for } i = 1, 2, \dots, p.$$

**COROLLARY 2.3.** *Let  $\{z_1, z_2, \dots, z_p\} \subset T$ ,  $p \in \mathbf{Z}^+$ . Then for each  $\varepsilon$ ,  $0 < \varepsilon < 1$ , there exist neighborhoods  $U_1, U_2, \dots, U_p$  and there exists  $m_0 \in \mathbf{Z}^+$  such that*

- (1)  $z_i \in U_i$  and  $U_i \subset D$ ,  $1 \leq i \leq p$ ,
- (2)  $|u - 1| < \varepsilon$  for all

$$u \in U_i^{m_0} = \{w_1 w_2 \cdots w_{m_0}: w_j \in U_i\}, \quad 1 \leq i \leq p.$$

*Proof.* Let  $z_j = e^{i\theta_j}$ ,  $1 \leq j \leq p$ . By Theorem 2.2, there exists  $m_0 \in \mathbf{Z}^+$  such that  $|m_0 \theta_j \pmod{2\pi}| < \varepsilon/2$  for all  $j$ . Now, for each  $j$ , let

$$U_j = \left\{ w = |w| e^{i\omega} \varepsilon D: |\omega - \theta_j| < \frac{\varepsilon}{4m_0} \text{ and } |w| > \left[ 1 - \frac{\varepsilon}{4} \right]^{1/m_0} \right\}.$$

Then if  $u \in U_j^{m_0}$ ,  $u = w_1 w_2 \cdots w_{m_0}$ ,  $w_k \in U_j$  for all  $k$ , so that  $|\omega_1 + \omega_2 + \cdots + \omega_{m_0} - m_0 \theta_j| < \varepsilon/4$  and  $|w_1| |w_2| \cdots |w_{m_0}| > 1 - \varepsilon/4$ . Thus, if  $u \in U_j^{m_0}$ , then

$$|u - 1| \leq |u - z_j^{m_0}| + |z_j^{m_0} - 1| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

After a technical lemma, the desired result will be proved.  $S$  continues to be the semigroup of positive rationals under addition.

**LEMMA 2.4.** *Let  $\{\chi_1, \chi_2, \dots, \chi_p\} \subset \hat{S}$ ,  $p \in \mathbf{Z}^+$ . Then there exists a subsequence  $\{1/n_k: k \in \mathbf{Z}^+\}$  of  $\{1/n: n \in \mathbf{Z}^+\}$  and there exist  $z_1, z_2, \dots, z_p \in T$  such that  $\chi_i(1/n_k) \rightarrow z_i$  for each  $i$ ,  $1 \leq i \leq p$ .*

*Proof.* Note that for each  $i$ ,  $\chi_i(1/n)$  is an  $n$ th root of  $\chi_i(1)$  and so  $|\chi_i(1/n)| \rightarrow 1$  as  $n \rightarrow \infty$ .

Now,  $\{\chi_i(1/n): n \in \mathbf{Z}^+\}$  is a subset of the closed unit disk  $D$ , and so by compactness has a convergent subsequence with limit  $z_i$ ;  $z_i \in T$  by the above remark. Further, if a subsequence  $\{1/n_\ell: \ell \in \mathbf{Z}^+\}$  exists such that  $\chi_i(1/n_\ell) \rightarrow z_i$  for  $i = 1, 2, \dots, j$ , then by compactness  $\{\chi_{j+1}(1/n_\ell): \ell \in \mathbf{Z}^+\}$  has a convergent subsequence  $\{\chi_{j+1}(1/n_k): k \in \mathbf{Z}^+\}$  with limit  $z_{j+1} \in T$ . Thus, the induction proof is complete.

**THEOREM 2.5.** *There exists a net  $\{q_d: d \in \mathcal{D}\} \subset S$  such that  $\chi(q_d) \rightarrow 1$  for each  $\chi \in \hat{S}$ . Therefore,  $\mathcal{I}_1(S)$  has a weak approximate identity of norm one.*

*Proof.* Let  $\mathcal{F}(\hat{S})$  denote the collection of all finite subsets of  $\hat{S}$  and let  $\mathcal{D} = \mathbf{Z}^+ \times \mathcal{F}(\hat{S})$  be directed by  $(n, A) \leq (m, B)$  if and only if  $n \leq m$  and  $A \subset B$ .

Now, define a mapping  $d \mapsto q_d$  of  $\mathcal{D}$  into  $S$  as follows: For each  $d = (n, A)$ ,  $A = \{\chi_1, \dots, \chi_p\}$ , fix a subsequence  $\{1/n_k: k \in \mathbf{Z}^+\}$  such that  $\chi_i(1/n_k) \rightarrow z_i \in T$  for all  $i$ . Then there exist  $m_0 \in \mathbf{Z}^+$  and neighborhoods  $U_1, \dots, U_p$  of  $z_1, \dots, z_p$ , respectively, such that  $|u - 1| < 1/n$  for all  $u \in U_j^{m_0}$ ,  $1 \leq i \leq p$ . Now, there exist  $K_i \in \mathbf{Z}^+$  such that  $k \geq K_i$  implies  $\chi_i(1/n_k) \in U_i$  for  $1 \leq i \leq p$ . Hence, for each  $i$ ,  $1 \leq i \leq p$ ,

$$|\chi_i(m_0/n_k) - 1| = |\chi_i(1/n_k)^{m_0} - 1| < \frac{1}{n}$$

for all  $k \geq K_i$ . Set  $K_0 = \max\{K_i: i = 1, 2, \dots, p\}$ . Then define  $q_d = m_0/n_{K_0}$ .

Finally, it remains to show that for each  $\chi \in \hat{S}$ ,  $\chi(q_d) \rightarrow 1$ . So, let  $\varepsilon > 0$  be given. Then choose  $n_0$  such that  $(1/n_0) < \varepsilon$ , and let  $A_0 = \{\chi\}$ . If  $d = (n, A) \geq (n_0, A_0) = d_0$ , then  $|\chi(q_d) - 1| < (1/n_0) < \varepsilon$ .

**COROLLARY 2.6.** *There exists a net  $\{1/n_d: d \in \mathcal{D}\} \subset \{1/n: n \in \mathbf{Z}^+\}$  such that  $\chi(1/n_d) \rightarrow 1$  for each  $\chi \in \hat{S}$ .*

*Proof.* Repeat the proofs of Lemma 2.4 and Theorem 2.5 with  $\{1/n: n \in \mathbf{Z}^+\}$  replaced by  $\{1/n!: n \in \mathbf{Z}^+\}$ . Then in the proof of Theorem 2.5 choose  $K_0$  such that

(1)  $K_0 \geq \max\{K_i: i = 1, 2, \dots, p\}$  and

(2)  $n_{K_0} \geq m_0$ . Thus,  $q_d = m_0/n_{K_0}!$  is of the form  $1/n_d$  for some  $n_d \in \mathbf{Z}^+$ .

Theorem 2.5 and Corollary 2.6 make it clear that  $\mathcal{I}_1(S)$  has a bounded weak approximate identity  $\{\delta_{1/n_d}: d \in \mathcal{D}\}$  [3]. However,  $S$  does not have relative units. That is, given  $m/n \in S$ , there is no  $v \in S$  such that  $v(m/n) = m/n$ . Thus,  $\mathcal{I}_1(S)$  does not have a norm approximate identity, bounded or unbounded.

3. A general result. The same techniques developed in § 2 can be used to prove a useful result about weak approximate identities of norm one for a commutative Banach algebra.

**THEOREM 3.1.** *Let  $A$  be a commutative Banach algebra. Then  $A$  has a weak approximate identity of norm one if and only if there exists a net  $\{v(\rho): \rho \in \mathcal{S}\}$  in  $A$ ,  $\|v(\rho)\| \leq 1$  for all  $\rho$ , such that  $|\chi(v(\rho))| \rightarrow 1$  for all  $\chi \in \Delta A$ .*

*Proof.* If  $A$  has a weak approximate identity of norm one, then there exists a net  $\{v(\rho): \rho \in \mathcal{S}\}$  in  $A$ ,  $\|v(\rho)\| \leq 1$  for all  $\rho$ , such that

$$\chi(v(\rho)a) \longrightarrow \chi(a) \quad \text{for all } a \in A, \chi \in \Delta A.$$

Thus, for each  $\chi \in \Delta A$ ,  $\chi(a) \neq 0$  for some  $a \in A$  implies that  $\chi(v(\rho)) \rightarrow 1$  and hence  $|\chi(v(\rho))| \rightarrow 1$ .

Conversely, assume that  $\{v(\rho)\}$  is such that  $|\chi(v(\rho))| \rightarrow 1$  for each  $\chi \in \Delta A$ . Let  $\mathcal{F}(\Delta A)$  be the collection of all finite subsets of  $\Delta A$  and let  $\Lambda = \mathbf{Z}^+ \times \mathcal{F}(\Delta A)$  be directed by  $(n, F) \leq (m, E)$  if and only if  $n \leq m$  and  $F \subset E$ .

Then define a mapping  $\lambda \mapsto e(\lambda)$  of  $\Lambda$  into  $A$  as follows: for each  $\lambda = (n, F)$ , where  $n \in \mathbf{Z}^+$  and  $F = \{\chi_1, \chi_2, \dots, \chi_r\}$ , there exists by compactness of  $D$  a subnet  $\{v(\rho')\}$  of  $\{v(\rho)\}$  such that  $\chi_i(\rho') \rightarrow z_i \in T$  for  $i, 1 \leq i \leq r$ . By Corollary 2.3, there exists  $m_0 \in \mathbf{Z}^+$  and neighborhoods  $U_i$  of  $z_i$  in  $D$  such that  $|z - 1| < 1/n$  for all  $z \in U_i^{m_0}$ ,  $1 \leq i \leq r$ . Now, let  $\rho'_0$  be such that  $\chi_i(v(\rho'_0)) \in U_i$  for all  $i, 1 \leq i \leq r$ , and define  $e(\lambda) = v(\rho'_0)^{m_0}$ . Note that for each  $i$ ,

$$\begin{aligned} |\chi_i(e(\lambda)) - 1| &= |\chi_i(v(\rho'_0)^{m_0}) - 1| \\ &= |(\chi_i(v(\rho'_0)))^{m_0} - 1| < \frac{1}{n}. \end{aligned}$$

Thus,  $\chi(e(\lambda)) \rightarrow 1$  for each  $\chi \in \Delta A$  and hence  $\chi(e(\lambda)a) \rightarrow \chi(a)$  for each  $\chi \in \Delta A, a \in A$ . Also,  $\|e(\lambda)\| = \|v(\rho'_0)^{m_0}\| \leq 1$  for all  $\lambda \in \Lambda$ .

**COROLLARY 3.2.** *Let  $S$  be a commutative semigroup for which  $\mathcal{L}_1(S)$  is semisimple. Then  $\mathcal{L}_1(S)$  has a weak approximate identity of norm one if and only if there exists a net  $\{s(\rho): (\rho) \in \mathcal{S}\}$  in  $S$  such that  $|\chi(s(\rho))| \rightarrow 1$  for all  $\chi \in \hat{S}$ .*

*Proof.* The Banach algebra  $\mathcal{L}_1(S)$  has a weak approximate identity of norm one if and only if there exists a net  $\{s(\lambda): \lambda \in \Lambda\}$  in  $S$  such that  $\chi(s(\lambda)) \rightarrow 1$  for all  $\chi \in \hat{S}$  [3]. Thus, the proof is completed by applying Theorem 3.1 with  $v(\rho) = \delta_{s(\rho)}$  for all  $\rho$ .

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