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Let M be a unimodular lattice on an indefinite hermitian space over an algebraic number field. The submodules of M invariant under the action of the special unitary group of M are classified. Generators for the local unitary groups of M are also determined.

1. Introduction. Let F be an algebraic number field of finite degree and K a quadratic extension of F. Let V be an indefinite hermitian space over K of finite dimension $n \geqq 3$ and $\Phi: V \times V \rightarrow K$ the associated nondegenerate hermitian form on V with respect to the nontrivial automorphism of K over F. Assume V supports a unimodular lattice M (in the sense of O'Meara [7; § 82G] for quadratic spaces). Denote by $U(V)$ the unitary group of V and by $U(M)$ the subgroup of isometries in $U(V)$ that leave M invariant. We will classify the sublattices of M that are invariant under the action of the special unitary group $SU(M)$. The problem is first solved locally; the global result is then obtained by applying the approximation theorem of Shimura [8; 5.12].

We now consider localization (see also [2; § 2] and [8]). Let $\mathfrak{p}$ be a finite prime spot of F and $F_\mathfrak{p}$ the corresponding local field. Put $K_\mathfrak{p} = K \otimes_F F_\mathfrak{p}$ and $V_\mathfrak{p} = V \otimes_F F_\mathfrak{p}$. Making the standard identifications, we have $K \subseteqq K_\mathfrak{p}$, $F_\mathfrak{p} \subseteqq K_\mathfrak{p}$ and $V \subseteqq V_\mathfrak{p}$. The hermitian form Φ on V extends naturally to an hermitian form on $V_\mathfrak{p}$. Let $\mathfrak{o}$ be the ring of integers in F, $\mathfrak{o}_\mathfrak{p}$ the (topological) closure of $\mathfrak{o}$ in $F_\mathfrak{p}$ and $\mathfrak{O}_\mathfrak{p}$ the integral closure of $\mathfrak{o}_\mathfrak{p}$ in $K_\mathfrak{p}$. Put $M_\mathfrak{p} = \mathfrak{O}_\mathfrak{p} M \subseteqq V_\mathfrak{p}$. Locally, we must study the submodules of $M_\mathfrak{p}$ invariant under the action of $SU(M_\mathfrak{p})$. Except when $K_\mathfrak{p}$ is a ramified extension of a dyadic field $F_\mathfrak{p}$, the classification will be trivial. For ramified dyadic extensions, it is necessary to determine a set of generators of $U(M_\mathfrak{p})$ before the classification can be determined.

We now state the main results.

THEOREM A. *Let M be a unimodular lattice on an indefinite hermitian space of dimension $n \geqq 3$ over an algebraic number field. Then a sublattice N of M is invariant under the action of the special unitary group $SU(M)$ if and only if for all finite prime spots $\mathfrak{p}$ of F, the localization $N_\mathfrak{p} = \mathfrak{O}_\mathfrak{p} N$ is invariant under the ac-*

tion of $SU(M_{\mathfrak{p}})$.

For x in $V_{\mathfrak{p}}$, define $2q(x) = \Phi(x, x)$, and let $M_{\mathfrak{p}*}$ be the sublattice of $M_{\mathfrak{p}}$ generated by the x in $M_{\mathfrak{p}}$ with $q(x)$ in $\mathfrak{o}_{\mathfrak{p}}$. Let

$$M_{\mathfrak{p}}^* = \{x \in V_{\mathfrak{p}} \mid \Phi(x, M_{\mathfrak{p}*}) \subseteqq \mathfrak{O}_{\mathfrak{p}}\}$$

be the dual lattice of $M_{\mathfrak{p}*}$. Then $M_{\mathfrak{p}*} \subseteqq M_{\mathfrak{p}} \subseteqq M_{\mathfrak{p}}^*$ and, except when $K_{\mathfrak{p}}$ is a ramified extension of a dyadic local field $F_{\mathfrak{p}}$, we will show later that $M_{\mathfrak{p}*} = M_{\mathfrak{p}}^*$. A sublattice $N_{\mathfrak{p}}$ of $M_{\mathfrak{p}}^*$ is called primitive if $N_{\mathfrak{p}}$ is not contained in $\pi M_{\mathfrak{p}}^*$ for any prime element $\pi \in \mathfrak{O}_{\mathfrak{p}}$. Clearly, if $N_{\mathfrak{p}}$ is invariant under $SU(M_{\mathfrak{p}})$, the lattice $\mathfrak{a}_{\mathfrak{p}} N_{\mathfrak{p}}$ is also invariant for any fractional ideal $\mathfrak{a}_{\mathfrak{p}}$ in $\mathfrak{O}_{\mathfrak{p}}$. It is therefore enough to classify locally the primitive invariant sublattices of $M_{\mathfrak{p}}^*$.

THEOREM B. *A primitive sublattice* $N_{\mathfrak{p}}$ *of* $M_{\mathfrak{p}}^*$ *is invariant under the action of* $SU(M_{\mathfrak{p}})$ *if and only if* $M_{\mathfrak{p}*} \subseteqq N_{\mathfrak{p}}$, *except when the following three conditions all apply*:

(i) $K_{\mathfrak{p}}$ *is a totally ramified extension of the* 2*-adic field* $\boldsymbol{Q}_2$,

(ii) $K_{\mathfrak{p}}$ *is a ramified prime extension of* $F_{\mathfrak{p}}$,

(iii) $\dim V_{\mathfrak{p}} = 3$ *or* 4.

In particular, except when $K_{\mathfrak{p}}$ *is a ramified extension of a dyadic field* $F_{\mathfrak{p}}$, *the only primitive invariant lattice is* $M_{\mathfrak{p}}$.

Theorem B will be proven for the various cases in §§ 2-4 and the exceptional 3 and 4 dimensional cases studied in § 5. Theorem A is established in the final section. The special case where F is the field of rational numbers is also studied in detail.

The approach here follows that given for quadratic spaces in [5] and [6].

2. Local isometries. In this and next three sections we are only concerned with local problems.

The structure of $\mathfrak{O}_{\mathfrak{p}}$ over $\mathfrak{o}_{\mathfrak{p}}$ depends on the prime $\mathfrak{p}$. If $\mathfrak{p}$ splits in K, then $K_{\mathfrak{p}} = F_{\mathfrak{p}} \times F_{\mathfrak{p}}$ and $\mathfrak{O}_{\mathfrak{p}} = \mathfrak{o}_{\mathfrak{p}} \times \mathfrak{o}_{\mathfrak{p}}$. In this case the involution $*$ on K becomes $(\alpha, \beta)^* = (\beta, \alpha)$ on $K_{\mathfrak{p}}$. If $\mathfrak{p}$ does not split in K, we may take $K_{\mathfrak{p}} = F_{\mathfrak{p}}(\zeta)$ where $\zeta^2 \in F_{\mathfrak{p}}$ and $\zeta^* = -\zeta$. Fix a prime π in $K_{\mathfrak{p}}$ and p in $F_{\mathfrak{p}}$ and let $e = \operatorname{ord}_p 2$. If $\mathfrak{p}$ is dyadic, there are now three possible types of extensions of $K_{\mathfrak{p}}$ over $F_{\mathfrak{p}}$; the details are an application of [7; 63.2, 63.3].

(i) $K_{\mathfrak{p}}$ is an unramified extension of $F_{\mathfrak{p}}$. Then $\zeta^2 = 1 + 4\delta$ with δ a unit in $F_{\mathfrak{p}}$ and $\mathfrak{O}_{\mathfrak{p}}$ consists of all the elements $(\alpha + \zeta\beta)/2$ with $\alpha, \beta \in \mathfrak{o}_{\mathfrak{p}}$ and $\alpha \equiv \beta \bmod 2\mathfrak{o}_{\mathfrak{p}}$.

(ii) $K_\mathfrak{p}$ is a ramified extension of $F_\mathfrak{p}$ and ζ is a prime in $K_\mathfrak{p}$—the ramified prime case. Now we may assume $\pi = \zeta$, $p = \pi\pi^*$ and $\mathfrak{O}_\mathfrak{p}$ is generated over $\mathfrak{o}_\mathfrak{p}$ by 1 and π.

(iii) $K_\mathfrak{p}$ is a ramified extension of $F_\mathfrak{p}$ and ζ is a unit in $K_\mathfrak{p}$—the ramified unit case. We now have $\zeta^2 = 1 + p^{2h+1}\delta$ for some unit δ in $F_\mathfrak{p}$ and some rational integer h with $0 \leqq h < e$. Put $\pi = (1 + \zeta)p^{-h}$ so that $\pi\pi^* = -p\delta$. Here $\mathfrak{O}_\mathfrak{p}$ consists of the elements $(\alpha + \zeta\beta)p^{-h}$ with $\alpha, \beta \in \mathfrak{o}_\mathfrak{p}$ and $\alpha \equiv \beta \bmod p^h\mathfrak{o}_\mathfrak{p}$.

In the nondyadic (nonsplit) case $\mathfrak{O}_\mathfrak{p}$ is generated over $\mathfrak{o}_\mathfrak{p}$ by 1 and ζ provided we choose ζ to be a prime or a unit according as the extension is ramified or not.

Thus if $K_\mathfrak{p}/F_\mathfrak{p}$ is a quadratic extension of fields, $\mathfrak{O}_\mathfrak{p}$ consists of the elements $(\alpha + \zeta\beta)p^{-h}$ with $\alpha, \beta \in \mathfrak{o}_\mathfrak{p}$ and $\alpha \equiv \beta \bmod p^h\mathfrak{o}_\mathfrak{p}$, where we define $h = 0$ in the nondyadic and ramified prime dyadic cases, and $h = e$ in the unramified dyadic case.

Since $M_\mathfrak{p}$ is a unimodular $\mathfrak{O}_\mathfrak{p}$-lattice with rank at least three, it is split by a hyperbolic plane (if $\mathfrak{p}$ splits in K this can be easily verified, otherwise see [4; 7.1, 8.1a, 10.3]). Hence $M_\mathfrak{p} = H_\mathfrak{p} \perp L_\mathfrak{p}$ where $H_\mathfrak{p} = \mathfrak{O}_\mathfrak{p}u + \mathfrak{O}_\mathfrak{p}v$ is a hyperbolic plane with $q(u) = q(v) = 0$ and $\Phi(u, v) = 1$. This choice of u and v will be fixed throughout the local discussion.

We now describe the standard isometries in the unitary group $U(M_\mathfrak{p})$ that are needed. The norm and trace mappings from $K_\mathfrak{p}$ to $F_\mathfrak{p}$ are denoted by $\mathscr{N}$ and $\mathscr{T}$, respectively, and our convention for the hermitian form Φ on $V_\mathfrak{p}$ is $\Phi(\alpha x, \beta y) = \alpha^*\Phi(x, y)\beta$.

Let λ in $\mathfrak{O}_\mathfrak{p}$ have $\mathscr{T}(\lambda) = 0$. The transvection $T_\lambda(u)$ is defined by

$$T_\lambda(u)(z) = z + \lambda\Phi(u, z)u\ , \quad z \in M_\mathfrak{p}\ .$$

Then $\det T_\lambda(u) = 1$ so that $T_\lambda(u)$ is in $SU(M_\mathfrak{p})$. Similarly, $T_\lambda(v) \in SU(M_\mathfrak{p})$.

Let λ in $K_\mathfrak{p}$ satisfy $\mathscr{T}(\lambda) = 2\mathscr{N}(\lambda)$. For x in $M_\mathfrak{p}$ with $\lambda q(x)^{-1}$ in $\mathfrak{O}_\mathfrak{p}$, define the symmetry $\Psi_\lambda(x)$ by

$$\Psi_\lambda(x)(z) = z - \lambda\Phi(x, z)q(x)^{-1}x\ , \quad z \in M_\mathfrak{p}\ .$$

Then $\det \Psi_\lambda(x) = 1 - 2\lambda$ and $\Psi_\lambda(x) \in U(M_\mathfrak{p})$.

Recall that $M_{\mathfrak{p}*}$ is the sublattice of $M_\mathfrak{p}$ generated by those x in $M_\mathfrak{p}$ with $q(x) \in \mathfrak{o}_\mathfrak{p}$. Since $2q(x) = \Phi(x, x)$, in the nondyadic case $M_{\mathfrak{p}*} = M_\mathfrak{p}$. This is also true when $\mathfrak{p}$ splits in K; for the involution on $K_\mathfrak{p} = F_\mathfrak{p} \times F_\mathfrak{p}$ is given by $(\alpha, \beta)^* = (\beta, \alpha)$, so that for x in $M_\mathfrak{p}$,

$$q((1, 0)x) = \mathscr{N}(1, 0)q(x) = 0\ .$$

Thus $(1, 0)x \in M_{\mathfrak{p}*}$ and $x = (1, 1)x$ is in $M_{\mathfrak{p}*}$.

PROPOSITION 2.1. *Let $F_{\mathfrak{p}}$ be a dyadic local field with $\mathfrak{p}$ not split in K. Then*

$$M_{\mathfrak{p}*} = \{x \in M_{\mathfrak{p}} \mid p^h q(x) \in \mathfrak{o}_{\mathfrak{p}}\} .$$

In particular, $M_{\mathfrak{p}*} = M_{\mathfrak{p}}$ when $K_{\mathfrak{p}}$ is an unramified extension of $F_{\mathfrak{p}}$.

Proof. Let S be the set of all elements x in $M_{\mathfrak{p}}$ with $p^h q(x)$ in $\mathfrak{o}_{\mathfrak{p}}$. Since $\mathscr{T}(\mathfrak{O}_{\mathfrak{p}}) \subseteqq 2p^{-h}\mathfrak{o}_{\mathfrak{p}}$ and

$$q(x + y) = q(x) + q(y) + \mathscr{T}(\Phi(x, y))/2 ,$$

it follows that S is an $\mathfrak{O}_{\mathfrak{p}}$-module. Hence $M_{\mathfrak{p}*} \subseteqq S$. We now prove the converse inclusion. For x in S, let $x = y + z$ with $y \in H_{\mathfrak{p}}$ and $z \in L_{\mathfrak{p}}$. Clearly, u, v and consequently y are in S. Therefore, $z = x - y$ is in S and $p^h q(z) \in \mathfrak{o}_{\mathfrak{p}}$. Let $w = u - \alpha v + z$ where $\alpha = q(z)(1 + \zeta)$ is in $\mathfrak{O}_{\mathfrak{p}}$. Then $q(w) = 0$ and $w \in M_{\mathfrak{p}*}$. Hence $z \in M_{\mathfrak{p}*}$ and $S \subseteqq M_{\mathfrak{p}*}$, proving the proposition.

Fix μ in $\mathfrak{O}_{\mathfrak{p}}$ such that $\mathscr{T}(\mu) = 2$. For x in $L_{\mathfrak{p}}$ with $\mu q(x)$ in $\mathfrak{O}_{\mathfrak{p}}$, define the Siegel transformation $E(u, x)$ by

$$E(u, x)(z) = z - \Phi(u, z)x + \Phi(x, z)u - \mu q(x)\Phi(u, z)u .$$

Then $\det E(u, x) = 1$ and $E(u, x)$ is in $SU(M_{\mathfrak{p}})$. Similarly, define $E(v, x)$. Fix $\mu = 1$ except when $F_{\mathfrak{p}}$ is dyadic and $K_{\mathfrak{p}}$ is either an unramified or a ramified unit extension of $F_{\mathfrak{p}}$. In these exceptional cases fix $\mu = 1 + \zeta \in p^h\mathfrak{O}_{\mathfrak{p}}$. Except for the split dyadic case, it is now sufficient to choose x in $L_{\mathfrak{p}} \cap M_{\mathfrak{p}*}$ for $E(u, x)$ to be an integral isometry. Let $\mathscr{E}$ be the subgroup of $SU(M_{\mathfrak{p}})$ generated by the Siegel transformations.

In the following three sections we classify locally the primitive sublattices of $M_{\mathfrak{p}}^*$ invariant under the action of the special unitary group $SU(M_{\mathfrak{p}})$. We conclude this section with three observations. Assume that $\mathfrak{p}$ does not split in K.

2.2. *Any lattice $N_{\mathfrak{p}}$ satisfying $M_{\mathfrak{p}*} \subseteqq N_{\mathfrak{p}} \subseteqq M_{\mathfrak{p}}^*$ is invariant under the action of $\mathscr{E}$.*

Proof. Let $z \in N_{\mathfrak{p}}$ and $x \in L_{\mathfrak{p}} \cap M_{\mathfrak{p}*}$. Then $\Phi(x, z) \in \mathfrak{O}_{\mathfrak{p}}$ and

$$E(u, x)(z) \equiv z \bmod M_{\mathfrak{p}*} .$$

Hence $E(u, x)(z)$ and, likewise, $E(v, x)(z)$ lies in $N_{\mathfrak{p}}$. The result follows.

2.3. *If $N_{\mathfrak{p}}$ is invariant under $SU(M_{\mathfrak{p}})$ and $u \in N_{\mathfrak{p}}$ or $v \in N_{\mathfrak{p}}$, then $M_{\mathfrak{p}*} \subseteqq N_{\mathfrak{p}}$.*

Proof. For any x in $L_{\mathfrak{p}}$ with $q(x)^{-1}$ in $\mathfrak{O}_{\mathfrak{p}}$, we have $\Psi_1(u - v)\Psi_1(x)$

is in $SU(M_{\mathfrak{p}})$. This isometry interchanges u and v, so that $H_{\mathfrak{p}} \subseteqq N_{\mathfrak{p}}$. Let $y \in L_{\mathfrak{p}} \cap M_{\mathfrak{p}*}$. Then $E(u, y)(v)$ is in $N_{\mathfrak{p}}$ and hence $y \in N_{\mathfrak{p}}$. Thus $M_{\mathfrak{p}*} \subseteqq N_{\mathfrak{p}}$.

2.4. *Assume either $K_{\mathfrak{p}}$ is an unramified extension of $F_{\mathfrak{p}}$ or $F_{\mathfrak{p}}$ is a nondyadic field. Then $M_{\mathfrak{p}}$ is the unique primitive sublattice invariant under the action of $SU(M_{\mathfrak{p}})$.*

Proof. Let $N_{\mathfrak{p}}$ be a primitive invariant sublattice. It suffices by 2.3 to show that $u \in N_{\mathfrak{p}}$, since under our assumptions $M_{\mathfrak{p}*} = M_{\mathfrak{p}}$. Since $N_{\mathfrak{p}} \nsubseteqq \pi M_{\mathfrak{p}}$, there exists z in $N_{\mathfrak{p}}$ with $z \notin \pi M_{\mathfrak{p}}$. Let $z = \alpha u + \beta v + t$ where $t \in L_{\mathfrak{p}}$. If α and β are nonunits, there exists r in $L_{\mathfrak{p}}$ such that $\Phi(r, t) = 1$ (since $z \notin \pi M_{\mathfrak{p}}^*$). The coefficient of v in $E(v, r)(z) \in N_{\mathfrak{p}}$ is now a unit. Assume, therefore, $\beta = 1$ (or symmetrically, $\alpha = 1$). If $K_{\mathfrak{p}} = F_{\mathfrak{p}}(\zeta)$ is an unramified extension of $F_{\mathfrak{p}}$, ζ is a unit. Then $T_{\zeta}(u)(z) = z + \zeta u$ is in $N_{\mathfrak{p}}$. Hence $u \in N_{\mathfrak{p}}$ and the result follows. Now assume $F_{\mathfrak{p}}$ is a nondyadic field. Then $E(u, t)(z) = \gamma u + v$ is in $N_{\mathfrak{p}}$ for some γ in $\mathfrak{O}_{\mathfrak{p}}$. Let $w \in L_{\mathfrak{p}}$ have $q(w)$ a unit. Applying $E(u, \rho w)$ to $\gamma u + v \in N_{\mathfrak{p}}$ with $\rho = 1, -1$ gives $\rho w + q(w)u$ is in $N_{\mathfrak{p}}$. Since 2 is now a unit, it follows that u is in $N_{\mathfrak{p}}$ and hence $N_{\mathfrak{p}} = M_{\mathfrak{p}}$.

Theorem B has now been established except when either $\mathfrak{p}$ splits in K, or $K_{\mathfrak{p}}$ is a ramified extension of a dyadic field $F_{\mathfrak{p}}$.

3. Split extensions. Assume $\mathfrak{p}$ splits in K so that $K_{\mathfrak{p}} = F_{\mathfrak{p}} \times F_{\mathfrak{p}}$ and $\mathfrak{O}_{\mathfrak{p}} = \mathfrak{o}_{\mathfrak{p}} \times \mathfrak{o}_{\mathfrak{p}}$. Let $N_{\mathfrak{p}}$ be a primitive invariant sublattice of $M_{\mathfrak{p}}^* = M_{\mathfrak{p}} = H_{\mathfrak{p}} \perp L_{\mathfrak{p}}$. We wish to prove $N_{\mathfrak{p}} = M_{\mathfrak{p}}$. Since $N_{\mathfrak{p}} \nsubseteqq \pi M_{\mathfrak{p}}$ for any prime element π in $\mathfrak{O}_{\mathfrak{p}}$, there exists $x \in N_{\mathfrak{p}}$ with $x \notin \pi M_{\mathfrak{p}}$. Let $x = \alpha u + \beta v + t$ with $t \in L_{\mathfrak{p}}$. If β (or α) is a unit in $\mathfrak{O}_{\mathfrak{p}}$, we may assume $\beta = 1$. Then, since $\mathscr{T}(1, -1) = 0$, it follows that

$$T_{(1,-1)}(u)(x) = x + (1, -1)u$$

is in $N_{\mathfrak{p}}$. Thus $(1, -1)u$ and u are in $N_{\mathfrak{p}}$. As in 2.3, we now have $H_{\mathfrak{p}} \subseteqq N_{\mathfrak{p}}$. Let $y \in L_{\mathfrak{p}}$. Then $E(u, (1, 0)y)(v)$ is in $N_{\mathfrak{p}}$. Hence $(1, 0)y$, and likewise $(0, 1)y$, are in $N_{\mathfrak{p}}$. Consequently, $y \in N_{\mathfrak{p}}$ and $N_{\mathfrak{p}} = M_{\mathfrak{p}}$.

Now assume that neither $\alpha = (\alpha_1, \alpha_2)$ nor $\beta = (\beta_1, \beta_2)$ is a unit. If α_1 is a unit in $\mathfrak{o}_{\mathfrak{p}}$, replacing x by $T_{(1,-1)}(v)(x)$ if necessary, we may assume β_1 is also a unit. Hence, unless both α_1 and β_1 are nonunits, or both α_2 and β_2 are nonunits, we arrange that β becomes a unit in $\mathfrak{O}_{\mathfrak{p}}$ and we are finished. Assume, therefore, $\alpha_1, \beta_1 \in p\mathfrak{o}_{\mathfrak{p}}$. Since $x \notin \pi M_{\mathfrak{p}}$, there exists y in $M_{\mathfrak{p}}$ such that $\Phi(x, y) = (1, 1)$. Hence, there exists $r \in L_{\mathfrak{p}}$ such that $\Phi(t, r) = (?, 1)$. In $E(u, (0, 1)r)(x)$ the new coefficient of u has first component a unit. The second component is unchanged. We can thus arrange that β becomes a unit in $\mathfrak{O}_{\mathfrak{p}}$, and consequently $N_{\mathfrak{p}} = M_{\mathfrak{p}}$.

4. Ramified dyadic extensions. Now let $K_\mathfrak{p}$ be a ramified extension of the dyadic field $F_\mathfrak{p}$. Before classifying the primitive invariant sublattices in this case it is necessary to determine a set of generators for $U(M_\mathfrak{p})$. Special cases have already appeared in the work of Baeza [1] and Hayakawa [3], but it appears better to modify the approach in [5].

By [4; 10.3], we can split hyperbolic planes and write

$$M_\mathfrak{p} = H_\mathfrak{p} \perp J_\mathfrak{p} \perp B_\mathfrak{p}$$

where $J_\mathfrak{p}$ is an orthogonal sum of hyperbolic planes and rank $B_\mathfrak{p} \leqq 2$. Then $J_\mathfrak{p}$ has dual bases $w_1, \cdots, w_m$ and $z_1, \cdots, z_m$ such that $\Phi(w_i, z_j) = \delta_{ij}$, $1 \leqq i, j \leqq m$. Recall that $\mathscr{E}$ is the subgroup of $SU(M_\mathfrak{p})$ generated by the Siegel transformations defined in § 2.

PROPOSITION 4.1. *$U(M_\mathfrak{p})$ is generated by $\mathscr{E}$ and $U(H_\mathfrak{p} \perp B_\mathfrak{p})$.*

Proof. Let $\varphi \in U(M_\mathfrak{p})$. We reduce φ to the identity using the given isometries. Let $w_1, \cdots, w_m$ and $z_1, \cdots, z_m$ be dual bases of $J_\mathfrak{p}$, as above, and assume for some $k \leqq m$ that $\varphi(w_j) = w_j$, $1 \leqq j \leqq k-1$ (at worst, $k = 1$). Let

$$\varphi(u + w_k) = \varepsilon u + \beta v + t$$

where $t \in J_\mathfrak{p} \perp B_\mathfrak{p}$. We want ε to be a unit. Assume ε is not a unit. If β is a unit, use the isometry in $U(H_\mathfrak{p})$ which interchanges u and v. If β is not a unit, let $\varphi(z_k)$ have component r in $J_\mathfrak{p} \perp B_\mathfrak{p}$. Then $\Phi(t, r)$ is a unit. Since $z_k \in M_{\mathfrak{p}*}$, it follows that $r \in M_{\mathfrak{p}*}$. Also, $\Phi(r, w_j) = \Phi(\varphi(z_k), \varphi(w_j)) = 0$ for $1 \leqq j \leqq k-1$. Now replace φ by $E(u, r)\varphi$ and the new coefficient of u is a unit.

We may now assume ε is a unit. Let $s = t - w_k$. Then

$$\Phi(s, w_j) = \Phi(\varphi(u + w_k) - w_k, w_j) = 0$$

for $1 \leqq j \leqq k-1$. Also, since $q(t) \equiv q(w_k) \bmod p^{-h}\mathfrak{o}_\mathfrak{p}$, we have $s \in M_{\mathfrak{p}*}$. Put

$$\psi = E(u, -\varepsilon^* z_k)T_\lambda(v)E(v, \varepsilon^{-1}s)\varphi E(u, z_k)$$

where $\lambda \in \mathfrak{O}_\mathfrak{p}$ is to be chosen subject to the restraint $\mathscr{T}(\lambda) = 0$. Then $\psi(w_j) = w_j$ for $1 \leqq j \leqq k-1$. Choose λ such that

$$E(v, \varepsilon^{-1}s)\varphi E(u, z_k)(w_k) = \varepsilon(u - \lambda v) + w_k\,.$$

Then $\mathscr{T}(\lambda) = 0$ and $\psi(w_k) = w_k$. If ψ is generated by the given isometries, so is φ. The result now follows by induction on k.

This proposition reduces the question of generators for $U(M_\mathfrak{p})$ to the cases rank $M_\mathfrak{p} = 3, 4$. It can be easily verified that $U(H_\mathfrak{p})$ is generated by symmetries and transvections. Also, if rank $B_\mathfrak{p} = 2$ the basis w, z of $B_\mathfrak{p}$ can be chosen such that $\Phi(w, z) = 1$ and $z \in M_{\mathfrak{p}*}$ (see [4; 9.2]).

THEOREM 4.2. *$U(M_\mathfrak{p})$ is generated by $\mathscr{E}$, $U(H_\mathfrak{p})$ and symmetries on $B_\mathfrak{p}$.*

Proof. We need only consider rank $M_\mathfrak{p} = 3, 4$.

(i) Let rank $M_\mathfrak{p} = 4$ and $M_\mathfrak{p} = H_\mathfrak{p} \perp B_\mathfrak{p}$ with $B_\mathfrak{p}$ having a basis as above. We reduce φ in $U(M_\mathfrak{p})$ to the identity using the given isometries. From the proof of Proposition 4.1, we may assume $\varphi(w) = w$. In fact, if $w \in M_{\mathfrak{p}*}$, the proposition proves the theorem. Now assume $w \notin M_{\mathfrak{p}*}$. Put $r = w - 2q(w)z$ so that $\Phi(r, w) = 0$. Then

$$\varphi(z) = \alpha u + \beta v + z + \gamma r$$

for some α, β in $\mathfrak{O}_\mathfrak{p}$ and γ in $\pi\mathfrak{O}_\mathfrak{p}$ ($\gamma r \in M_{\mathfrak{p}*}$). Let

$$\mathscr{M}_z = \{x \in M \mid \Phi(x, z) = 1\} = w + H_\mathfrak{p} \perp \mathfrak{O}_\mathfrak{p}(z - 2q(z)w)$$

be the characteristic set of z (cf. [5; p. 429]). Then

$$q(\mathscr{M}_{\varphi(z)}) = q(\mathscr{M}_z) \equiv q(w) \bmod p^{-h}\mathfrak{o}_\mathfrak{p} .$$

Since $(1 - \alpha^*)w + v$ is in $\mathscr{M}_{\varphi(z)}$, it follows that $q(\alpha w) \in p^{-h}\mathfrak{o}_\mathfrak{p}$ and hence $\alpha w \in M_{\mathfrak{p}*}$. Similarly, $\beta w \in M_{\mathfrak{p}*}$. Interchanging u and v if necessary, we have $\beta = \alpha\lambda$ with $\lambda = (\lambda_1 + \lambda_2\zeta)p^{-h}$ in $\mathfrak{O}_\mathfrak{p}$ and $\lambda_1 \equiv \lambda_2 \bmod p^h$. Using a transvection, we can then arrange that $\lambda \in \mathfrak{o}_\mathfrak{p}$ in the ramified prime case and $\lambda \in \pi\mathfrak{o}_\mathfrak{p}$ in the ramified unit case. In the ramified prime case the proof can be completed by modifying the argument in [5; 2.4]; the symmetry on $B_\mathfrak{p}$ needed is $\Psi_\delta(r)$ with $\delta \in \mathscr{O}_\mathfrak{p}$. In the ramified unit case we proceed as follows. The coefficient of v in $E(v, \xi r)\varphi(z)$ is zero if

$$\alpha\lambda + \xi^*\Phi(r, z + \gamma r) = \mu q(\xi r)\alpha .$$

Here $\mu = 1 + \zeta = \pi p^h$ and $\varepsilon = \Phi(r, z + \gamma r)$ is a unit. By Hensel's lemma there exists a root ξ of the form $\xi = \varepsilon\pi^*\alpha^*\rho$ with ρ in $\mathfrak{o}_\mathfrak{p}$. Similarly, the coefficient of u can be made zero and we may assume $\varphi(z) = z + \gamma r$. Put $\delta = \gamma q(w) = -\gamma q(r)\Phi(z, r)^{-1}$. Then $\mathscr{T}(\delta) = 2\mathscr{N}(\delta)$ and $\Psi_\delta(r)^{-1}\varphi$ acts as the identity on both w and z. This completes the proof in this case.

(ii) Let rank $M_\mathfrak{p} = 3$ and $M_\mathfrak{p} = H_\mathfrak{p} \perp \mathfrak{O}_\mathfrak{p} w$ where $2q(w)$ is a unit. Again, we can reduce φ in $U(M_\mathfrak{p})$ to the identity by the isometries. Let

$$\varphi(w) = \alpha(u + \lambda v) + \eta w$$

where η is a unit. Moreover, as in the previous case, we may assume λ is in $\pi\mathfrak{o}_\mathfrak{p}$ (resp. $\mathfrak{o}_\mathfrak{p}$) in the ramified unit (resp. prime) case. Since

$$q(\mathfrak{O}_\mathfrak{p}\varphi(w)^\perp) = q(\mathfrak{O}_\mathfrak{p}w^\perp) = q(H_\mathfrak{p}) \subseteqq p^{-h}\mathfrak{o}_\mathfrak{p} ,$$

it follows that $\alpha w \in M_{\mathfrak{p}*}$. Using Siegel transformations we can reduce to the case $\varphi(w) = \varepsilon w$, although in the ramified prime case it is necessary to use the fact that $\mathscr{N}(\eta) \equiv 1 \bmod 4$ and hence $\mathscr{N}(\eta)$ is a square. Finally, since $\mathscr{N}(\varepsilon) = 1$, putting $\delta = (1 - \varepsilon)/2$ gives $\mathscr{T}(\delta) = 2\mathscr{N}(\delta)$ and $\Psi_\delta(w)^{-1}\varphi$ fixes w. This completes the proof.

COROLLARY 4.3. *Except in the ramified unit case with the rank of* $M_\mathfrak{p}$ *even, all lattices* $N_\mathfrak{p}$ *satisfying*

$$M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p} \subseteqq M_\mathfrak{p}^*$$

are invariant under the action of $U(M_\mathfrak{p})$.

Proof. This follows from 2.2 and the easily verified fact that $U(H_\mathfrak{p})$ and the symmetries used in the proof of the theorem preserve such $N_\mathfrak{p}$.

COROLLARY 4.4. *In the ramified unit case with* $\operatorname{rank} M_\mathfrak{p}$ *even, all lattices between* $M_{\mathfrak{p}*}$ *and* $M_\mathfrak{p}^*$ *are* $SU(M_\mathfrak{p})$*-invariant.*

Proof. Symmetries Ψ_δ in $U(H_\mathfrak{p})$ have $p^h\delta \in \mathfrak{O}_\mathfrak{p}$ and $\det \Psi_\delta \equiv 1 \bmod 2p^{-h}$. Hence, for φ in $SU(M_\mathfrak{p})$ in the proof of Theorem 2.2, the only symmetries $\Psi_\delta(r)$ on $B_\mathfrak{p}$ needed will also have $p^h\delta \in \mathfrak{O}_\mathfrak{p}$. These symmetries leave invariant lattices between $M_{\mathfrak{p}*}$ and $M_\mathfrak{p}^*$.

We now investigate the converse. Let $N_\mathfrak{p}$ be a primitive $SU(M_\mathfrak{p})$-invariant sublattice of $M_\mathfrak{p}^*$. As in 2.4, there exists $x = \alpha u + v + t$ in $N_\mathfrak{p}$ with $t \in L_\mathfrak{p}^*$ (letting $M_\mathfrak{p}^* = H_\mathfrak{p} \perp L_\mathfrak{p}^*$). In the ramified unit case ζ is a unit and $\mathscr{T}(\zeta) = 0$. Since $T_\zeta(u)(x) \in N_\mathfrak{p}$, it follows that $\zeta u \in N_\mathfrak{p}$. By 2.3, $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$, completing the proof of Theorem B in this case. Finally, the ramified prime case. If $\dim V_\mathfrak{p} \geqq 5$, then $L_\mathfrak{p}$ is split by a hyperbolic plane $H'_\mathfrak{p} = \mathfrak{O}_\mathfrak{p}u' + \mathfrak{O}_\mathfrak{p}v'$. Applying $E(u, u')$ to x, we obtain $u' - \Phi(u', t)u$ is in $N_\mathfrak{p}$. Applying $E(u, v')$ now gives $u \in N_\mathfrak{p}$ and hence $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Assume, therefore, the rank of $M_\mathfrak{p}$ is 3 or 4 and that the residue class field of $F_\mathfrak{p}$ has at least four elements. Let ε be a unit in $F_\mathfrak{p}$ with $\varepsilon^2 \not\equiv 1 \bmod p$. The proof of Theorem B is now easily completed by using the isometry $u \mapsto \varepsilon u$, $v \mapsto \varepsilon^{-1}v$ on x to obtain $v \in N_\mathfrak{p}$. The exceptional case is studied in the next section.

5. Exceptional invariant lattices. In this section $F_\mathfrak{p}$ is a totally ramified extension of the 2-adic field $\boldsymbol{Q}_2$ and $K_\mathfrak{p}$ is a ramified prime extension of $F_\mathfrak{p}$. Thus the residue class fields of both $F_\mathfrak{p}$ and $K_\mathfrak{p}$ have only two elements.

We consider first the case with $\dim V_\mathfrak{p} = 3$ so that $M_\mathfrak{p} = H_\mathfrak{p} \perp \mathfrak{O}_\mathfrak{p} w$. Then $M_{\mathfrak{p}*} = H_\mathfrak{p} \perp \mathfrak{O}_\mathfrak{p}\pi^e w$ and $M_\mathfrak{p}^* = H_\mathfrak{p} \perp \mathfrak{O}_\mathfrak{p}\pi^{-e} w$ where $e = \operatorname{ord}_p 2$. There are now two new invariant lattices

$$E_\mathfrak{p} = \pi M_\mathfrak{p}^* + \mathfrak{O}_\mathfrak{p}(u + v + \pi^{-e}w)$$

and its dual $E_\mathfrak{p}^\sharp$. It can be easily verified using the generators in Theorem 4.2 that $E_\mathfrak{p}$ is a $SU(M_\mathfrak{p})$-invariant lattice; it follows that the dual $E_\mathfrak{p}^\sharp$ is also invariant.

Let $N_\mathfrak{p}$ be a primitive invariant sublattice of $M_\mathfrak{p}^*$. As in the proof of 2.4, there exists an element $x = \alpha u + v + \beta w$ in $N_\mathfrak{p}$ with α and $\pi^e\beta$ in $\mathfrak{O}_\mathfrak{p}$. Since $\pi = \zeta$, $T_\pi(u)(x)$ is in $N_\mathfrak{p}$. Hence $\pi M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Assume first that $\pi^e\beta$ is a unit. Then $\pi x \in N_\mathfrak{p}$ forces $\pi^{1-e}w \in N_\mathfrak{p}$ and $\pi M_\mathfrak{p}^* \subseteqq N_\mathfrak{p}$. If α is not a unit, then the image of $v + \pi^{-e}w$ under $E(v, \pi^e w)$ is in $N_\mathfrak{p}$. Hence $v \in N_\mathfrak{p}$ and $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Assume, therefore, $\alpha \equiv 1 \bmod \pi$. We have now shown, when $\pi^e\beta$ is a unit, $E_\mathfrak{p} \subseteqq N_\mathfrak{p}$. Moreover, $E_\mathfrak{p} \neq N_\mathfrak{p}$ forces $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Now assume $\pi^e\beta$ is not a unit and apply $E(u, \pi^e w)$ to x. This gives $u + \pi^e w$ is in $N_\mathfrak{p}$. The isometry $u \mapsto v$, $v \mapsto u$, $w \mapsto -w$ is in $SU(M_\mathfrak{p})$. Hence both $v - \pi^e w$ and $u + v$ are in $N_\mathfrak{p}$. Define

$$G_\mathfrak{p} = \pi M_{\mathfrak{p}*} + \mathfrak{O}_\mathfrak{p}(u + v) + \mathfrak{O}_\mathfrak{p}(v + \pi^e w) .$$

Then $\pi^{-1}G_\mathfrak{p} = E_\mathfrak{p}^\sharp$, the dual lattice of $E_\mathfrak{p}$. Now, if $\pi^e\beta$ is not a unit, $G_\mathfrak{p} \subseteqq N_\mathfrak{p}$ and if $G_\mathfrak{p} \neq N_\mathfrak{p}$, necessarily $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. In summary,

5.1. *The only exceptional three dimensional invariant lattices are of the form* $\mathfrak{a}_\mathfrak{p}E_\mathfrak{p}$ *and* $\mathfrak{a}_\mathfrak{p}E_\mathfrak{p}^\sharp$, *with* $\mathfrak{a}_\mathfrak{p}$ *a fractional ideal in* $K_\mathfrak{p}$.

Now consider the more complicated situation when $\dim V = 4$ and $M_\mathfrak{p} = H_\mathfrak{p} \perp B_\mathfrak{p}$ with w, z a basis of $B_\mathfrak{p}$ having $\Phi(w, z) = 1$ and $z \in M_{\mathfrak{p}*}$. Let f be the minimal integer such that $\pi^f w$ is in $M_{\mathfrak{p}*}$. Then

$$M_{\mathfrak{p}*} = H_\mathfrak{p} \perp (\mathfrak{O}_\mathfrak{p}\pi^f w + \mathfrak{O}_\mathfrak{p} z) .$$

If $f = 0$, then $M_{\mathfrak{p}*} = M_\mathfrak{p}$ and it is easily verified that $M_\mathfrak{p}$ is the only primitive invariant lattice. Assume, therefore, $1 \leqq f \leqq e$. Now z can be chosen with $q(z)$ in $p\mathfrak{o}_\mathfrak{p}$. For $1 \leqq g \leqq f$, define

$$E(g)_\mathfrak{p} = \pi M_{\mathfrak{p}*} + \mathfrak{O}_\mathfrak{p}\pi^g w + \mathfrak{O}_\mathfrak{p}(u + v + \pi^{-f} z)$$

and

$$G(g)_\mathfrak{p} = \pi M_{\mathfrak{p}*} + \mathfrak{O}_\mathfrak{p}(u + v) + \mathfrak{O}_\mathfrak{p}\pi^{1-g} z + \mathfrak{O}_\mathfrak{p}(u + \pi^f w) .$$

Then $G(g)_\mathfrak{p} = \pi^{-1}E(g)_\mathfrak{p}^\sharp$ and using Theorem 4.2 we can check that these lattices are all $SU(M_\mathfrak{p})$-invariant. However, except when $f=1$, these are not the only new invariant lattices that arise. We shall only consider $f=1$ in detail; this includes the case where 2 is prime in $F_\mathfrak{p}$.

Let $N_\mathfrak{p}$ be a primitive $SU(M_\mathfrak{p})$-invariant sublattice of $M_\mathfrak{p}^*$. Again $N_\mathfrak{p}$ contains an element $x = \alpha u + v + \beta w + \gamma z$ with α, β and $\pi^f\gamma$ in $\mathfrak{O}_\mathfrak{p}$. Applying $T_\pi(u)$ to x gives $\pi u \in N_\mathfrak{p}$ and hence $\pi M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Since $E(u, z)(x)$ is in $N_\mathfrak{p}$, we can conclude that β is in $\pi\mathfrak{O}_\mathfrak{p}$ and z is in $N_\mathfrak{p}$, for otherwise $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Assume first that γ is in $\pi^{1-f}\mathfrak{O}_\mathfrak{p}$. Then $E(u, \pi^f w)(x) \in N_\mathfrak{p}$ gives $u + \pi^f w$ and $u + v$ are both in $N_\mathfrak{p}$. Hence $G(1)_\mathfrak{p} \subseteqq N_\mathfrak{p}$. If $f = 1$ and $G(1)_\mathfrak{p} \neq N_\mathfrak{p}$, necessarily $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Now assume $\pi^f\gamma$ is a unit. Then $E(u, \pi^f w)(x) \in N_\mathfrak{p}$ gives $\pi^f w \in N_\mathfrak{p}$. If α is a nonunit, applying $E(v, \pi^f w)$ to x leads to $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Hence $\alpha \equiv 1 \bmod \pi$ and now $u + v + \beta w + \pi^{-f} z$ is in $N_\mathfrak{p}$ with $\beta \in \pi\mathfrak{O}_\mathfrak{p}$. Again, if $f = 1$, this gives $E(1)_\mathfrak{p} \subseteqq N_\mathfrak{p}$ and, if $E(1)_\mathfrak{p} \neq N_\mathfrak{p}$, necessarily $M_{\mathfrak{p}*} \subseteqq N_\mathfrak{p}$. Hence,

5.2. *For $f = 1$ the only exceptional four dimensional invariant lattices are of the form $\mathfrak{a}_\mathfrak{p}E(1)_\mathfrak{p}$ and $\mathfrak{a}_\mathfrak{p}E(1)_\mathfrak{p}^\#$, with $\mathfrak{a}_\mathfrak{p}$ a fractional ideal in $K_\mathfrak{p}$.*

For $f \geqq 2$, the analysis of the exceptional lattices is more complicated, but could be carried out in the above manner.

6. **Global results.** We start by proving Theorem A; in fact, this result remains valid even if M is not unimodular.

First let N be a $SU(M)$-invariant sublattice of M. We must prove $N_\mathfrak{p} = \mathfrak{O}_\mathfrak{p}N$ is $SU(M_\mathfrak{p})$-invariant at all finite prime spots $\mathfrak{p}$ of F. Fix a finite prime spot $\mathfrak{q}$ and an isometry $\psi_\mathfrak{q}$ in $SU(M_\mathfrak{q})$. By the approximation theorem of Shimura [8; 5.12], there exists a φ in $SU(V)$ with local extension $\varphi_\mathfrak{q}$ close to $\psi_\mathfrak{q}$ at the spot $\mathfrak{q}$ and $\varphi_\mathfrak{p}(M_\mathfrak{p}) = M_\mathfrak{p}$ elsewhere. Since $\psi_\mathfrak{q}(M_\mathfrak{q}) = M_\mathfrak{q}$, we have $\varphi_\mathfrak{q}(M_\mathfrak{q}) = M_\mathfrak{q}$ if $\varphi_\mathfrak{q}$ is sufficiently close to $\psi_\mathfrak{q}$ and hence $\varphi(M) = M$. Thus φ is in $SU(M)$ and hence $\varphi(N) = N$. Therefore, $\varphi_\mathfrak{q}(N_\mathfrak{q}) = N_\mathfrak{q}$ and if $\varphi_\mathfrak{q}$ is sufficiently close to $\psi_\mathfrak{q}$, necessarily $N_\mathfrak{q}$ is invariant under $\psi_\mathfrak{q}$.

Conversely, let N be a lattice in M with $N_\mathfrak{p} = \mathfrak{O}_\mathfrak{p}N$ a $SU(M_\mathfrak{p})$-invariant lattice at all finite prime spots $\mathfrak{p}$. We must prove $\varphi(N) = N$ for all φ in $SU(M)$. Clearly, however, $\varphi_\mathfrak{p} \in SU(M_\mathfrak{p})$ so that $\varphi(N)_\mathfrak{p} = \varphi_\mathfrak{p}(N_\mathfrak{p}) = N_\mathfrak{p}$. The result now follows as in O'Meara [7; § 81E]. Notice that this half of the proof does not require that Φ be indefinite. This completes the proof of Theorem A.

We can also construct global invariant lattices from local ones as follows.

Proposition 6.1. *At each finite spot $\mathfrak{p}$ of F assume given a*

$SU(M_\mathfrak{p})$-invariant sublattice $J_\mathfrak{p}$ of $M_\mathfrak{p}$ with $J_\mathfrak{p} = M_\mathfrak{p}$ almost always. Then there exists a sublattice N of M such that for each spot $\mathfrak{p}$

$$N_\mathfrak{p} = \mathfrak{O}_\mathfrak{p} N = J_\mathfrak{p} .$$

Proof. This is an immediate consequence of [2; 2.4].

We conclude this paper by giving more explicitly the invariant lattices when F is the rational field $\boldsymbol{Q}$. Now $K = \boldsymbol{Q}(\sqrt{m})$ with m a square free integer. Let p be a rational prime. Then p splits in K if either $p = 2$ and $m \equiv 1 \bmod 8$, or p is odd and $(m/p) = 1$. Otherwise, for $p = 2$, we have an unramified extension if $m \equiv 5 \bmod 8$, a ramified unit extension with $h = 0$ if $m \equiv 3 \bmod 4$, and a ramified prime extension if m is even.

Let M be a unimodular lattice on an indefinite hermitian space V over $\boldsymbol{Q}(\sqrt{m})$. Except when $\boldsymbol{Q}_2(\sqrt{m})$ is a ramified extension of $\boldsymbol{Q}_2$, the only primitive invariant sublattice is $M_\mathfrak{p}$. Hence, when $m \equiv 1 \bmod 4$, the $SU(M)$-invariant lattices are the $\mathfrak{a}M$ with $\mathfrak{a}$ a fractional ideal in $\boldsymbol{Q}(\sqrt{m})$.

When $m \equiv 3 \bmod 4$ or m is even, $\boldsymbol{Q}_2(\sqrt{m})$ is a ramified extension of $\boldsymbol{Q}_2$ and M_2 can support other local invariant lattices. If the rank of M is odd, the invariant lattices are the $\mathfrak{a}N$ with $\mathfrak{a}$ a fractional ideal and N_2 one of the lattices M_{2*}, M_2 or M_2^*, together with E_2 and $E_2^\#$ when m is even and $\dim V = 3$.

Finally, when the rank of M is even there are a number of possibilities. If Φ is an even form, namely if $M_{2*} = M_2$, the only invariant sublattices are the $\mathfrak{a}M$ with $\mathfrak{a}$ a fractional ideal. If Φ is an odd form and $m \equiv 3 \bmod 4$ or m is even, there are five lattices N_2 lying between M_{2*} and M_2^*. If $M_2 = H_2 \perp J_2 \perp (\mathfrak{O}_2 w + \mathfrak{O}_2 z)$ with $\Phi(w, z) = 1$, $2q(w)$ a unit and $q(z) \in \mathfrak{o}_\mathfrak{p}$, these five lattices are M_2, M_{2*}, M_2^*,

$$H_2 \perp J_2 \perp (\mathfrak{O}_2 \pi w + \mathfrak{O}_2 \pi^{-1} z)$$

and

$$H_2 \perp J_2 \perp (\mathfrak{O}_2 \pi w + \mathfrak{O}_2(w + \pi^{-1} z)) .$$

For $\dim V \geqq 6$ and for $\dim V = 4$ when $m \equiv 3 \bmod 4$, the invariant lattices are the $\mathfrak{a}N$ with $\mathfrak{a}$ a fractional ideal, N_2 one of these five lattices and $N_p = M_p$ for p odd. When $\dim V = 4$ and m is even, N_2 can also be one of the dual pair of exceptional lattices $E(1)_2$ and $E(1)_2^\#$ obtained in the previous section.

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