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**TRANSCENDENTAL CONSTANTS OVER THE COEFFICIENT  
FIELDS IN DIFFERENTIAL ELLIPTIC FUNCTION FIELDS**

KEIJI NISHIOKA

# TRANSCENDENTAL CONSTANTS OVER THE COEFFICIENT FIELDS IN DIFFERENTIAL ELLIPTIC FUNCTION FIELDS

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Let  $k$  be a differential field of characteristic 0, and  $\Omega$  be a universal extension of  $k$ . Suppose that the field of constants  $k_0$  of  $k$  is algebraically closed. Consider the following differential polynomial of the first order over  $k$  in a single indeterminate  $y$ :

$$T(y) = (y')^2 - \lambda S(y; \kappa); \quad \lambda \in k; \quad \lambda \neq 0;$$

here

$$S(y; \kappa) = y(1 - y)(1 - \kappa^2 y); \\ \kappa \in k; \quad \kappa^2 \neq 0, 1; \quad \kappa' = 0.$$

Take a generic point  $z$  of the general solution of  $T$ . Then,  $z$  is transcendental over  $k$ , and  $k(z, z')$  is called a differential elliptic function field.

We prove the following:

**THEOREM.** Let  $k(z, z')$  be a differential elliptic function field over  $k$ . Then, there exists a finitely generated differential extension field  $k^*$  of  $k$  such that the following three conditions are satisfied:

- (i)  $z$  is transcendental over  $k^*$ ;
- (ii) the field of constants of  $k^*$  is the same as  $k_0$ ;
- (iii) there exists an element  $\zeta$  of  $\Omega$  such that  $k^*(z, z') = k^*(\zeta, \zeta')$  and  $(\zeta')^2 = 4S(\zeta; \kappa)$  with the same modulus as  $\kappa$ .

Matsuda [3] gave an example of a differential elliptic function field such that  $k = \bar{k}$  and we can not take  $k$  as  $k^*$  (cf. [5]).

**REMARK.** Matsuda [3] gave a differential algebraic proof of the following theorem essentially due to Poincaré: Suppose that a differential algebraic function field  $K$  over an algebraically closed coefficient field  $k$  is free from parametric singularities. Then,  $K$  is a differential elliptic function field over  $k$  if the genus of  $K$  is 1.

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1. Two lemmas. The following theorem is due to Kolchin [1]:

**LEMMA 1.** Let  $\Sigma$  be a perfect differential ideal in the differential polynomial algebra  $k\{y\}$ , and let  $J$  be a differential polynomial

in  $k\langle y \rangle$  which is not in  $\Sigma$ . Then,  $\Sigma$  has a zero  $\eta$  in  $\Omega$  such that  $J(\eta) \neq 0$  and the field of constants of  $k\langle \eta \rangle$  is  $k_0$ .

We shall prove the following:

**LEMMA 2.** *Let  $F$  be an element of  $k\langle y \rangle$  of the first order and let  $\xi$  be a zero of  $F$  which is transcendental over  $k$ . Suppose that  $F$  is algebraically irreducible over the algebraic closure  $\bar{k}$  of  $k$ , and that the field of constants of  $k\langle \xi \rangle$  is  $k_0$ . Then, there exists a nonsingular zero  $\eta$  of  $F$  such that  $\xi$  is transcendental over  $k\langle \eta \rangle$  and the field of constants of  $k\langle \eta \rangle$  is  $k_0$ .*

*Proof.* Let  $\eta$  be a generic point of the general solution of  $F$  over  $\overline{k\langle \xi \rangle}$ . Then,  $\eta \notin \overline{k\langle \xi \rangle}$  and  $\eta \notin \bar{k}$ . Hence,  $\xi \notin \overline{k\langle \eta \rangle}$ . By Gourin's theorem (cf. [4, p. 49]) both  $\xi$  and  $\eta$  are generic points of the general solution of  $F$  over  $k$ . Hence, there exists an isomorphism of  $k\langle \xi \rangle$  onto  $k\langle \eta \rangle$  over  $k$ . Therefore, the field of constants of  $k\langle \eta \rangle$  is  $k_0$ .

**2. Proof of Theorem.** We shall prove that there exists a nonsingular zero  $w$  of  $T$  such that  $z$  is transcendental over  $k\langle w \rangle$  and the field of constants of  $k\langle w \rangle$  is  $k_0$ . First we shall assume that the field of constants of  $k\langle z \rangle$  contains properly  $k_0$ . Let  $\Sigma$  be the prime differential ideal in  $k\langle y \rangle$  associated with the general solution of  $T$ . Then, the separant  $2y'$  of  $T$  does not belong to  $\Sigma$ . By Lemma 1, there exists a nonsingular zero  $w$  of  $T$  such that the field of constants of  $k\langle w \rangle$  is  $k_0$ . Suppose that  $z$  is algebraic over  $k\langle w \rangle$ . Then, the field of constants of  $k\langle z \rangle$  is contained in  $k_0$ , since  $k\langle z \rangle \subseteq \overline{k\langle w \rangle}$ . This contradicts our assumption. Hence,  $z$  is transcendental over  $\overline{k\langle w \rangle}$ . Secondly, let us assume that the field of constants of  $k\langle z \rangle$  is the same as  $k_0$ . Then, there exists a nonsingular zero  $w$  of  $T$  such that the field of constants of  $k\langle w \rangle$  is  $k_0$  and  $z$  is transcendental over  $k\langle w \rangle$  by Lemma 2, since  $T$  is algebraically irreducible over  $\bar{k}$ .

We shall denote  $k\langle w \rangle$  by  $k_1$ . Let us define an element  $a$  of  $k_1\langle z \rangle$  by

$$a = \{B(z, w) - 2\lambda^{-1}w'z'\}/A(z, w)^2,$$

where

$$A(y_1, y_2) = 1 - \kappa^2 y_1 y_2$$

$$B(y_1, y_2) = y_1(1 - y_2)(1 - \kappa^2 y_2) + y_2(1 - y_1)(1 - \kappa^2 y_1).$$

The polynomials  $A$ ,  $B$  and  $S$  satisfy a relation:

$$(1) \quad B(y_1, y_2)^2 = 4S(y_1)S(y_2) + (y_1 - y_2)^2 A(y_1, y_2)^2$$

which is verified in the following:

$$\begin{aligned}
 & B(y_1, y_2)^2 - 4S(y_1)S(y_2) \\
 &= \{y_1y_2^{-1}S(y_2) + y_2y_1^{-1}S(y_1)\}^2 - 4S(y_1)S(y_2) \\
 &= \{y_1y_2^{-1}S(y_2) - y_2y_1^{-1}S(y_1)\}^2 \\
 &= \{y_1(1 - y_2)(1 - \kappa^2y_2) - y_2(1 - y_1)(1 - \kappa^2y_1)\}^2 \\
 &= \{y_1 - y_2 - \kappa^2(y_1^2y_2 - y_1y_2^2)\}^2 \\
 &= (y_1 - y_2)^2A(y_1, y_2)^2.
 \end{aligned}$$

By the definition of  $a$

$$\{A(z, w)^2a - B(z, w)\}^2 - 4\lambda^{-2}(w')^2(z')^2 = 0.$$

Since  $w$  and  $z$  are solutions of  $T = 0$  and (1), the left hand side is

$$\begin{aligned}
 & \{A(z, w)^2a - B(z, w)\}^2 - 4S(w)S(z) \\
 &= A(z, w)^4a^2 - 2A(z, w)^2B(z, w)a + B(z, w)^2 - 4S(w)S(z) \\
 &= A(z, w)^4a^2 - 2A(z, w)^2B(z, w)a + (z - w)^2A(z, w)^2 \\
 &= A(z, w)^2\{A(z, w)^2a^2 - 2B(z, w)a + (z - w)^2\}.
 \end{aligned}$$

Since  $A(z, w) \neq 0$ , we have an algebraic relation over  $k_1$  between  $a$  and  $z$ :

$$(2) \quad A(z, w)^2a^2 - 2B(z, w)a + (z - w)^2 = 0.$$

The left hand side of (2) is

$$\begin{aligned}
 & (1 - \kappa^2zw)^2a^2 - 2z(1 - w)(1 - \kappa^2w) + w(1 - z)(1 - \kappa^2z)a \\
 & \quad + (z - w)^2 \\
 &= a^2(\kappa^4z^2w^2 - 2\kappa^2zw + 1) \\
 & \quad - 2a[\kappa^2wz^2 + \{\kappa^2w^2 - 2(1 + \kappa^2)w + 1\}z + w] \\
 & \quad + z^2 - 2zw + w^2 \\
 &= z^2(\kappa^4a^2w^2 - 2\kappa^2aw + 1) \\
 & \quad - 2z[\kappa^2wa^2 + \{\kappa^2w^2 - 2(1 + \kappa^2)w + 1\}a + w] \\
 & \quad + a^2 - 2wa + w^2.
 \end{aligned}$$

Hence we have a relation equivalent to (2):

$$(3) \quad A(a, w)^2z^2 - 2B(a, w)z + (a - w)^2 = 0.$$

Since  $z$  is transcendental over  $k_1$ ,  $a$  is transcendental over  $k_1$  and satisfies  $[k_1(a, z):k_1(z)] = 2$ . For the discriminant of (2) is  $16S(z)S(w)$  by (1). We have  $k_1\langle z \rangle = k_1(a, z)$ . We shall prove that  $a$  is a constant (cf. [2, p. 805]). Let us take an element  $\alpha$  of  $\bar{k}$  such that  $\alpha^2 = 4/\lambda$  and define a new differentiation signed by the dot in  $k_1\langle \alpha, z \rangle$  by  $\dot{x} = \alpha x'$ . Then,

$$(4) \quad a = \{B(z, w) - 2^{-1}\dot{w}\dot{z}\}/A(z, w)^2, \\ (\dot{z})^2 = 4S(z), \quad (\dot{w})^2 = 4S(w).$$

In what follows, we denote  $A(z, w)$  and  $B(z, w)$  by  $A$  and  $B$  respectively for simplicity. Differentiating both sides of  $(\dot{w})^2 = 4S(w)$ , we have  $2\dot{w}\ddot{w} = 4S_w\dot{w}$  and  $\ddot{w} = 2S_w$  since  $\dot{w} \neq 0$ . Hence,

$$\begin{aligned} B_z - \ddot{w}/2 &= B_z - S_w \\ &= (1-w)(1-\kappa^2w) + w\{2\kappa^2z - (1+\kappa^2)\} \\ &\quad - \{3\kappa^2w^2 - 2(1+\kappa^2)w + 1\} \\ &= -2\kappa^2w^2 + 2\kappa^2wz \\ &= 2\kappa^2w(z-w). \end{aligned}$$

On the other hand

$$\begin{aligned} 2A_zB - (\dot{w})^2A_w &= -2\kappa^2wB + 4\kappa^2zw(1-w)(1-\kappa^2w) \\ &= 2\kappa^2w\{2z(1-w)(1-\kappa^2w) - B\} \\ &= 2\kappa^2w\{z(1-w)(1-\kappa^2w) - w(1-z)(1-\kappa^2z)\} \\ &= 2\kappa^2w(z-w)A. \end{aligned}$$

Therefore

$$A(B_z - \ddot{w}/2) = 2A_zB - (\dot{w})^2A_w = 2\kappa^2w(z-w)A.$$

Similarly we have

$$A(B_w - \ddot{z}/2) = 2A_wB - (\dot{z})^2A_z = 2\kappa^2z(w-z)A.$$

From the above equalities and (4)

$$\begin{aligned} A^3\dot{a} &= \dot{z}\{A(B_z - \ddot{w}/2) - 2A_zB + (\dot{w})^2A_w\} \\ &\quad + \dot{w}\{A(B_w - \ddot{z}/2) - 2A_wB + (\dot{z})^2A_z\} \\ &= 0. \end{aligned}$$

Hence,  $\dot{a} = 0$ , and  $a' = 0$ .

Let  $k_2$  denote  $k_1(\alpha)$  and  $b$  be an element of  $k_2\langle z \rangle$  defined by

$$b = \{A(a, w)^2z - B(a, w)\}/(\alpha w').$$

Then, we have  $b^2 = S(a)$ . In fact from (1) and (3) we have

$$\begin{aligned} \{A(a, w)^2z - B(a, w)\}^2 &= B(a, w)^2 - (a-w)^2A(a, w)^2 \\ &= 4S(a)S(w), \end{aligned}$$

and  $(\alpha w')^2 = 4S(w)$  since  $w$  is a solution of  $T = 0$ . Hence,  $k_2\langle z \rangle = k_2(\alpha, b)$  because  $[k_2\langle z \rangle : k_2(\alpha)] = [k_2(\alpha, b) : k_2(\alpha)] = 2$  and  $b \in k_2\langle z \rangle$ .

By Lemma 1, there exists a nonsingular solution  $v$  of  $(y')^2 = 4S(y)$  such that the field of constants of  $k_2\langle v \rangle$  is  $k_0$ . Since  $a$  is a constant,

$$\text{trans. deg } k^*(a)/k^* = \text{trans. deg } k_0(a)/k_0 = 1,$$

where  $k^* = k_2\langle v \rangle$  (cf. [2, p. 767]). Hence,  $a$  is transcendental over  $k^*$ . Therefore,  $z$  is transcendental over  $k^*$  by (3).

Let us define an element  $\zeta$  of  $k^*\langle z \rangle$  by

$$\zeta = \{B(a, v) + bv'\}/A(a, v)^2.$$

Matsuda [3] proved that  $\zeta$  is a solution of  $(y')^2 = 4S(y)$  and  $k^*(\zeta, \zeta') = k^*(a, b)$ : We may take elements  $s_i, c_i, d_i$  ( $1 \leq i \leq 3$ ) of  $\Omega$  such that

$$\begin{aligned} s_1^2 &= v, & c_1^2 &= 1 - v, & d_1^2 &= 1 - \kappa^2 v, & s_1' &= c_1 d_1; \\ s_2^2 &= a, & c_2^2 &= 1 - a, & d_2^2 &= 1 - \kappa^2 a, & b &= s_2 c_2 d_2; \\ s_3 &= (s_1 c_2 d_2 + s_2 c_1 d_1)(1 - \kappa^2 s_1^2 s_2^2)^{-1}; \\ c_3 &= (c_1 c_2 - s_1 s_2 d_1 d_2)(1 - \kappa^2 s_1^2 s_2^2)^{-1}; \\ d_3 &= (d_1 d_2 - \kappa^2 s_1 s_2 c_1 c_2)(1 - \kappa^2 s_1^2 s_2^2)^{-1}. \end{aligned}$$

We shall prove that

$$(5) \quad c_3^2 = 1 - s_3^2, \quad d_3^2 = 1 - \kappa^2 s_3^2, \quad s_3' = c_3 d_3.$$

In fact by the definitions

$$c_1' = -s_1 d_1, \quad d_1' = -\kappa^2 s_1 c_1, \quad c_2' = d_2' = 0.$$

Since

$$1 - \kappa^2 s_1^2 s_2^2 = c_1^2 + s_1^2 d_2^2 = c_2^2 + s_2^2 d_1^2,$$

we have

$$\begin{aligned} (1 - s_3^2)(1 - \kappa^2 s_1^2 s_2^2)^2 &= (1 - \kappa^2 s_1^2 s_2^2)^2 - (s_1 c_2 d_2 + s_2 c_1 d_1)^2 \\ &= (c_1^2 + s_1^2 d_2^2)(c_2^2 + s_2^2 d_1^2) - (s_1 c_2 d_2 + s_2 c_1 d_1)^2 \\ &= c_1^2 c_2^2 + s_1^2 s_2^2 d_1^2 d_2^2 - 2s_1 s_2 c_1 c_2 d_1 d_2 \\ &= (c_1 c_2 - s_1 s_2 d_1 d_2)^2. \end{aligned}$$

Hence,  $c_3^2 = 1 - s_3^2$ . Similarly, we have  $d_3^2 = 1 - \kappa^2 s_3^2$ , since

$$1 - \kappa^2 s_1^2 s_2^2 = d_1^2 + \kappa^2 s_1^2 c_2^2 = d_2^2 + \kappa^2 s_2^2 c_1^2.$$

We have  $s_3' = c_3 d_3$  according to the following:

$$\begin{aligned} (1 - \kappa^2 s_1^2 s_2^2)^2 s_3' &= (1 - \kappa^2 s_1^2 s_2^2)(s_1 c_2 d_2 + s_2 c_1 d_1)' \\ &= (1 - \kappa^2 s_1^2 s_2^2)(s_1 c_2 d_2 + s_2 c_1 d_1)' \end{aligned}$$

$$\begin{aligned}
& - (1 - \kappa^2 s_1^2 s_2^2)' (s_1 c_2 d_2 + s_2 c_1 d_1) \\
= & (1 - \kappa^2 s_1^2 s_2^2) (s_1' c_2 d_2 + s_2 c_1' d_1 + s_2 c_1 d_1') \\
& + 2\kappa^2 s_1 s_1' s_2^2 (s_1 c_2 d_2 + s_2 c_1 d_1) \\
= & (1 - \kappa^2 s_1^2 s_2^2) (c_1 c_2 d_1 d_2 - s_1 s_2 d_1^2 - \kappa^2 s_1 s_2 c_1^2) \\
& + 2\kappa^2 s_1 s_2^2 c_1 d_1 (s_1 c_2 d_2 + s_2 c_1 d_1) \\
= & c_1 c_2 d_1 d_2 - s_1 s_2 d_1^2 - \kappa^2 s_1 s_2 c_1^2 - \kappa^2 s_1^2 s_2^2 c_1 c_2 d_1 d_2 \\
& + \kappa^2 s_1^3 s_2^3 d_1^2 + \kappa^4 s_1^3 s_2^3 c_1^2 + 2\kappa^2 s_1^2 s_2^2 c_1 c_2 d_1 d_2 + 2\kappa^2 s_1 s_2^2 c_1^2 d_1^2 \\
= & c_1 c_2 d_1 d_2 + \kappa^2 s_1^2 s_2^2 c_1 c_2 d_1 d_2 \\
& - s_1 s_2 (d_1^2 + \kappa^2 c_1^2 - \kappa^2 s_1^2 s_2^2 d_1^2 - \kappa^4 s_1^2 s_2^2 c_1^2 - 2\kappa^2 s_2^2 c_1^2 d_1^2) ;
\end{aligned}$$

here

$$\begin{aligned}
d_1^2 + \kappa^2 c_1^2 - \kappa^2 s_1^2 s_2^2 d_1^2 - \kappa^4 s_1^2 s_2^2 c_1^2 - 2\kappa^2 s_2^2 c_1^2 d_1^2 \\
= & d_1^2 (1 - \kappa^2 s_1^2 s_2^2 - \kappa^2 s_2^2 c_1^2) \\
& + \kappa^2 c_1^2 (1 - \kappa^2 s_1^2 s_2^2 - s_2^2 d_1^2) \\
= & d_1^2 \{1 - \kappa^2 s_2^2 (s_1^2 + c_1^2)\} + \kappa^2 c_1^2 \{1 - s_2^2 (\kappa^2 s_1^2 + d_1^2)\} \\
= & d_1^2 d_2^2 + \kappa^2 c_1^2 c_2^2 .
\end{aligned}$$

Hence,

$$\begin{aligned}
(1 - \kappa^2 s_1^2 s_2^2) s_3' \\
= & c_1 c_2 d_1 d_2 + \kappa^2 s_1^2 s_2^2 c_1 c_2 d_1 d_2 - s_1 s_2 (d_1^2 d_2^2 + \kappa^2 c_1^2 c_2^2) \\
= & (c_1 c_2 - s_1 s_2 d_1 d_2) (d_1 d_2 - \kappa^2 s_1 s_2 c_1 c_2) ,
\end{aligned}$$

and we have  $s_3' = c_3 d_3$ .

By the definition of  $\zeta$  we have irreducible equations over  $k^*$ :

$$\begin{aligned}
A(a, v)^2 \zeta^2 - 2B(a, v) + (a - v)^2 = 0 , \\
A(\zeta, v)^2 a^2 - 2B(\zeta, v) a + (\zeta - v)^2 = 0 ,
\end{aligned}$$

as we get (2) and (3). Hence,  $k^*(\zeta, \zeta') = k^*(a, b) = k^*(z, z')$ . For we have  $[k^*(\zeta, \zeta') : k^*(\zeta)] = [k^*(a, \zeta) : k^*(\zeta)] = [k^*(a, \zeta) : k^*(a)] = [k^*(a, b) : k^*(a)] = 2$  by above equalities.

We remark that the adopting of the  $s$ ,  $c$  and  $d$  gives an expository verification of the identity  $(\zeta')^2 = 4S(\zeta)$  proved by Matsuda [3].

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