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λ -LARGE SUBGROUPS OF C_λ -GROUPS

RONALD CHARLES LINTON

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RONALD C. LINTON

If L is a fully invariant subgroup of the p -primary group G , and if $G = B + L$ for all basic subgroups B of G , then L is called a large subgroup of G ; this definition is due to R. Pierce. In light of K. Wallace's generalization of the concept of basic subgroup to that of a λ -basic subgroup, we extend Pierce's definition by defining the fully invariant subgroup L to be a λ -large subgroup of G if $G = B + L$ for all λ -basic subgroups B of G . Our main theorems are: (1) L is a λ -large subgroup of the C_λ -group G if and only if $L = G(v)$ where v denotes an increasing sequence of ordinals less than λ satisfying the gap condition. (2) If L is a λ -large subgroup of the C_λ -group G , then G/L is a totally projective group, and L is a C_μ -group where μ denotes the length of $L/p^\lambda G$. (3) If L is a λ -large subgroup of the C_λ -group G , then L is a totally projective group only if G is a totally projective group.

1. Preliminaries. All our groups are additively written, abelian, p -primary groups for some prime p . Most of the terminology and notation we use can be found in [2].

DEFINITION 1. [10] Let λ denote a limit ordinal and B a subgroup of the p -primary group G . Then B is called a λ -basic subgroup of G if B is a totally projective group of length at most λ , B is a p^λ -pure [8] subgroup of G , and G/B is divisible. A reduced p -primary group G is a C_λ -group if $G/p^\alpha G$ is a totally projective group for all α less than λ .

Wallace has shown in [10] that the p -primary group G contains a proper λ -basic subgroup if and only if λ is cofinal with ω (the first infinite ordinal) and G is a C_λ -group. Thus λ will henceforth denote a limit ordinal cofinal with ω . If G is a C_λ -group of length less than λ , then G is necessarily a totally projective group. Since properties of these groups are well-known, we shall restrict our attention to C_λ -groups of length at least λ . By applying results in [8], we can prove the following.

PROPOSITION 1. A subgroup B of G is a λ -basic subgroup if and only if (1) B is a totally projective group of length λ , (2) $G[p] \subseteq p^\alpha G + B[p]$ for all α less than λ , and (3) there is no subgroup H of G properly containing B such that $H[p] = B[p]$.

DEFINITION 2. If L is a fully invariant subgroup of the C_λ -group G , then L is called a λ -large subgroup of G if $G = B + L$ for all λ -basic subgroups B of G .

Note that the ω -large subgroups are just the large subgroups that Pierce studies in [9]. It follows from Proposition 1 that $p^\alpha G$ is a λ -large subgroup of G whenever α is less than λ . In addition, a straightforward argument shows that $p^\alpha L$ is a λ -large subgroup of G whenever L itself possesses this property and n is a positive integer; further on, we shall show that $p^\alpha L$ is also a λ -large subgroup if α is less than the length of $L/p^\alpha G$.

DEFINITION 3. [2] Let $v = (\sigma(0), \sigma(1), \dots, \sigma(n), \dots)$ denote a sequence of ordinals and perhaps symbols ∞ such that for any k and t , $\sigma(k) < \sigma(k+1)$ if $\sigma(k)$ is an ordinal and $\sigma(t+1) = \infty$ if $\sigma(t) = \infty$. We say that v satisfies the gap condition (for the p -primary group K) if $\sigma(n)+1 < \sigma(n+1)$ for some n implies that the Ulm invariant of K corresponding to $\sigma(n)$ is nonzero. If v satisfies the gap condition for the reduced p -primary group K and if each ordinal is less than the length of K , then v is called a U -sequence for K .

DEFINITION 4. [3] If $v = (\sigma(0), \sigma(1), \dots, \sigma(n), \dots)$ is a sequence of ordinals and perhaps symbols ∞ , and if K is a p -primary group, then $K(v)$ denotes $\{x \in K : h_k^*(p^n x) \geq \sigma(n) \text{ for each } n\}$. Note that $\sigma(n) = \infty$ for all n larger than some fixed integer k if and only if $p^{k+1}(K(v)) = 0$.

In [3] Kaplansky shows that each fully invariant subgroup of a fully-transitive, p -primary group K has the form $K(v)$ where v is a U -sequence for K . In [9] Pierce proves that a fully invariant subgroup is a large subgroup of K if and only if it has the form $K(v)$ where v is a U -sequence for K consisting of nonnegative integers. In the next two sections, we shall show that λ -large subgroups are similarly determined by U -sequences of ordinals less than λ .

2. C_λ -groups of length λ . Our immediate objective is to show that C_λ -groups of length λ are fully transitive.

DEFINITION 5. [7] Call a reduced, p -primary group G of length β σ -summable if $G[p] = \cup \{S(n) : n < \omega\}$ where $S(n) \subseteq S(n+1)$ and $S(n) \cap p^{\alpha(n)} G = 0$ for some increasing sequence of ordinals $\{\alpha(n) : n < \omega\}$ having supremum β .

The following generalized Kulikov Criterion plays a crucial role in the development of results in this section and in the study of the structure of λ -large subgroups which we begin in Section 4.

PROPOSITION 2. [7] *A p -primary group G of length λ (cofinal with ω) is a totally projective group if and only if G is a σ -summable C_λ -group.*

PROPOSITION 3. *If H is a subgroup of a C_λ -group G and $H \cap p^\alpha G = 0$ for some α less than λ , then H is contained in some λ -basic subgroup of G .*

Proof. If α is the first ordinal satisfying $H \cap p^\alpha G = 0$, then we let $\{\alpha(n) : n < \omega\}$ denote an increasing sequence of ordinals greater than α having supremum λ . We construct an increasing sequence of subgroups $\{S(n) : n < \omega\}$ with the property that $S(n)$ is maximal in $G[p]$ with respect to the property $S(n) \cap p^{\alpha(n)} G = 0$. If B is maximal in $G[p]$ with respect to the properties $B \supseteq H$ and $B[p] = \cup \{S(n) : n < \omega\}$, then B is σ -summable and satisfies the second and third conditions of Proposition 1. For each α less than λ , $B/p^\alpha B$ is isomorphic to $G/p^\alpha G$ and thus B is a C_λ -group. Proposition 2 implies that B is a totally projective group.

PROPOSITION 4. *Let H denote a finite subgroup of the C_λ -group G . If $H \cap p^\alpha G = 0$ for some α less than λ , then $G = A \oplus K$ where $A \supseteq H$ and A is a direct summand of some λ -basic subgroup of G .*

Proof. According to the preceding proposition, H is contained in a λ -basic subgroup B of G . Since B is of length λ and λ is a limit ordinal, we can write $B = \bigoplus \{B(i) : i \in I\}$ where, for each $i \in I$, $B(i)$ is a totally projective group of length less than λ . There is a finite subset J of I such that $H \subseteq \bigoplus \{B(j) : j \in J\}$; let A denote this sum. If $K = (\bigoplus \{B(i) : i \in I - J\}) + p^\beta G$, where β is the maximum of the lengths of the groups $B(j)$ for $j \in J$, then $G = A \oplus K$.

PROPOSITION 5. *Every C_λ -group of length λ is fully transitive.*

Proof. Suppose that x and y are elements in the C_λ -group G of length λ , where $h_G^*(p^n x) \leq h_G^*(p^n y)$ for all n . We need only show the existence of an endomorphism of G sending x to y . By Proposition 4, $G = A \oplus K$ where $\langle x, y \rangle \subseteq A$ and A is a direct summand of a λ -basic subgroup of G ; thus A is a totally projective group. Since totally projective groups are fully transitive and since $h_A^*(a) = h_G^*(a)$ for all $a \in A$, $f(x) = y$ for some endomorphism f of A . There is an obvious extension of f to an endomorphism of G .

The next proposition can be proved by applying Proposition 3 and generalizing the proof of Lemma (1.2) in [9].

PROPOSITION 6. *If B is a λ -basic subgroup of the C_λ -group G , where*

the length of G is not less than λ , and if A and C are fully invariant subgroups of G , then $(A + B) \cap C = (A \cap C) + (B \cap C)$.

COROLLARY 1. *Suppose that B is a λ -basic subgroup of G , $x \in G$, and α is less than λ . If $o(x)$ denotes the exponential order of x , then $x = b + g$ for some $g \in p^\alpha G$ and $b \in B$ satisfying $o(b) \leq o(x)$.*

Proof. If $o(x) = m$, set $A = p^\alpha G$ and $C = G[p^m]$ and then apply the preceding proposition.

DEFINITION 6. Let $v = (\sigma(0), \sigma(1), \dots, \sigma(n), \dots)$ denote a U -sequence for the p -primary group K . Then v is called a U_β -sequence for K if each $\sigma(n)$ is an ordinal less than β .

THEOREM 1. *Suppose that G is a C_λ -group of length λ . Then L is a λ -large subgroup of G if and only if $L = G(v)$ where v is a U_λ -sequence for G .*

Proof. Suppose first that L is λ -large in G . Since G is fully transitive and L is fully invariant, then $L = G(v)$ where $v = (\sigma(0), \sigma(1), \dots, \sigma(n), \dots)$ is a U -sequence for G . Thus, if $\sigma(n)$ is an ordinal for some n , then $\sigma(n)$ is less than λ ; however, all of the symbols $\sigma(n)$ are ordinals since λ -large subgroups must be unbounded.

Conversely, suppose that $L = G(v)$ where v is a U_λ -sequence for G . It suffices to show that $G \subseteq B + G(v)$. If $x \in G$ and $o(x) = m$, then by Corollary 1, we can write $x = b + g$ where $b \in B$, $g \in p^{\sigma(m)}G$, and $o(b) \leq o(x)$. It follows that $g \in G(v)$.

Note that in the preceding proof we have shown that $G(v)$ is λ -large whenever v is a U_λ -sequence, even when the length of G exceeds λ . The following corollary is useful in the study of λ -large subgroups of C_λ -groups having length greater than λ which we begin in §3.

COROLLARY 2. *If G is a C_λ -group of length λ , then L is a λ -large subgroup of G if and only if L is an unbounded, fully invariant subgroup of G .*

3. C_λ -groups of length greater than λ . Whenever L is a λ -large subgroup of a C_λ -group G , L contains $p^\lambda G$; this follows from Proposition 6 by setting $A = L$ and $C = p^\lambda G$. Now, if $p^n(L/p^\lambda G) = 0$, we can show that $p^\lambda G = p^n L$; however this gives us a decomposition $G = B \oplus p^\lambda G$ for any λ -basic subgroup B of G since $p^n L$ is also a λ -large subgroup. Since C_λ -groups are reduced, we must conclude that $L/p^\lambda G$ is an unbounded subgroup of $G/p^\lambda G$. It is important to our

development to show that $L/p^\lambda G$ is a λ -large subgroup of $G/p^\lambda G$ whenever L is a λ -large subgroup of G ; according to Corollary 2, we need only show that $L/p^\lambda G$ is a fully invariant subgroup of $G/p^\lambda G$. Most of this section is devoted to accomplishing that goal.

Many of our subsequent results can best be formulated in topological language. Therefore we introduce the following definition.

DEFINITION 7. [6] The λ -topology is defined on the p -primary group K by taking the family of subgroups $\{p^\alpha K : \alpha < \lambda\}$ as neighborhoods of the identity. If H is a subset of K , then H'_λ will denote the closure of H in K with respect to the λ -topology on K ; whenever the containing group is obvious, we will simply write H' .

PROPOSITION 7. *If F is a fully invariant subgroup of G , and if B is a λ -basic subgroup of G , then $F \subseteq (F \cap B)'$. Moreover, if G has length λ and F is unbounded, then $F = (F \cap B)'$.*

Proof. If α is less than λ , then we set $A = p^\alpha G$ and $C = F$ and apply Proposition 6 to get $F \subseteq (F \cap B) + p^\alpha G$, and thus $F \subseteq (F \cap B)'$. If G has length λ , then we write $F = G(v)$ where $v = (\sigma(0), \sigma(1), \dots, \sigma(n), \dots)$ is a U_λ -sequence for G . Now if $x \in (F \cap B)'$ where $o(x) = m$, say, then by applying Corollary 1, we can write $x = b + g$ where $g \in p^{\sigma(m)} G$, $b \in B(v)$. It follows that $x \in G(v)$.

In general, fully invariant subgroups are not closed in the λ -topology. For example, $p^{\lambda+1} G$ is a fully invariant subgroup of G and has $p^\lambda G$ as its closure. On the other hand, the following proposition shows that λ -large subgroups of G are closed even when G has length exceeding λ .

PROPOSITION 8. *If B is a λ -basic subgroup of G , and if L is a λ -large subgroup of G , then $L = (L \cap B)'$.*

Proof. Since $G = L + B$ and $L \subseteq (L \cap B)'$, by the modular law $(L \cap B)' = (L + B) \cap (L \cap B)' = L + B \cap (L \cap B)'$. Thus it suffices to prove that $B \cap (L \cap B)' \subseteq L \cap B$. But by purity, $B \cap (L \cap B)' = (L \cap B)'_B$. Thus the result follows from Proposition 7, once we see that $L \cap B$ is fully invariant subgroup of B . Now if $z \in L \cap B$ and f is an endomorphism of B , then by applying the technique of Proposition 4, we can obtain a subgroup A of B where $\langle z_2, f(z_2) \rangle \subseteq A$ and $G = A \oplus K$. Thus there is an endomorphism of A mapping z_2 to $f(z_2)$ which extends to an endomorphism of G . Since L is a fully invariant subgroup of G , $f(z_2) = b$ is in L . Thus $x = b + z_1$ is in L , and $(L \cap B)'$ is contained in L .

COROLLARY 3. *If F is an unbounded, fully invariant subgroup of B , where B is a λ -basic subgroup of G , then F'_G is a fully invariant subgroup of G .*

Proof. We can write $F = B(v)$ where v is a U -sequence for B . But v is also a U -sequence for G . By Proposition 8, $G(v) = (G(v) \cap B)' = B(v)'$.

Note that the proof of Proposition 8 shows that $F \cap B$ is a fully invariant subgroup of B whenever F is a fully invariant subgroup of G and B is a λ -basic subgroup of G . This observation is important to our study which now turns to the quotients $L/p^\lambda G$.

PROPOSITION 9. *L is a λ -large subgroup of G if and only if $L/p^\lambda G$ is a λ -large subgroup of $G/p^\lambda G$.*

Proof. Suppose first that L is a λ -large subgroup of G . We have seen that $L/p^\lambda G$ is unbounded; since $G/p^\lambda G$ is a C_λ -group of length λ , we need only show that $L/p^\lambda G$ is a fully invariant subgroup. If B is a λ -basic subgroup of G , then $(L/p^\lambda G) \cap ((B + p^\lambda G)/p^\lambda G)$ is equal to $((L \cap B) + p^\lambda G)/p^\lambda G$. The latter quotient is an isomorphic copy of $L \cap B$, which is unbounded by Proposition 8, while $(B + p^\lambda G)/p^\lambda G$ is isomorphic to B . Thus $(L/p^\lambda G) \cap ((B + p^\lambda G)/p^\lambda G)$ is an unbounded, fully invariant subgroup of $(B + p^\lambda G)/p^\lambda G$, a λ -basic subgroup of $G/p^\lambda G$. By Proposition 8, $(L \cap B)' = L$ and thus the closure of $((L \cap B) + p^\lambda G)/p^\lambda G$ in $G/p^\lambda G$ is just $L/p^\lambda G$. Since $(B + p^\lambda G)/p^\lambda G$ is a λ -basic subgroup of $G/p^\lambda G$, Corollary 3 guarantees that $L/p^\lambda G$ is a fully invariant subgroup of $G/p^\lambda G$.

On the other hand, if $L/p^\lambda G$ is a λ -large subgroup of $G/p^\lambda G$, then we can easily show that L is a fully invariant subgroup of G . If B is a λ -basic subgroup of G , then $G = B + L$ since $G/p^\lambda G = ((B + p^\lambda G)/p^\lambda G) + (L/p^\lambda G)$.

THEOREM 2. *L is a λ -large subgroup G if and only if $L = G(v)$, where v is a U_λ -sequence for G .*

Proof. L is λ -large in G if and only if $L/p^\lambda G$ is λ -large in $G/p^\lambda G$. Thus L is λ -large in G if and only if $L/p^\lambda G = (G/p^\lambda G)(v)$ for some U_λ -sequence for $G/p^\lambda G$; however v is also a U_λ -sequence for G , and $(G/p^\lambda G)(v) = G(v)/p^\lambda G$. Hence L is λ -large in G if and only if $L/p^\lambda G = G(v)/p^\lambda G$.

4. The structure of λ -large subgroups. It is shown in [1] that some of the solutions to the open statement "A large subgroup L

of G has property P if and only if G has property P'' are these properties: direct sum of cyclic groups, direct sum of countable groups, and totally projective group. In this section we study the relation between the structure of λ -large subgroups and the structure of the containing groups. Note that if G is a totally projective group of length $\Omega + \omega_2$ where Ω denotes the first uncountable ordinal, then $p^{\Omega+\omega}G$ is a direct sum of cyclic groups and $p^\alpha G$ is a direct sum of countable groups. Since each of these subgroups is a λ -large subgroup of G , we see that inheritance of structure in our general C_λ -theory is not as widespread as that in the classical theory.

PROPOSITION 10. *If L is a λ -large subgroup of G and if α is less than the length of $L/p^\alpha G$, then $p^\alpha L$ is also a λ -large subgroup of G .*

Proof. Let μ denote the length of $L/p^\alpha G$. We can assume that α is not less than ω and write $\alpha = \omega + \sigma$ and $\mu = \omega + \beta$, where $\sigma < \beta$. If $L = G(v)$, where $v = (\sigma(0), \sigma(1), \dots, \sigma(n), \dots)$, and if $\delta = \sup\{\sigma(n) : n < \omega\}$, then $p^\omega L = p^\delta G$; hence $p^\alpha L = p^\sigma(p^\omega L) = p^{\delta+\sigma}G$ where $\delta + \sigma$ is less than λ .

PROPOSITION 11. ([4], [5], [2]). *Let F denote a fully invariant subgroup of the totally projective group K . Then F and K/F are totally projective and the length of K/F does not exceed the length of K .*

COROLLARY 4. *G/L is a totally projective group whenever L is a λ -large subgroup of G .*

Proof. If B is a λ -basic subgroup of G , then G/L is isomorphic to $B/(L \cap B)$ where $L \cap B$ is a fully invariant subgroup of the totally projective group B .

THEOREM 3. *If L is a λ -large subgroup of G , then L is a totally projective group only if G is a totally projective group.*

Proof. We first consider the case where G has length λ . Our proof is inductive on λ ; if $\lambda = \omega$, then the result follows from Theorem 4.3 in [1]. Thus we assume the conclusion for all limit ordinals β less than λ where β is a limit ordinal cofinal with ω . If $L = G(v)$ where $v = (\sigma(0), \sigma(1), \dots, \sigma(n), \dots)$ is a U_λ -sequence for G , then we set $\delta = \sup\{\sigma(n) : n < \omega\}$ and consider two cases.

Case 1. $\delta < \lambda$. In this case we note that $(G/p^\delta G)(v) = L/p^\delta G = L/p^\alpha L$ is a totally projective group and is a δ -large subgroup of the

C_δ -group $G/p^\delta G$. By the induction hypothesis, $G/p^\delta G$ is a totally projective group as is $p^\delta G = p^\omega L$. Hence G is a totally projective group.

Case 2. $\delta = \lambda$. In this case, $p^\omega L = p^\delta G = 0$ and, according to the generalized Kulikov Criterion, L is σ -summable; thus $L[p] = \cup \{S(n): n < \omega\}$ where $S(n) \subseteq S(n+1)$ and $S(n) \cap p^n L = 0$ for each $n < \omega$. Since $p^{\sigma(0)}G[p] = L[p]$, we can show that $p^{\sigma(0)}G$ is a σ -summable C_μ -group of length μ , where μ is a limit ordinal cofinal with ω . For each positive integer n , there is an ordinal $\mu(n)$ less than μ , the length of $p^{\sigma(0)}G$, such that $\sigma(n) = \sigma(0) + \mu(n) < \sigma(0) + \mu = \lambda$. From familiar properties of ordinals, it follows that $\mu = \sup\{\mu(n): n < \omega\}$ and hence μ is cofinal with ω . Since $S(n) \cap p^{\mu(n)}(p^{\sigma(0)}G)[p] \subseteq S(n) \cap p^{\sigma(n)}G[p] \subseteq S(n) \cap p^n L = 0$, we see that $p^{\sigma(0)}G$ is σ -summable. Let β denote an ordinal less than μ . Since G is a C_λ -group, $G/p^\beta(p^{\sigma(0)}G)$ and $p^{\sigma(0)}(G/p^\beta(p^{\sigma(0)}G))$ are totally projective groups. Hence $p^{\sigma(0)}G/p^\beta(p^{\sigma(0)}G)$ is a totally projective group and $p^{\sigma(0)}G$ is a C_λ -group. Thus, by the generalized Kulikov Criterion, $p^{\sigma(0)}G$ is a totally projective group as is $G/p^{\sigma(0)}G$. So G possess this property.

In general, if G has length greater than λ , then $0 \neq p^\delta G = p^\omega L$ where $\delta = \sup\{\sigma(n): n < \omega\}$. Thus $L/p^\omega L = L/p^\delta G$ is a totally projective group and δ -large in $G/p^\delta G$. By the argument given above, $G/p^\delta G$ is a totally projective group as is $p^\delta G = p^\omega L$.

PROPOSITION 12. *If L is a λ -large subgroup of G and B is a λ -basic subgroup of G , then $L \cap B$ is a μ -basic subgroup of B , where μ denotes the length of $L/p^\lambda G$.*

Proof. Let $L = G(v)$ where v is a U_λ -sequence for G . Then $L \cap B = B(v)$ is a fully invariant subgroup of the totally projective group B . By Proposition 11, $L \cap B$ is a totally projective group. If $\delta = \sup\{\sigma(n): n < \omega\}$ and β has the property that $\lambda = \delta + \beta$ and $\mu = \omega + \beta$, then $p^\mu(B(v)) = p^\beta(p^\omega B(v)) = p^\beta(p^\delta B) = p^\lambda B = 0$. Thus the length of $B(v) = L \cap B$ does not exceed μ .

In order to show that $L[p] \subseteq (L \cap B)[p] + p^\beta L$ for all β less than μ , we first note that $p^\beta L$ is λ -large in G . Thus, by Proposition 6, $p^\beta G[p] = p^\beta G[p] \cap (B + p^\beta L) = p^\beta B[p] + p^\beta L[p]$. Since B is a λ -basic subgroup of G , $G[p] = p^\beta G[p] + B[p] = B[p] + p^\beta L[p]$. Suppose now that $x \in L[p]$. Then $L[p] = L \cap G[p] = L \cap (B[p] + p^\beta L[p]) = (L \cap B)[p] + p^\beta L[p]$ (by the modular law).

All that remains is to show that there is no subgroup H of L properly containing $L \cap B$ such that $H[p] = (L \cap B)[p]$. It suffices to show that $pL \cap (L \cap B) \subseteq p(L \cap B)$ or $pL \cap B \subseteq p(B(v))$. So suppose that $pz \in pL \cap B$ for some $z \in L$. Then $pz \in p^{\sigma(1)}G \cap B \subseteq p^{\sigma(0)+1}B$, and

$pz = pb$ for some $b \in p^{\sigma(0)}B$. It follows that $b \in B(v)$ and that $pL \cap B \subseteq p(B(v))$.

COROLLARY 5. *If L is a λ -large subgroup of the C_λ -group G and if μ denotes the length of $L/p^\lambda G$, then L is a C_μ -group.*

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