A VANISHING THEOREM FOR THE $\text{mod } p$ MASSEY-PETERSON SPECTRAL SEQUENCE

Masamitsu Mori
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SEQUENCE

MASAMITSU MORI

A vanishing theorem and periodicity theorem for the classical mod 2 Adams spectral sequence were originally proved by Adams [1]. The results were extended to the unstable range by Bousfield [2]. The purpose of this paper is to show the analogue of Bousfield's work for the mod $p$ unstable Adams spectral sequence of Massey-Peterson type (called the mod $p$ Massey-Peterson spectral sequence), where $p$ is an odd prime. The results generalized those obtained by Liulevicius [5], [6] to the unstable range. As an immediate topological application we have the estimation of the upper bounds of the orders of elements in the $p$-primary component of the homotopy groups of, for example, an odd dimensional sphere, Stiefel manifold, or $H$-space.

1. The vanishing theorem. Let $A$ denote the mod $p$ Steenrod algebra. Let $A_\text{un}$ the category of unstable left $A$-modules and $\text{un}A$ the category of unstable right $A$-modules. We may define $\text{Ext}^s_{A_\text{un}}$, $s \geq 0$, as the $s$th right derived functor of $\text{Hom}_{A_\text{un}}$, and similarly define $\text{Ext}^s_{\text{un}A}$, since $A_\text{un}$ and $\text{un}A$ are abelian categories with enough projectives. Note that, if $M \in A_\text{un}$ is of finite type, then

$$\text{Ext}^s_{A_\text{un}}(M, Z_p) = \text{Ext}^s_{\text{un}A}(Z_p, M^*) .$$

Recall the mod $p$ Massey-Peterson spectral sequence (see, for example, [4]). Let $X$ be a simply connected space with $\pi_*(X)$ of finite type. Suppose that $H^*(X; Z_p) \cong U(M)$, $M \in A_\text{un}$, where $U(M)$ is the free unstable $A$-algebra generated by $M$. Then there is a spectral sequence $\{E_r(X)\}$ with

$$d_r : E_r^{s,t}(X) \longrightarrow E_r^{s+r, t+r-1}(X) ,$$

such that

$$E_r^{s,t}(X) \cong \text{Ext}_r^{s,t}(M, Z_p) ,$$

and

$$E_\infty(X) \cong \text{Gr}_* \pi_*(X)/(\text{torsion prime to } p) .$$

Let $A$ be the bigraded differential algebra over $Z_p$ introduced by Bousfield et al [3], which has multiplicative generators $\lambda_i$ of
bidegree \((1, 2i(p - 1) - 1)\) for \(i > 0\) and \(\mu_i\) of bidegree \((1, 2i(p - 1))\) for \(i \geq 0\).

For \(N \in \mathcal{M} A\), let \(V(N)^*\) be the subspace of \(N \otimes A^*\) generated by all \(x \otimes \nu_I\) with \(\nu_I = \nu_{i_1} \cdots \nu_{i_s}\) allowable and \(\deg x \geq 2i\) if \(\nu_{i_1} = \lambda_{i_1}\) and \(\deg x \geq 2i + 1\) if \(\nu_{i_1} = \mu_{i_1}\). Then \(V(N)\) is the cochain complex with

\[
\delta(x \otimes \nu_I) = (-1)^{\deg x} \sum_{i \geq 0} x \beta^i \otimes \lambda_{i} \nu_I
\]

\[
+ \sum_{i \geq 0} x \beta^i \otimes \mu_i \nu_I + (-1)^{\deg x} x \otimes \partial \nu_I.
\]

Here \(x \otimes \nu_I\) is of bidegree \((s, t)\) with \(t = s + \deg x + \deg \nu_I\). Recall that for \(N \in \mathcal{M} A\)

\[
\Ext^s_A(Z_p, N) = H^s(V(N))_{t-s}.
\]

Let \(O(N)\) be the subcomplex of \(V(N)\) generated by all \(x \otimes \nu_I \in V(N)^*\) with \(\nu_I = \nu_{i_1} \cdots \nu_{i_s}\) allowable and \(\nu_{i_s} = \lambda_{i_s}\). Let \(T(N)\) be the quotient complex of \(V(N)\) such that

\[
T(N)^s = \begin{cases} 
N \otimes \mu_0^s & \text{for } s = 0, 1, \\
N \otimes \mu_0^s + \sum_{i \geq 1} N^i \otimes \lambda_i \mu_i^{t-1} & \text{for } s \geq 2.
\end{cases}
\]

Then we have a long exact sequence

\[
\cdots \longrightarrow H^{s-1}(T(N)) \overset{\delta}{\longrightarrow} H^s(O(N)) \overset{j^*}{\longrightarrow} H^s(V(N)) \overset{q^*}{\longrightarrow} H^s(T(N)) \longrightarrow \cdots,
\]

which is induced from the natural isomorphism

\[
H^*(O(N)) \cong H^*(\Ker q),
\]

where \(j: O(N) \rightarrow V(N)\) and \(q: V(N) \rightarrow T(N)\) are the natural maps. Remark that \(H^*(T(N))\) consists of towers in the sense that

\[
H^s(T(N)) \cong H^{s+1}(T(N)) ,
\]

for \(s \geq 2\), and thus \(H^*(T(N))\) is easily determined.

**Definition.** A function \(\varphi_n(k), n \geq 2, k \geq 0\), is defined as follows. If \(n = 2, 3, 4\),

\[
\varphi_n(k) = \begin{cases} 
\lceil (k + 2)/2(p - 1) \rceil & \text{for } k \geq 2(p - 1) - 1 \\
0 & \text{for } k < 2(p - 1) - 1
\end{cases}
\]

where \([x]\) is the integer part of \(x\), and if \(n \geq 5\),

\[
\varphi(k) = \varphi_n(k) = i,
\]
where

\[ 2i(p - 1) \leq k < 2(i + 1)(p - 1) - 1 \quad \text{if} \quad i \not\equiv -1, 0 \mod p , \]
\[ 2i(p - 1) \leq k < 2(i + 1)(p - 1) - 2 \quad \text{if} \quad i \equiv -1 \mod p , \]
\[ 2i(p - 1) - 2 \leq k < 2(i + 1)(p - 1) - 1 \quad \text{if} \quad i \equiv 0 \mod p . \]

Now we state our main theorem.

**Theorem 1 (Vanishing).** Let \( N \in \mathcal{A} \) with \( N_i = 0 \) for \( i < n \), where \( n \geq 2 \). Then

\[
\text{Ext}_{A}^{s+s+k+n}(Z_p, N) \cong H^*(V(N))_{k+n} \xrightarrow{q^*} H^*(T(N)) ,
\]

is an isomorphism for \( s > \phi_s(k) \).

This will be proved in §4.

By virtue of our vanishing theorem the calculation of \( H^*(V(N)) \) is reduced to that of \( H^*(O(N)) \) in a large extent. Note that \( q^* \) is epimorphic when \( U(M) \) is generated by a single element, where \( M = N \).

As an immediate topological corollary we have:

**Corollary 2.** Let \( X \) be a simply connected space with \( \pi_*(X) \) of finite type. Suppose that \( H^*(X; Z_p) \cong U(M) \), where \( M \) is an unstable \( A \)-module. If \( M^i = 0 \) for \( i < n \), then the orders of elements in the \( p \)-primary component of \( \pi_{k+n}(X) \) are at most \( p^{\phi_s(k)} \).

This may be applied, for example, when \( X \) is an odd dimensional sphere, Stiefel manifold, or \( H \)-space.

**Remark.** If \( N_i = 0, i > m \), for some \( m \), then \( H^*(N)_t \) is zero for dimensional reason when \( t \) is large with respect to \( s \). Hence in this case Corollary 2 is slightly improved.

2. **Periodicity theorems.** For a module \( M \in A_{\mathcal{A}} \) we define the \( \beta \)-cohomology by \( H_{\beta}(M) = \ker \beta / \text{Im} \beta \).

**Definition.** A module \( M \in A_{\mathcal{A}} \) is called \( \beta \)-trivial if

\[
\rho^i: M^{2i} \longrightarrow H^{2i+p}_{\beta}(M) ,
\]

is an isomorphism for all \( i \) and \( H^{k}_{\beta}(M) = 0 \) for \( k \not\equiv 0 \mod 2p \).

Remark that \( M \in A_{\mathcal{A}} \) is \( \beta \)-trivial if and only if \( N = M^* \in \mathcal{A} A \) is towerless, i.e., \( H^s(T(N)) = 0 \) for \( s > 0 \).

Let \( \mathcal{C} \) denote the category of graded \( Z_p \)-modules. Let \( LsF \) denote the \( s \)th left derived functor of a functor \( F: A_{\mathcal{A}} \rightarrow \mathcal{C} \).
THEOREM 3 (Periodicity). Let $F: \mathcal{A} \to \mathcal{G}$ be a functor such that $F(M) = M/\beta M + \rho^i M$. If $M \in \mathcal{A}$ is $\beta$-trivial, then there is a natural map

$$P: L_* F(M)^t \to L_{*+2} F(M)^{t+2p(p-1)+p},$$

such that $P$ is an isomorphism for $s \geq 2$ and a monomorphism for $s = 1$.

This will be proved in §3.

Additionally, we give here such a kind of periodicity theorems.

THEOREM 4. Let $G_{i}(M) = M/\rho^i M$ for $M \in \mathcal{A}$, where $0 < i < p$. Then there is a natural map

$$Q: L_* G_{i}(M)^t \to L_{*+2} G_{i}(M)^{t+2p(p-1)},$$

such that $Q$ is an isomorphism for $s \geq 2$ and a monomorphism for $s = 1$.

THEOREM 5. Let $G_{i}(M) = M/\beta M + \rho^i M$ for $M \in \mathcal{A}$, where $0 < i < p$. If $M \in \mathcal{A}$ is $\beta$-trivial, then there is a natural map

$$Q: L_* G_{i}(M)^t \to L_{*+2} G_{i}(M)^{t+2p(p-1)},$$

such that $Q$ is an isomorphism for $s \geq 2$ and a monomorphism for $s = 1$.

THEOREM 6. Let $G(M) = M/\rho^i + \beta^i M$ for $M \in \mathcal{A}$. If $M \in \mathcal{A}$ is $\beta$-trivial, then there is a natural map

$$R: L_* G(M)^t \to L_{*+2} G(M)^{t+4p},$$

such that $R$ is an isomorphism for $s \geq 2$ and a monomorphism for $s = 1$.

3. Proofs of periodicity theorems. Suppose given a circular sequence of functors from $\mathcal{A}$ to $\mathcal{G}$ and natural transformations,

(§)

$$\begin{array}{c}
A_{k-2} \\ R_{k-2} \\
A_{k-3} \\
R_{k-1} \\
A_{k-1} \\
\cdots \\
A_1 \\
R_1 \\
A_0 = A_k \\
R_0 = R_k \\
A_{k-1}
\end{array}$$

satisfying $R_i R_{i+1} = 0$ for $i = 0, \cdots, k - 1$. Define functors Ker $R_i$, Im $R_i$, Coker $R_i$, $H_i = \text{Ker } R_i/\text{Im } R_{i+1}$ in a usual way.

DEFINITION. A module $M \in \mathcal{A}$ is called trivial for the diagram (§) if
\[ L_sA_i(M) = L_sH_i(M) = 0, \]
for all \( s > 0 \) and \( i = 0, \ldots, k - 1 \).

**Lemma.** If \( M \in \mathcal{A} / \mathcal{M} \) is trivial for the diagram (\#), then there is a natural map

\[ P : L_s \text{Coker } R_0(M)^t \longrightarrow L_{s+k} \text{Coker } R_0(M)^{t+k}, \]
such that \( P \) is an isomorphism for \( s \geq 2 \) and a monomorphism for \( s = 1 \). Here \( h = \sum_{i=0}^{k-1} h_i, h_i = \deg R_i \).

**Proof.** Let \( h(a) = \sum_{i=0}^{k-1} h_i. \) Since \( M \) is trivial for (\#), we have the following natural isomorphism

\[ L_{s+k} \text{Coker } R_0(M)^{t+k} \cong L_{s+k-1} \text{Im } R_0(M)^{t+k} \]
\[ \cong L_{s+k-2} \text{Ker } R_0(M)^{t+k(1)} \cong \cdots \]
\[ \cong L_s \text{Ker } R_k(M)^{t+k(k-1)} \cong L_s \text{Im } R_{k-1}(M)^{t+k(k-1)}. \]

On the other hand the natural map

\[ L_s \text{Coker } R_0(M)^t \longrightarrow L_s \text{Im } R_{k-1}(M)^{t+k(k-1)}, \]
is an isomorphism for \( s \geq 2 \) and a monomorphism for \( s = 1 \).

We shall use the following circular sequence due to Toda [9] (see, also, Oka [8]) to prove the periodicity theorems.

\[ (3.1) \quad R \leftarrow M \rightarrow R' \leftarrow \cdots \rightarrow M \rightarrow R \]
\[ \text{M/βM + M/βM} \]
where \( R_i = (i + 1)β^i - iβ^iβ, R = (β^iβ, \rho^i) \) and \( R' = ρ^iβ - β^iβ \).

\[ (3.2) \quad M \xrightarrow{\rho^i} M \quad \text{for } 0 < i < p, \]
\[ (3.3) \quad M/\rho_i M \xrightarrow{\rho^i} M/βM \quad \text{for } 0 < i < p, \]
\[ (3.4) \quad M/\rho_i M \xrightarrow{\rho^i} M/\rho^i M. \]

Here \( M \in \mathcal{A} / \mathcal{M} \) and the maps are induced from the left actions.

**Proof of Theorem 3.** We shall use the diagram (3.1). For convenience, we put \( R_0 = R_p = R, R_{p-1} = R' \). Let \( H_i(M) \) denote the cohomology \( \text{Ker } R_i/\text{Im } R_{i+1} \). If \( M \) is a free unstable \( A \)-module, then:

\[ (i) \quad \rho^* : (M/βM)^{2s} \cong H_0^{2s}(M) \quad \text{if } s \equiv -1 \mod p, \]
\[ \rho^* : M \cong H_0^{2s+1}(M) \quad \text{if } s \not\equiv -1 \mod p, \]
\[ \rho^* : (βM)^{2s+1} \cong H_0^{p+1}(M) \quad \text{if } s \not\equiv -1 \mod p, \]
(ii) for $i = 1, \cdots, p - 2$,
\[ \rho^*: (M/\beta M)^{2s} \cong H_i^{2sp}(M), \]
\[ \rho^* + \beta \rho^*: (\beta M)^{2s+2} + (M/\beta M)^{2s+1} \cong H_i^{2sp+2}(M), \]
\[ \rho^*: (M/\beta M)^{2s+2} \cong H_i^{2sp+3}(M) \]

(iii) $\rho^* + 0: (M/\beta M)^{2s} + 0 \cong H_{p-1}^{2sp}(M),$
\[ 0 + \rho^*: 0 + (M/\beta M)^{2s+j} \cong H_{p-1}^{2sp+j+1}(M) \]
for $j = 0, 1$ if $s \equiv 0 \mod p$,
\[ \rho^* + \rho^*: (\beta M)^{2s+1+j} + (M/\beta M)^{2s+j} \cong H_{p-1}^{2sp+j+1}(M) \]
for $j = 0, 1$ if $s \not\equiv 0 \mod p$,
\[ 0 + \rho^*: 0 + (M/\beta M)^{2s+2} \cong H_{p-1}^{2sp+3}(M). \]

(iv) otherwise $H_i^k(M) = 0$.

This unstable version of Toda's exactness theorem is shown by long but straightforward computations. Now Theorem 3 is proved by applying lemma.

By using the diagrams (3.2), (3.3) and (3.4), Theorems 4, 5 and 6 follow in a similar way, and thus we only state the following facts.

Let $M \in \mathcal{A}$ be a free unstable module. Fix $i$ such that $0 < i < p$.
If $H(M) = \text{Ker} (\rho^i: M \to M)/\text{Im} (\rho^{p-i}: M \to M)$, then:

(i) $\rho^*: M^{2s+j} \cong H_i^{2sp+j}(M)$ for $j = 0, 1$,
\[ \beta \rho^* + \rho^*: M^{2s+j-1} + M^{2s+j} \cong H_i^{2sp+j}(M) \]
for $j = 2, \cdots, 2i - 1$, 
\[ \beta \rho^*: M^{2s+j-1} \cong H_i^{2sp+j}(M) \]
for $j = 2i, 2i + 1$.

(ii) otherwise $H_i^k(M) = 0$.

If $H(M) = \text{Ker} (\rho^{p-i}: M/\beta M \to M/R_i M)/\text{Im} (\rho^i: M/R_i M \to M/\beta M)$, then:

(i) $\rho^*: (M/\beta M)^{2s+j} \cong H_i^{2sp+j}(M)$ for $j = 0, 1$,
\[ \beta \rho^* + \rho^*: (M/\beta M)^{2s+j-1} + (M/\beta M)^{2s+j} \cong H_i^{2sp+j}(M) \]
for $j = 2, \cdots, 2i - 1$, 
\[ \beta \rho^*: (M/\beta M)^{2s+j-1} \cong H_i^{2sp+j}(M) \]
for $j = 2i, 2i + 1$.

(ii) if $s \equiv 0 \mod p$,
\[ \rho^*: (M/\beta M)^{2s+j} \cong H_i^{2sp+j}(M) \]
for $j = 0, 1$,
\[ \beta \rho^* + \rho^*: (M/\beta M)^{2s+j-1} + (M/\beta M)^{2s+j} \cong H_i^{2sp+j}(M) \]
for $j = 2, \cdots, 2i - 1$, 
\[ \beta \rho^*: (M/\beta M)^{2s+j-1} \cong H_i^{2sp+j}(M) \]
for $j = 2i, 2i + 1$.
\[\rho^*: (M/\beta M)^{2s+j} \cong H^{2sp+j}(M) \quad \text{for } j = 0, 1,\]
\[\beta \rho^* + \rho^*: M^{2s+j-1} + (M/\beta M)^{2s+j} \cong H^{2sp+j}(M) \quad \text{for } j = 2, \cdots, 2i - 1,\]
\[\beta \rho^*: M^{2s+j-1} \cong H^{2sp+j}(M) \quad \text{for } j = 2i, 2i + 1.\]

(iii) if \( s \equiv p - i \mod p, \)
\[\beta \rho^*: M^{2s+j-1} \cong H^{2sp+j}(M) \quad \text{for } j = 2, \cdots, 2i - 1,\]

(iv) otherwise \( H^i(M) = 0. \)

Finally, if \( H(M) = \text{Ker}(\beta \rho^*: M/\rho^i M \to M/\rho^i M)/\text{Im}(\beta \rho^*: M/\rho^i M \to M/\rho^i M), \) then:

(i) \[\rho^{ps}: M^{2s+j} \cong H^{2sp^2+j}(M) \quad \text{for } j = 0, 1,\]
\[\beta \rho^{ps} + \rho^{ps}: (M/\beta M)^{2s+1} + M^{2s+2} \cong H^{2sp^2+j}(M),\]

(ii) otherwise \( H^i(M) = 0. \)

4. Proof of the vanishing theorem. Let \( F(n) \) denote a free unstable \( A \)-module on one generator \( \iota_n \). We define an unstable \( A \)-module \( N(n) \) to be the quotient of \( F(n) \) by the relation \( \beta \iota_n = 0 \). Next define \( M(n) \) to be the subcomplex of \( N(n) \) by omitting the \( \iota_n \) from \( N(n) \) if \( n \) odd and omitting the \( \iota_n, (\iota_n)^{p}, \cdots, (\iota_n)^{p^t}, \cdots \) from \( N(n) \) if \( n \) even. Note that \( M(n) \) is \( \beta \)-trivial.

First we suppose that \( n \) is odd. Then by the long exact sequence induced from a short exact sequence

\[0 \to M(n) \to N(n) \to \mathbb{Z}_p \to 0,\]

we have an isomorphism

\[E^{t,s}_{t,s+n}(S^n) = \text{Ext}^{t,s+n}_{A}(\mathbb{Z}_p, \mathbb{Z}_p) \cong \text{Ext}^{t-1,s+n}_{A}(M(n), \mathbb{Z}_p),\]

for \( t \neq s \). Let \( C(n) \) be a minimal resolution of \( M(n) \). By virtue of Theorem 3 we can prove the vanishing theorem for \( Z_p \) by analysing \( C(n) \). Namely, \( \text{Ext}_{A}^{t-1,s+n}(M(n), \mathbb{Z}_p)(t \neq s) \) vanishes for \( s > \varphi_n(t - s) \).

Furthermore we can observe the periodicity phenomenon in a range near the vanishing line. In fact, by Theorems 3, 4 and 5 we have two periodicity operators \( P \) and \( Q \) of bidegree \((p, 2p(p - 1) + p)\) and \((2, 2p(p - 1))\), respectively.

For lower dimensional sphere we shall give periodic families. Let \( 1 < m \leq p + 1 \). In \( E^{t,s}_{2,s+m-1}(S^{2m-1}) \) there appear nontrivial elements when \((s, t - s)\) is as follows:

(i) \((1, q - 1)\)

\((1, pq - 1)\) for \( m = p + 1,\)

(ii) \((s, sq - 1), (s, (m + s - 2)q - 2)\)

for \( s = 2, \cdots, p - m + 1 \) and \( m \neq p, p + 1,\)
(s, sq - 1), (s, pq - 2), (s, pq - 1)
for s = p - m + 2 and m \neq p + 1,
(s, sq - 1), (s, pq - 2), (s, pq - 1), (s, (m + s - 2)q - 2)
for s = p - m + 3, \ldots, p - 1 and p \neq 3,
(p, pq - 2), (p, pq - 1), (p, (p + m - 2)q - 2),
(p + 1, (p + 1)q - 1), (p + 1, (p + m - 1)q - 2),
where q = 2(p - 1). Applying the periodicity operators \( P \) and \( Q \) repeatedly, we can determine the behavior of all \( E_2(S^{2m-1}) \) near the vanishing line. (Possibly other elements appear in a range apart from the vanishing line when we apply the iteration of the operator \( Q \).)

We next suppose that \( n \) is even. Let \( L(n; t)(0 < t \leq \infty) \) be an unstable \( A \)-module with elements \( \sigma^*_n, (\sigma^*_n)^p, \ldots, (\sigma^*_n)^{pt} \) where \( \deg \sigma^*_n = n \). By the long exact sequence induced from short exact sequences

\[
0 \rightarrow M(n) + L(p^{t+1}; \infty) \rightarrow N(n) \rightarrow L(n; t) \rightarrow 0,
0 \rightarrow M(p^{t+1}n) \rightarrow N(p^{t+1}n) \rightarrow L(p^{t+1}n; \infty) \rightarrow 0,
\]
we have an isomorphism

\[
\Ext^{s,t+n}_{s,t+n}(L(n; t), Z_p) \\
\cong \Ext^{s-1,t+n}_{s-1,t+n}(M(n), Z_p) + \Ext^{s-2,t+n}_{s-2,t+n}(M(p^{t+1}n), Z_p),
\]
for \( t \neq s, s + (p^{t+1} - 1)n - 1 \). Thus in a similar way we have the required results for \( L(n; t) \).

Now we have shown that

\[
q^*: H^*(V(N))_{k+n} \rightarrow H^*(T(N)_{k+n},
\]
is an isomorphism for \( s > \varphi_n(k) \), when \( N^* = H^*(S^*; Z_p) = Z_p(n \text{ odd}) \) and \( N^* = L(n; t)(n \text{ even}) \). The general case follows inductively using the five lemma.

References


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