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A VANISHING THEOREM FOR THE mod *p* MASSEY-PETERSON SPECTRAL SEQUENCE

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A vanishing theorem and periodicity theorem for the classical mod 2 Adams spectral sequence were originally proved by Adams [1]. The results were extended to the unstable range by Bousfield [2]. The purpose of this paper is to show the analogue of Bousfield's work for the mod p unstable Adams spectral sequence of Massey-Peterson type (called the mod p Massey-Peterson spectral sequence), where p is an odd prime. The results generalized those obtained by Liulevicius [5], [6] to the unstable range. As an immediate topological application we have the estimation of the upper bounds of the orders of elements in the p-primary component of the homotopy groups of, for example, an odd dimensional sphere, Stiefel manifold, or H-space.

1. The vanishing theorem. Let A denote the mod p Steenrod algebra. Let $A \mathscr{M}$ the category of unstable left A-modules and $\mathscr{M}A$ thecategory of unstable right A-modules. We may define $\operatorname{Ext}_{A\mathscr{M}}^{s}, s \geq 0$, as the sth right derived functor of $\operatorname{Hom}_{A\mathscr{M}}$, and similarly define $\operatorname{Ext}_{\mathscr{M}}^{s}$, since $A\mathscr{M}$ and $\mathscr{M}A$ are abelian categories with enough projectives. Note that, if $M \in A\mathscr{M}$ is of finite type, then

$$\operatorname{Ext}_{\mathcal{A}\mathcal{M}}(M, Z_p) = \operatorname{Ext}_{\mathcal{M}\mathcal{A}}(Z_p, M^*)$$
.

Recall the mod p Massey-Peterson spectral sequence (see, for example, [4]). Let X be a simply connected space with $\pi_*(X)$ of finite type. Suppose that $H^*(X; Z_p) \cong U(M)$, $M \in A_{\mathcal{M}}$, where U(M) is the free unstable A-algebra generated by M. Then there is a spectral sequence $\{E_r(X)\}$ with

$$d_r: E_r^{s,t}(X) \longrightarrow E_r^{s+r,t+r-1}(X)$$
,

such that

$$E_2^{s,t}(X) \cong \operatorname{Ext}_{A,\mathscr{M}}^{s,t}(M, Z_p)$$
,

and

 $E_{\infty}(X) \cong \operatorname{Gr} \pi_*(X)/(\operatorname{torsion \ prime \ to \ } p)$.

Let Λ be the bigraded differential algebra over Z_p introduced by Bousfield et al [3], which has multiplicative generators λ_i of bidegree (1, 2i(p-1)-1) for i > 0 and μ_i of bidegree (1, 2i(p-1)) for $i \ge 0$.

For $N \in \mathscr{M}A$, let $V(N)^{s}$ be the subspace of $N \otimes \Lambda^{s}$ generated by all $x \otimes \nu_{I}$ with $\nu_{I} = \nu_{i_{1}} \cdots \nu_{i_{s}}$ allowable and deg $x \ge 2i$ if $\nu_{i_{1}} = \lambda_{i_{1}}$ and deg $x \ge 2i_{1} + 1$ if $\nu_{i_{1}} = \mu_{i_{1}}$. Then V(N) is the cochain complex with

$$egin{aligned} \delta(x\otimes oldsymbol{
u}_I) &= (-1)^{ ext{deg}x}\sum\limits_{i>0} x
ho^i \otimes \lambda_i oldsymbol{
u}_I \ &+ \sum\limits_{i\geq 0} x eta
ho^i \otimes \mu_i oldsymbol{
u}_I + (-1)^{ ext{deg}x} x \otimes \partial oldsymbol{
u}_I \ . \end{aligned}$$

Here $x \otimes \nu_I$ is of bidegree (s, t) with $t = s + \deg x + \deg \nu_I$. Recall that for $N \in \mathcal{M}A$

$$\operatorname{Ext}_{\mathscr{A}A}^{s,t}(Z_p, N)\cong H^s(V(N))_{t-s}$$
 .

Let O(N) be the subcomplex of V(N) generated by all $x \otimes \nu_I \in V(N)^s$ with $\nu_I = \nu_{i_1} \cdots \nu_{i_s}$ allowable and $\nu_{i_s} = \lambda_{i_s}$. Let T(N) be the quotient complex of V(N) such that

$$T(N)^s = egin{cases} N \otimes \mu_0^s & ext{for} \quad s = 0, 1 \ N \otimes \mu_0^s + \sum\limits_{t > 0} N_{2t}^s \otimes \lambda_t \mu_0^{s-1} & ext{for} \quad s \geqq 2 \ . \end{cases}$$

Then we have a long exact sequence

$$\begin{array}{ccc} & & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ &$$

which is induced from the natural isomorphism

$$H^*(O(N))\cong H^*(\operatorname{Ker} q)$$
 ,

where $j: O(N) \to V(N)$ and $q: V(N) \to T(N)$ are the natural maps. Remark that $H^*(T(N))$ consists of towers in the sense that

$$H^{s}(T(N))\cong H^{s+1}(T(N))$$
 ,

for $s \ge 2$, and thus $H^*(T(N))$ is easily determined.

DEFINITION. A function $\varphi_n(k)$, $n \ge 2$, $k \ge 0$, is defined as follows. If n = 2, 3, 4,

$$arphi_{\mathtt{n}}(k) = egin{cases} [(k+2)/2(p-1)] & ext{ for } k \geq 2(p-1) - 1 \ 0 & ext{ for } k < 2(p-1) - 1 \end{cases}$$

where [x] is the integer part of x, and if $n \ge 5$,

$$\varphi(k) = \varphi_n(k) = i$$
,

where

 $egin{array}{ll} 2i(p-1) \leq k < 2(i+1)(p-1) - 1 & ext{if} \quad i
ot = -1, \ 0 \ ext{mod} \ p \ , \ 2i(p-1) \leq k < 2(i+1)(p-1) - 2 & ext{if} \quad i \equiv -1 \ ext{mod} \ p \ , \ 2i(p-1) - 2 \leq k < 2(i+1)(p-1) - 1 & ext{if} \quad i \equiv 0 \ ext{mod} \ p \ . \end{array}$

Now we state our main theorem.

THEOREM 1 (Vanishing). Let $N \in \mathcal{M}A$ with $N_i = 0$ for i < n, where $n \geq 2$. Then

$$\operatorname{Ext}_{\mathscr{A}^{s,s+k+n}}^{s,s+k+n}(Z_p,N)\cong H^s(V(N))_{k+n} \xrightarrow{q^*} H^s(T(N))$$
 ,

is an isomorphism for $s > \varphi_n(k)$.

This will be proved in $\S4$.

By virtue of our vanishing theorem the calculation of $H^*(V(N))$ is reduced to that of $H^*(O(N))$ in a large extent. Note that q^* is epimorphic when U(M) is generated by a single element, where $M=N.^*$

As an immediate topological corollary we have.

COROLLARY 2. Let X be a simply connected space with $\pi_*(X)$ of finite type. Suppose that $H^*(X; Z_p) \cong U(M)$, where M is an unstable A-module. If $M^i = 0$ for i < n, then the orders of elements in the p-primary component of $\pi_{k+n}(X)$ are at most $p^{\varphi_n(k)}$.

This may be applied, for example, when X is an odd dimensional sphere, Stiefel manifold, or H-space.

REMARK. If $N_i = 0$, i > m, for some m, then $H^s(N)_t$ is zero for dimensional reason when t is large with respect to s. Hence in this case Corollary 2 is slightly improved.

2. Periodicity theorems. For a module $M \in A_{\mathscr{M}}$ we define the β -cohomology by $H_{\beta}(M) = \operatorname{Ker} \beta/\operatorname{Im} \beta$.

DEFINITION. A module $M \in A_{\mathscr{M}}$ is called β -trivial if

$$\rho^i: M^{2i} \longrightarrow H^{2ip}_{\beta}(M)$$
 ,

is an isomorphism for all i and $H^k_{\beta}(M) = 0$ for $k \not\equiv 0 \mod 2p$.

Remark that $M \in A \mathscr{M}$ is β -trivial if and only if $N = M^* \in \mathscr{M} A$ is towerless, i.e., $H^s(T(N)) = 0$ for s > 0.

Let \mathcal{G} denote the category of graded Z_p -modules. Let L_sF denote the sth left derived functor of a functor $F: A \mathcal{M} \to \mathcal{G}$.

THEOREM 3 (Periodicity). Let $F: A \mathcal{M} \to \mathcal{G}$ be a functor such that $F(M) = M/\beta M + \rho^{1}M$. If $M \in A \mathcal{M}$ is β -trivial, then there is a natural map

$$P \colon L_s F(M)^t \longrightarrow L_{s+p} F(M)^{t+2p(p-1)+p}$$
 ,

such that P is an isomorphism for $s \ge 2$ and a monomorphism for s = 1.

This will be proved in §3.

Additionally, we give here such a kind of periodicity theorems.

THEOREM 4. Let $G_i(M) = M/\rho^i M$ for $M \in A$ *M*, where 0 < i < p. Then there is a natural map

$$Q: L_s G_i(M)^t \longrightarrow L_{s+2} G_i(M)^{t+2p(p-1)}$$

such that Q is an isomorphism for $s \ge 2$ and a monomorphism for s = 1.

THEOREM 5. Let $G_i(M) = M/\beta M + \rho^i M$ for $M \in A_{\mathscr{M}}$, where 0 < i < p. If $M \in A_{\mathscr{M}}$ is β -trivial, then there is a natural map

 $Q: L_s G_i(M)^t \longrightarrow L_{s+2} G_i(M)^{t+2p(p-1)}$,

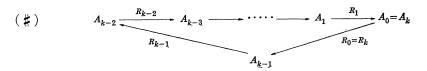
such that Q is an isomorphism for $s \ge 2$ and a monomorphism for s = 1.

THEOREM 6. Let $G(M) = M/\rho^1 + \beta \rho^1 M$ for $M \in A \mathcal{M}$. If $M \in A \mathcal{M}$ is β -trivial, then there is a natural map

 $R: L_s G(M)^t \longrightarrow L_{s+2} G(M)^{t+4p-2}$,

such that R is an isomorphism for $s \ge 2$ and a monomorphism for s = 1.

3. Proofs of periodicity theorems. Suppose given a circular sequence of functors from $A \mathcal{M}$ to \mathcal{G} and natural transformations,



satisfying $R_i R_{i+1} = 0$ for $i = 0, \dots, k-1$. Define functors Ker R_i , Im R_i , Coker R_i , $H_i = \text{Ker } R_i/\text{Im } R_{i+1}$ in a usual way.

DEFINITION. A module $M \in A \mathscr{M}$ is called trivial for the diagram (#) if

$$L_sA_i(M) = L_sH_i(M) = 0$$
,

for all s > 0 and $i = 0, \dots, k - 1$.

LEMMA. If $M \in A \mathscr{M}$ is trivial for the diagram (#), then there is a natural map

$$P: L_s \operatorname{Coker} R_0(M)^t \longrightarrow L_{s+k} \operatorname{Coker} R_0(M)^{t+k}$$
,

such that P is an isomorphism for $s \ge 2$ and a monomorphism for s = 1. Here $h = \sum_{i=0}^{k-1} h_i$, $h_i = \deg R_i$.

Proof. Let $h(a) = \sum_{i=a}^{k-1} h_i$. Since M is trivial for (#), we have the following natural isomorphism

$$egin{aligned} &L_{s+k}\operatorname{Coker} R_{\scriptscriptstyle 0}(M)^{t+h}\cong L_{s+k-1}\operatorname{Im} R_{\scriptscriptstyle 0}(M)^{t+h}\ &\cong L_{s+k-2}\operatorname{Ker} R_{\scriptscriptstyle 0}(M)^{t+h(1)}\cong\cdots\ &\cong L_s\operatorname{Ker} R_{k-2}(M)^{t+h(k-1)}\cong L_s\operatorname{Im} R_{k-1}(M)^{t+h(k-1)} \end{aligned}$$

On the other hand the natural map

$$L_s \operatorname{Coker} R_0(M)^t \longrightarrow L_s \operatorname{Im} R_{k-1}(M)^{t+h(k-1)}$$

is an isomorphism for $s \ge 2$ and a monomorphism for s = 1.

We shall use the following circular sequence due to Toda [9] (see, also, Oka [8]) to prove the periodicity theorems.

$$(3.1) \qquad \qquad M \xrightarrow{R_{p-2}} M \xrightarrow{\dots \dots \dots} M \xrightarrow{R_1} M$$

where $R_i = (i+1)\beta\rho^1 - i\rho^1\beta$, $R = (\rho^1\beta, \rho^1)$ and $R' = \rho^1\beta - \beta\rho^1\beta$.

$$(3.2) M \xleftarrow[p^{p^i}]{\rho^i} M for 0 < i < p ,$$

$$(3.3) \hspace{1cm} M/R_iM \stackrel{
ho^i}{\xleftarrow{}{\leftarrow}{\leftarrow}{\rightarrow}{\rightarrow}} M/etaM \hspace{1cm} ext{for} \hspace{1cm} 0 < i < p$$
 ,

(3.4)
$$M/\rho^{1}M \xleftarrow{\rho^{1}}{\longrightarrow} M/\rho^{1}M$$

Here $M \in A \mathscr{M}$ and the maps are induced from the left actions.

Proof of Theorem 3. We shall use the diagram (3.1). For convenience, we put $R_0 = R_p = R$, $R_{p-1} = R'$. Let $H_i(M)$ denote the cohomology Ker $R_i/\operatorname{Im} R_{i+1}$. If M is a free unstable A-module, then:

 $egin{array}{lll} ({\,\,\mathrm{i}}\,) &
ho^{s\!\!:}\,(M/eta M)^{2s}\cong H^{2sp}_{\scriptscriptstyle 0}(M) & {
m if} \quad s\equiv -1\,{
m mod}\,p \ , \
ho^{s\!\!:}\,M\cong H^{2sp}_{\scriptscriptstyle 0}(M) & {
m if} \quad s\not\equiv -1\,{
m mod}\,p \ , \
ho^{s\!\!:}\,(eta M)^{2s+1}\cong H^{2t\,p+1}_{\scriptscriptstyle 0}(M) & {
m if} \quad s\not\equiv -1\,{
m mod}\,p \ , \end{array}$

(ii) for
$$i = 1, \dots, p - 2$$
,
 $\rho^{s}: (M/\beta M)^{2s} \cong H_{i}^{2sp}(M)$,
 $\rho^{s} + \beta \rho^{s}: (\beta M)^{2s+2} + (M/\beta M)^{2s+1} \cong H_{i}^{2sp+2}(M)$,
 $\rho^{s}: (M/\beta M)^{2s+2} \cong H_{i}^{2sp+3}(M)$
(iii) $\rho^{s} + 0: (M/\beta M)^{2s} + 0 \cong H_{p-1}^{2sp}(M)$,
 $0 + \rho^{s}: 0 + (M/\beta M)^{2s+j} \cong H_{p-1}^{2sp+j+1}(M)$
for $j = 0, 1$ if $s \equiv 0 \mod p$,
 $\rho^{s} + \rho^{s}: (\beta M)^{2s+j+1} + (M/\beta M)^{2s+j} \cong H_{p-1}^{2sp+j+1}(M)$
for $j = 0, 1$ if $s \not\equiv 0 \mod p$,
 $0 + \rho^{s}: 0 + (M/\beta M)^{2s+2} \cong H_{p-1}^{2sp+3}(M)$.

(iv) otherwise $H_i^k(M) = 0$.

This unstable version of Toda's exactness theorem is shown by long but straightforward computations. Now Theorem 3 is proved by applying lemma.

By using the diagrams (3.2), (3.3) and (3.4), Theorems 4, 5 and 6 follow in a similar way, and thus we only state the following facts.

Let $M \in A \mathscr{M}$ be a free unstable module. Fix i such that 0 < i < p.

If $H(M) = \operatorname{Ker} (\rho^i: M \to M) / \operatorname{Im} (\rho^{p-i}: M \to M)$, then:

$$egin{array}{lll} ({\,\, {
m i}}\,) &
ho^{s}{
m :}\, M^{2s+j}\cong H^{2sp+j}(M) & {
m for} & j=0,\,1\,\,, \ & eta
ho^{s}{
m :}\, M^{2s+j-1} + M^{2s+j}\cong H^{2sp+j}(M) & {
m for} & j=2,\,\cdots,\,2i-1\,\,, \ & eta
ho^{s}{
m :}\, M^{2s+j-1}\cong \, H^{2sp+j}(M) & {
m for} & j=2i,\,2i+1\,\,. \end{array}$$

(ii) otherwise $H^k(M) = 0$.

If $H(M) = \text{Ker}(\rho^{p-i}: M/\beta M \to M/R_iM)/\text{Im}(\rho^i: M/R_iM \to M/\beta M)$, then:

(i) $\rho^{s}: (M/\beta M)^{2s+j} \cong H^{2sp+j}(M)$ for $j = 0, 1, \dots, i-1$ and $s \equiv 0, 1, \dots, i-1 \mod p$, (ii) otherwise $H^{k}(M) = 0$. Next put $H(M) = \text{Ker}(\rho^{i}: M/R_{i}M \to M/\beta M)/\text{Im}(\rho^{p-i}: M/\beta M \to M/R_{i}M)$, then: (i) if $s \equiv 0 \mod p$,

$$egin{aligned} &
ho^{s\colon} (M/eta M)^{2s+j} \cong H^{2sp+j}(M) & ext{for} \quad j=0,1 \ , \ η
ho^{s}\colon (M/eta M)^{2s+j-1} + (M/eta M)^{2s+j} \cong H^{2sp+j}(M) & ext{for} \quad j=2,\,\cdots,\,2i-1 \ , \ η
ho^{s\colon} (M/eta M)^{2s+j-1} \cong H^{2sp+j}(M) & ext{for} \quad j=2i,\,2i+1 \ , \ &(ext{ii}) & ext{if} \ s\equiv 1,\,\cdots,\,p-i-1 \ ext{mod} \ p, \end{aligned}$$

$$egin{aligned} &
ho^{s}\colon (M/eta M)^{2s+j} \cong H^{2sp+j}(M) & ext{for} \quad j=0,\,1\;,\ η
ho^{s} +
ho^{s}\colon M^{2s+j-1} + (M/eta M)^{2s+j} \cong H^{2p+i}(M) & ext{for} \quad j=2,\,\cdots,\,2i-1\;,\ η
ho^{s}\colon M^{2s+j-1} \cong H^{2sp+j}(M) & ext{for} \quad j=2i,\,2i+1\;, \end{aligned}$$

(iii) if
$$s \equiv p - i \mod p$$
,
 $\beta \rho^s \colon M^{2s+j-1} \cong H^{2sp+j}(M)$ for $j = 2, \dots, 2i - 1$,

(iv) otherwise $H^k(M) = 0$.

Finally, if $H(M) = \text{Ker} (\beta \rho^1 \colon M/\rho^1 M \to M/\rho^1 M)/\text{Im} (\beta \rho^1 \colon M/\rho^1 M \to M/\rho^1 M)$, then:

- (i) $ho^{ps}: M^{2s+j} \cong H^{2sp^2+j}(M)$ for j = 0, 1, $eta
 ho^{ps} +
 ho^{ps}: (M/eta M)^{2s+1} + M^{2s+2} \cong H^{2sp^2+2}(M)$,
- (ii) otherwise $H^k(M) = 0$.

4. Proof of the vanishing theorem. Let F(n) denote a free unstable A-module on one generator ι_n . We define an unstable Amodule N(n) to be the quotient of F(n) by the relation $\beta \iota_n = 0$. Next define M(n) to be the subcomplex of N(n) by ommitting the ι_n from N(n) if n odd and ommitting the ι_n , $(\iota_n)^p$, \cdots , $(\iota_n)^{p^t}$, \cdots from N(n) if n even. Note that M(n) is β -trivial.

First we suppose that n is odd. Then by the long exact sequence induced from a short exact sequence

$$0 \longrightarrow M(n) \longrightarrow N(n) \longrightarrow Z_p \longrightarrow 0$$
,

we have an isomorphism

$$E_2^{s,t+n}(S^n) = \operatorname{Ext}_{A^{\mathscr{M}}}^{s,t+n}(Z_p, Z_p) \cong \operatorname{Ext}_{A^{\mathscr{M}}}^{s-1,t+n}(M(n), Z_p),$$

for $t \neq s$. Let C(n) be a minimal resolution of M(n). By virtue of Theorem 3 we can prove the vanishing theorem for Z_p by analysing C(n). Namely, $\operatorname{Ext}_{A,\mathscr{A}}^{s-1,t+n}(M(n), Z_p)(t \neq s)$ vanishes for $s > \varphi_n(t-s)$. Furthermore we can observe the periodicity phenomenon in a range near the vanishing line. In fact, by Theorems 3, 4 and 5 we have two periodicity operators P and Q of bidegree (p, 2p(p-1) + p)and (2, 2p(p-1)), respectively.

For lower dimensional sphere we shall give periodic families. Let $1 < m \leq p+1$. In $E_2^{s,t+2m-1}(S^{2m-1})$ there appear nontrivial elements when (s, t-s) is as follows:

(i) (1, q - 1)

$$(1, pq - 1)$$
 for $m = p + 1$,

(ii)
$$(s, sq - 1), (s, (m + s - 2)q - 2)$$

for $s = 2, \dots, p - m + 1$ and $m \neq p, p + 1$,

$$\begin{array}{l}(s,\,sq\,-\,1),\,(s,\,pq\,-\,2),\,(s,\,pq\,-\,1)\\ \mbox{ for }s\,=\,p\,-\,m\,+\,2 \ \ \mbox{and } \ \ m\neq p\,+\,1 \ ,\\(s,\,sq\,-\,1),\,(s,\,pq\,-\,2),\,(s,\,pq\,-\,1),\,(s,\,(m\,+\,s\,-\,2)q\,-\,2)\\ \mbox{ for }s\,=\,p\,-\,m\,+\,3,\,\cdots,\,p\,-\,1 \ \ \ \mbox{and }p\neq 3 \ ,\\(p,\,pq\,-\,2),\,(p,\,pq\,-\,1),\,(p,\,(p\,+\,m\,-\,2)q\,-\,2) \ ,\\(p\,+\,1,\,(p\,+\,1)q\,-\,1),\,(p\,+\,1,\,(p\,+\,m\,-\,1)q\,-\,2) \ ,\end{array}$$

where q = 2(p-1). Applying the periodicity operators P and Q repeatedly, we can determine the behavior of all $E_2(S^{2m-1})$ near the vanishing line. (Possibly other elements appear in a range apart from the vanishing line when we apply the iteration of the operator Q.)

We next suppose that *n* is even. Let $L(n; t)(0 < t \leq \infty)$ be an unstable *A*-module with elements $\sigma_n, (\sigma_n)^p, \dots, (\sigma_n)^{p^t}$ where deg $\sigma_n = n$. By the long exact sequence induced from short exact sequences

$$\begin{array}{l} 0 \longrightarrow M(n) + L(p^{t+1}; \ \infty) \longrightarrow N(n) \longrightarrow L(n; \ t) \longrightarrow 0 \\ 0 \longrightarrow M(p^{t+1}n) \longrightarrow N(p^{t+1}n) \longrightarrow L(p^{t+1}n; \ \infty) \longrightarrow 0 \end{array},$$

we have an isomorphism

$$\mathrm{Ext}^{s,t+n}_{A\mathscr{A}}(L(n;t), Z_p) \ \cong \mathrm{Ext}^{s-1,t+n}_{A\mathscr{A}}(M(n), Z_p) + \mathrm{Ext}^{s-2,t+n}_{A\mathscr{A}}(M(p^{t+1}n), Z_p)$$

for $t \neq s, s + (p^{t+1} - 1)n - 1$. Thus in a similar way we have the required results for L(n; t).

Now we have shown that

$$q^*: H^s(V(N))_{k+n} \longrightarrow H^s(T(N)_{k+n})$$

is an isomorphism for $s > \varphi_n(k)$, when $N^* = H^*(S^n; Z_p) = Z_p(n \text{ odd})$ and $N^* = L(n; t)(n \text{ even})$. The general case follows inductively using the five lemma.

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