

Pacific Journal of Mathematics

**ADDENDUM TO: "FIXED POINTS OF AUTOMORPHISMS OF
COMPACT LIE GROUPS"**

ROBERT F. BROWN

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THEOREM. Let G be a compact, connected Lie group and let h be an endomorphism of G . Then the rank of the Lie group $\Phi(h)$ is equal to the dimension of the graded vector space $\Phi(Ph^*)$.

The statement of the main result (Theorem 1.1) of [1] is unnecessarily restrictive. The result is stated only for automorphisms, but the theorem is in fact true for all endomorphisms.

The proof is the same as in [1] except for the following, which replaces the argument on pages 82-83. Let $\eta = dh: \mathfrak{G} \rightarrow \mathfrak{G}$ be the differential of h , where \mathfrak{G} is the Lie algebra of G . Then η induces $Q\eta_*: QH_*(\mathfrak{G}) \rightarrow QH_*(\mathfrak{G})$ on the indecomposables in the homology of \mathfrak{G} . By de Rham's theorem (see [2]) and Proposition 3.10 of [3], it is sufficient to prove

$$(*) \quad \text{rank } \Phi(\eta) = \dim \Phi(Q\eta_*).$$

For the same reasons, we already know that (*) is true if \mathfrak{G} is abelian (Propositions 2.2 and 2.3 of [1]) or if \mathfrak{G} is semisimple and η is an automorphism (Lemma 3.1). Write $\mathfrak{G} \cong \mathfrak{Z} \oplus \mathcal{S}\mathfrak{G}$ where \mathfrak{Z} is the center of \mathfrak{G} and $\mathcal{S}\mathfrak{G}$ is semisimple. Then write $\mathfrak{Z} \cong \mathfrak{Z}_a \oplus \mathfrak{Z}_b$ where $\mathfrak{Z}_a = \eta^{-1}(\mathfrak{Z})$. Let $\eta_a: \mathfrak{Z}_a \rightarrow \mathfrak{G}$ be the restriction of η to \mathfrak{Z}_a .

For $p: \mathfrak{G} \rightarrow \mathfrak{Z}_a$ the projection, $p\eta_a$ is an endomorphism of an abelian Lie algebra. So (*) is true for $p\eta_a$ - and therefore for η_a . Since $\mathfrak{Z}_b \cap \eta(\mathfrak{Z}_b) = 0$, we conclude that $\dim \Phi(Q\eta_{b*}) = 0$. Let $\mathcal{S}\eta: \mathcal{S}\mathfrak{G} \rightarrow \mathcal{S}\mathfrak{G}$ be the restriction of η . We can write $\mathcal{S}\mathfrak{G} \cong \mathfrak{U}_1 \oplus \dots \oplus \mathfrak{U}_N \oplus \mathfrak{B}$ where the restriction η_i of $\mathcal{S}\eta$ to each \mathfrak{U}_i is an automorphism and the behavior of $\mathcal{S}\eta$ on the fixed points is determined by the η_i . Since (*) is true for each η_i , it holds for $\mathcal{S}\eta$ as well. Finally,

$$\text{rank } \Phi(\eta) = \text{rank } \Phi(\eta_a) + \text{rank } \Phi(\mathcal{S}\eta)$$

which completes the proof.

I thank the referee for correcting an error in an earlier version of this paper.

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