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This paper is concerned with the M-ideal structure of the algebras $B(l_p)$ of bounded operators on the sequence spaces l_p, $1 < p < \infty$. The M-summands are completely determined, but the M-ideals are only partially characterized. However evidence is presented to support the conjecture that the only nontrivial M-ideal is the ideal $C(l_p)$ of compact operators on l_p.

1. **Introduction.** It has been observed by several authors that various structure theorems for $B(H)$, H a separable Hilbert space, can be extended to the spaces $B(l_p)$, $1 < p < \infty$. For instance, it is known that the ideal $C(l_p)$ of compact operators in $B(l_p)$, $1 < p < \infty$ is the only closed nontrivial two sided ideal [9], and Các [5] has shown that the second dual of this space is isometrically isomorphic with $B(l_p)$. In another direction Hennefeld [10] proved that $C(l_p)$ is an M-ideal in $B(l_p)$, $1 < p < \infty$. The notion of an M-ideal generalizes the two sided ideals in a C^*-algebra; due to the geometric characterization of the ideals in these special algebras the M-ideals have been identified with the two sided ideals [13].

The present work arose from an attempt to extend the latter result on M-ideals to $B(l_p)$, $1 < p < \infty$. Although the M-ideals in $B(l_p)$ are not yet completely characterized, certain positive results are obtained. For instance, in $B(l_p)$ the M-summands, a special subclass of M-ideals, are described. Moreover, it is shown that $C(l_p)$ is a minimal M-ideal in $B(l_p)$, $1 < p < \infty$, in the sense that every nontrivial M-ideal in $B(l_p)$ contains the ideal of compact operators. The techniques developed herein yield a new proof that the M-ideals must be two sided ideals in a C^*-algebra. In addition, certain structure theorems on the state space of $B(l_p)$, $1 < p < \infty$ and on the hermitian elements of $B(l_p)^{**}$ are derived.

2. **Preliminaries.** A closed subspace N of a Banach space X is said to be an L-ideal if there exists a closed subspace N' such that $X = N \oplus N'$ and $\|n + n'\| = \|n\| + \|n'\|$ for all $n \in N$ and $n' \in N'$. A closed subspace J is said to be an M-ideal if the annihilator $J^\perp$ is an L-ideal in X^*. A closely related concept is that of an M-summand which is defined to be an M-ideal J with a complementary closed subspace J' such that $\|j + j'\| = \max\{\|j\|, \|j'\|\}$ for all $j \in J$ and $j' \in J'$. It should be noted that M-ideals need not be M-summands. The detailed properties of these objects have been studied in [2],

and in particular the annihilator of an L-ideal is an M-summand, while the dual statement is true for the annihilator of an M-summand.

The M-ideal structure of Banach algebras was investigated in [13] and the results relevant to this paper are summarized below. Let A be a Banach algebra with identity e and let J be an M-ideal in A. Denote by S the state space of A defined to be $\{\phi \in A^*: ||\phi|| = \phi(e) = 1\}$. Then $J^\perp$ and its complementary L-ideal, when intersected with S, yield a pair of complementary split faces F and F' respectively of S [13]. $J^{\perp\perp}$ is an M-summand in A^{**} with complementary M-summand $J^{\perp\perp\prime}$ and $Pe = z$ is an hermitian projection in A^{**}, where P is the projection of A^{**} onto $J^{\perp\perp}$. If z is regarded as a real valued affine function on S then $z|F = 0$ and $z|F' = 1$. In general z is not the identity on the algebra $J^{\perp\perp}$ although if A is commutative then this is the case [12]. However the following relations hold.

THEOREM 2.1. *For an M-ideal J, $zA^{**}z \subset J^{\perp\perp}$ and $(e-z)A^{**}(e-z) \subset J^{\perp\perp\prime}$.*

If z is not the identity on $J^{\perp\perp}$ then z does not commute with every element of A^{**}. However there is a class of elements for which z is central, and this will be useful for later work.

LEMMA 2.2. *Let J be an M-ideal in A with associated projection $z \in A^{**}$. Then z commutes with every hermitian element of A^{**}.*

Proof. Let ϕ be a state in F' so that $z(\phi) = 1$, and define a linear functional $\phi_z \in A^*$ by

$$\phi_z(a) = \phi(za)$$

for all $a \in A$. Since $\phi_z(e) = 1$ it is clear that $\phi_z \in S$. If h is any hermitian element of A^{**} then $\phi_z(h) \in \boldsymbol{R}$ and so

$$\phi(zh) = \phi_z(h) \in \boldsymbol{R} .$$

For a state $\psi \in F$, $\psi_{(e-z)} \in S$ and thus

$$\psi(zh) = \psi(h) - \psi((e - z)h) \in \boldsymbol{R} .$$

The element zh is seen to take real values on F and F' and it follows then zh is hermitian since $S = \operatorname{conv}(F \cup F')$. Similar arguments imply that hz is also hermitian and so $hz - zh$ is hermitian. However $i(hz - zh)$ is hermitian by [4, p. 47] and the only way to reconcile these statements is to conclude that $hz = zh$.

In [13] it was shown that if A is a C^*-algebra then the M-ideals are closed algebraic ideals. It is interesting to note that this is an easy consequence of the preceding lemma.

COROLLARY 2.3. *The M-ideals in a C^*-algebra A are closed algebraic ideals.*

Proof. The hermitian elements span A and so z is central in A^{**}, by Lemma 2.2. The result follows from Theorem 2.1.

3. M-summands in $B(l_p)$. Henceforth the study of M-ideals will be concentrated on the classical Banach spaces of bounded operators on the sequence spaces l_p. The restrictions will be made that $1 < p < \infty$ and that $p \neq 2$. For $p = 2$, $B(l_2)$ is a C^*-algebra and so the results to be obtained in the general case are trivial consequences of [13, §5] for this space. The spaces with indices 1 and ∞ differ markedly from those considered here, and some indication of this will be given in a later section.

The first results concern M-summands in $B(l_p)$ and for these a theorem due to Tam will be needed.

THEOREM 3.1 (Tam [15]). *The hermitian operators in $B(l_p)$, $1 < p < \infty$ $p \neq 2$, are precisely the diagonal operators with respect to the canonical basis $\{e_i\}_{i=1}^{\infty}$ possessing real entries.*

THEOREM 3.2. *There are no nontrivial M-summands in $B(l_p)$, $1 < p < \infty$.*

Proof. The case $p = 2$ will be considered later and so suppose that $p \neq 2$. Let J and J' be complementary M-summands in $B(l_p)$, let $z \in B(l_p)$ be the hermitian projection associated with J, and denote by F and F' the pair of split faces in the state space of $B(l_p)$ obtained from J and J'. The projection z takes the values 1 on F' and 0 on F. The object is to show that z is the identity for J.

Consider $\phi \in F'$, and suppose that $\phi_{(e-z)} \neq 0$. Then there exists an operator $A \in B(l_p)$ of norm less than or equal to one and there exists $\delta \in (0, 1)$ such that $\phi_{(e-z)}(A) = \delta$. For each integer n define

$$X_n = z + \delta^n(e - z)A .$$

From Theorem 3.1 the matrix of z consists only of zeros and ones on the diagonal and so for any $y \in l_p$ the vectors zy and $\delta^n(e - z)Ay$ possess disjoint supporting sets from the canonical basis. Thus

$$\begin{aligned}||X_n y|| &= (||xy||^p + ||\delta^n(e-z)Ay||^p)^{1/p} \\ &\leqq (1+\delta^{np})^{1/p}||y|| \,.\end{aligned}$$

Hence

$$||X_n|| \leqq (1+\delta^{np})^{1/p}$$

and, since $||\phi|| = 1$, this leads to the inequalities

$$(1+\delta^{np})^{1/p} \geqq ||X_n|| \geqq |\phi(X_n)| = 1 + \delta^{n+1} \,.$$

From the binomial expansion

$$1 + \delta^{n+1} \leqq (1+\delta^{np})^{1/p} \leqq 1 + \delta^{np}/p \,,$$

which is equivalent to

$$p \leqq \delta^{n(p-1)-1} \,,$$

since $\delta > 0$. However this inequality holds for all n. As n tends to infinity $\delta^{n(p-1)-1}$ tends to zero, since $p > 1$, and this gives a contradiction. Thus $\phi_{(e-z)} = 0$.

This relation implies that, for $\phi \in F'$ and $A \in B(l_p)$,

$$\phi(zA) = \phi(A) \,,$$

while similar reasoning shows that, for $\psi \in F$,

$$\psi((e-z)A) = \psi(A) \,.$$

Now consider $j \in J$. If $\phi \in F'$ then

$$\phi(zj) = \phi(j) \,,$$

while if $\psi \in F$ then both

$$\psi(j) = 0 \quad \text{and} \quad \psi(zj) = \psi((e-z)zj) = 0 \,.$$

Thus $j = zj$ and so $J \subset zB(l_p)$. Similarly $J' \subset (e-z)B(l_p)$ and, since $B(l_p) = J \oplus J'$, it is clear that equality holds in these inclusions. Thus J and J' are right sided ideals in $B(l_p)$.

The adjoint is an isometric isomorphism between $B(l_p)$ and $B(l_q)$ where $1/p + 1/q = 1$, and so the image of J in $B(l_q)$ is an M-summand and thus a right sided ideal in $B(l_q)$. However the adjoint reverses multiplication and so J and J' are also left sided ideals. This shows that any M-summand in $B(l_p)$ is a two sided ideal. Now the only two sided ideals in $B(l_p)$ are 0, $B(l_p)$ and $C(l_p)$ [9] and in order that the condition $B(l_p) = J \oplus J'$ be satisfied it is clear that $J = 0$ or $J = B(l_p)$. This completes the proof.

REMARK 1. The above result is strict in the sense that there are many M-summands in $B(l_1)$. The subspace of matrices in $B(l_1)$ which have a prescribed set of column vectors identically zero is a nontrivial M-summand.

REMARK 2. The proof of Theorem 3.2 was motivated by some work of Prosser [11] who characterized the one sided ideals of a C^*-algebra.

REMARK 3. For $p = 2$ Theorem 3.1 fails and so the proof in Theorem 3.2 is no longer valid. However the M-ideals in a C^*-algebra are the closed two sided ideals [13] and the argument of the last paragraph is still true.

The ideal $C(l_p)$ of compact operators in $B(l_p)$ is known to be an M-ideal [10] and a natural conjecture is that this is the only nontrivial M-ideal, by analogy with the case $p = 2$. It has not proved possible to obtain this result, but this ideal can at least be shown to be contained in any nontrivial M-ideal.

LEMMA 3.3. *Let J be an M-ideal in $B(l_p)$. Then either $J \cap C(l_p) = 0$ or $J \cap C(l_p) = C(l_p)$.*

Proof. Suppose that the conclusion is false. Then there exists an M-ideal J such that $J \cap C(l_p)$ is a nontrivial M-ideal in $C(l_p)$. The second dual $C(l_p)^{**}$ is isometrically isomorphic to $B(l_p)$ [5], and $J \cap C(l_p)$ induces a pair of nontrivial complementary M-summands in $B(l_p)$. This contradicts Theorem 3.2.

THEOREM 3.4. *Let J be a nonzero M-ideal in $B(l_p)$. Then J contains $C(l_p)$.*

Proof. From Lemma 3.3, $J \cap C(l_p)$ is either 0 or $C(l_p)$. In the second case the theorem is proved, and so assume that $J \cap C(l_p) = 0$.

Let z be the hermitian projection associated with J, and for each n let P_n be the projection onto the span of the first n elements of the canonical basis. Consider a net $(e_\alpha)_{\alpha \in A}$ from $B(l_p)$ which converges in the w^*-topology of $B(l_p)^{**}$ to z. For each n it is clear that

$$\lim_{\alpha} P_n e_\alpha P_n = P_n z P_n$$

in the w^*-topology, while elements of the net $(P_n e_\alpha P_n)_{\alpha \in A}$ are compact and all lie in a finite dimensional subspace of $C(l_p)$. Thus convergence takes place in the norm topology, and it follows that $P_n z P_n \in C(l_p)$ for all n.

From Lemma 2.2, P_n and z commute, and so

$$zP_n z = zP_n = P_n z = P_n z P_n \in C(l_p) .$$

However $zP_n z \in J^{\perp\perp}$, by Theorem 2.1, and thus $zP_n z \in J \cap C(l_p)$. By hypothesis

$$zP_n = P_n z = zP_n z = 0$$

for all n. Let K be a compact operator. Given $\varepsilon > 0$ there exists n such that $||P_n K P_n - K|| < \varepsilon$, and the inequalities

$$||Kz|| = ||Kz - P_n K P_n z|| \leqq ||K - P_n K P_n|| \, ||z|| < \varepsilon$$

and

$$||zK|| = ||zK - zP_n K P_n|| \leqq ||K - P_n K P_n|| \, ||z|| < \varepsilon$$

show that

$$zK = Kz = 0 .$$

For every $K \in C(l_p)$,

$$(e - z)K(e - z) = K ,$$

and thus

$$C(l_p) = (e - z)C(l_p)(e - z) \subset (e - z)B(l_p)^{**}(e - z) \subset J^{\perp\perp\prime}$$

by Theorem 2.1. Now it is clear that J and $C(l_p)$ lie in complementary M-summands in $B(l_p)^{**}$ and so, for $K \in C(l_p)$ and $A \in J$,

$$||K + A|| = \max \{||K||, ||A||\} .$$

Choose a nonzero element $A \in J$ of unit norm. After multiplication by a suitable constant it may be assumed that the matrix of A has a strictly positive entry δ occuring in some position (i, j). Let K be the compact operator whose matrix has 1 in the (i, j) position and zeros elsewhere. Then

$$||A|| = 1 , \quad ||K|| = 1 \quad \text{and} \quad ||K + A|| \geqq 1 + \delta ,$$

which contradicts the defining equation for M-summands. The original assumption is seen to be incorrect, and this forces the conclusion that J contains $C(l_p)$.

REMARK. The behavior of $C(l_p)$ in the last theorem is uncharacteristic of that of M-ideals in general. For example the C^*-

algebra $C[0, 3]$ of continuous function on $[0, 3]$ possesses no nontrivial minimal M-ideals. In this example the ideals of functions which vanish on $[0, 2]$ and $[1, 3]$ respectively are nontrivial M-ideals which have trivial intersection.

4. Some structure theorems. In this section, a result on singular states of $B(l_p)$ is derived which is reminiscent of some work of Glimm [8]. This points out the similarity of the respective state spaces of $B(l_p)$ and $B(H)$. In addition, the hermitian elements of the second dual of $B(l_p)$ are partially characterized. The fact that the hermitian projections of $B(l_p)$ are exactly the diagonal operators with only zero and one entries was central to the arguments used in Theorem 3.2. Since determining the M-ideals of a space is equivalent to characterizing the M-summands of its second dual it is natural to investigate the hermitian elements of $B(l_p)^{**}$. By the Goldstine density theorem H is an hermitian element of a dual space X^{**} if and only if H is real valued on the state space of X. This fact coupled with Theorem 3.2 reformulates the problem to that of determining the M-ideal structure of $B(l_p)/C(l_p) \equiv A(l_p)$ and the corresponding state space of $A(l_p)$. A useful result along these lines is Proposition 4.3 which generalizes a lemma of Glimm [8].

In the sequel $\bar{Q}$ will denote the closure of a set Q, $\overline{\text{conv}}\, Q$ will be the closed, convex hull of Q and $\partial_e K$ will designate the extreme boundary of K.

LEMMA 4.1. *Let K be a compact convex set and let Q be a subset satisfying $\overline{\text{conv}}\, Q = K$. Then $\bar{Q}$ contains $\partial_e K$.*

Proof. Suppose that the conclusion is false. Then there exists $x \in \partial_e K/\bar{Q}$. Let f be a continuous function such that

$$f(x) = 1\,, \quad f\,|\,\bar{Q} = 0$$

and consider the lower envelope $\check{f}$ of f defined, for $y \in K$, by

$$\check{f}(y) = \sup\{a(y)\colon a \in A(K) \text{ and } a \leqq f\}\,.$$

Clearly $\check{f}\,|\,Q \leqq 0$, and $\check{f}(x) = f(x) = 1$ since x is an extreme point [1, I.4.1]. Hence there exists $a \in A(K)$ such that $a\,|\,\bar{Q} \leqq 0, a(x) \geqq 1/2$, and $a^{-1}((-\infty, 0])$ is a closed convex set containing Q but not containing x. It follows that $x \notin \overline{\text{conv}}\, Q$, which is a contradiction.

The above lemma is relevant in light of the following.

LEMMA 4.2 (*Stampfli, Williams* [14]). *Let $B(X)$ denote the set* *of bounded linear operators on the Banach space X. Then the convex hull of the set of vector states is w^*-dense in the state space of $B(X)$.*

PROPOSITION 4.3. *Let f be a state on $A(l_p)$. Then f is a w^*-limit of vector states on $B(l_p)$.*

Proof. By the Krein-Milman theorem f is the w^*-limit of convex combinations of pure states of $A(l_p)^*$. Therefore f is the w^*-limit of states of the form $\lambda_1 f_1 + \cdots + \lambda_n f_n$ where $\lambda_i \geqq 0$, $\sum_{i=1}^n \lambda_i = 1$ and where $f_1, \cdots, f_n$ are pure states of $A(l_p)$ which are regarded as lying in $B(l_p)^*$. So it suffices to study the case where f has the form $\lambda_1 f_1 + \cdots + \lambda_n f_n$ with the preceding properties. Let $x_1, \cdots, x_s$ be elements of $A(l_p)$ and construct unit vectors $\xi_1, \cdots, \xi_n$ in l_p having finite support so that $\langle x_i \xi_j, \xi_k \rangle < \varepsilon$ for $1 \leqq j,\ k \leqq n$ and $|f_j(x_i) - w_{\xi_j}(x_i)| < 1$ for all i and j. Suppose that the ξ_j's have been constructed for $j < m$. If $E_M = \operatorname{sp}\{e_1, \cdots, e_M\}$, pick $E_M^\perp$ so that for any unit vector v in $E_M^\perp$,

$$(4.3) \qquad \begin{aligned} &\langle x_i \xi_j, v \rangle \leqq \varepsilon/2 \\ &\langle \xi_j, x_i v \rangle \leqq \varepsilon/2\,, \quad 1 \leqq i, j \leqq m-1\,. \end{aligned}$$

Let P'_M denote the projection onto $E_M^\perp$ and $f_{m'}$ the singular state given by

$$f_{m'}(T) = f_m(P_{M'} T P_{M'})\,.$$

An easy argument shows that $f_{m'}$ remains a singular pure state. Since $f_{m'}$ may be viewed as a pure state on the space $P_{M'} B(l_p) P_{M'}$, Lemmas 4.1 and 4.2 apply and one concludes that $f_{m'}$ is the w^*-limit of functionals w_{ξ_α} where the ξ_α are unit vectors $E_M^\perp$. One thus can find $\xi_m \in E_M^\perp$ of finite support such that

$$|f_m(x_i) - w_{\xi_m}(x_i)| < 1 \quad \text{for} \quad 1 \leqq i \leqq s\,.$$

In addition, ξ_m must satisfy condition (4.3). This construction of the ξ_j can thus proceed by induction. This completed, set

$$\xi = \lambda_1^{1/p} \xi_1 + \cdots + \lambda_n^{1/p} \xi_n\,.$$

Since the ξ_i have disjoint supports, ξ is a unit vector. Since conditions (4.3) hold for $1 \leqq j,\ k \leqq n$, then

$$\begin{aligned} \left| \sum_{j=1}^n \lambda_j f_j(x_i) - w_\xi(x_i) \right| &= \left| \sum_{j=1}^n \lambda_j f_j(x_i) - \sum_{j,k=1}^n (x_i \lambda_j^{1/p} \xi_j,\ \lambda_k^{1-1/p} \xi_k) \right| \\ &\leqq \left| \sum_{j=1}^n \lambda_j f_j(x_j) - \sum_{j=1}^n \lambda_j (x_i \xi_j,\ \xi_j) \right| + n^2 \varepsilon\,. \end{aligned}$$

Since n is fixed, ε may be chosen so that the latter expression is less than one. This proves that $\sum_{j=1}^{n} \lambda_j f_j$ is the w^*-limit of vector states which in turn completes the proof.

It can be shown that if the set of hermitian elements of $B(l_p)$ is w^*-dense in the set of hermitian elements of $B(l_p)^{**}$ then the M-ideals in $B(l_p)$ are necessarily two sided ideals. This, in turn, would imply that $C(l_p)$ is the only nontrivial M-ideal in $B(l_p)$. This appears to be a difficult question. For instance, in a C^*-algebra the set of hermitian elements is w^*-dense in the set of hermitian elements of the second dual space. The result is also true, rather trivially, for $C(l_p)$ and its second dual space $B(l_p)$. On the other hand, the assertion is false for the disk algebra $A(D)$. The hermitian elements of $A(D)$ are just the real multiples of the constant function 1 [6], whereas $A(D)^{**}$ contains all the hermitian projections associated with M-ideals of $A(D)$ (cf. [7] and [12]). The following two propositions lend evidence that the assertion is indeed true for $B(l_p)$.

In the sequel, P will denote any projection whose range is spanned by some subset of the canonical basis vectors.

PROPOSITION 4.4. *If H is hermitian in $B(l_p)^{**}$, then PHP is also hermitian.*

Proof. Let ω be a vector state and consider the functional ω_P defined by

$$\omega_P(T) = \omega(PTP) = (PTPx, x') = (TPx, (Px)') .$$

Clearly ω_P is a real multiple of a state. Since this reasoning remains true for convex combinations of vector states, it also holds for any state ϕ. Thus there exists $\lambda \in \boldsymbol{R}$, $s \in S(B(l_p))$ so that $\phi_P = \lambda s$. Thus

$$\phi(PHP) = \phi_P(H) = \lambda s(H) \in \boldsymbol{R}$$

so PHP is hermitian. This concludes the proof.

If the hermitian elements in $B(l_p)$ are dense in those of the second dual, then these sets can be identified with the self-adjoint parts of the C^*-algebras l_∞ and l_∞^{**} respectively. In this case the hermitian elements form a commutative algebra, and thus the following two results point positively in this direction.

PROPOSITION 4.5. *Let $H \in B(l_p)^{**}$ be hermitian and let P be an hermitian projection in $B(l_p)$. Then $P^\perp HP = 0$ on vector states.*

Proof. This follows immediately from Lemma 1 of [3].

COROLLARY 4.6. *If P is a finite dimensional projection then $P^{\perp}HP = 0$ for all hermitian elements of $B(l_p)^{**}$.*

Proof. It suffices to consider the case where P is the projection onto the span of the first n basis elements. Consider the vector subspace V of $B(l_p)^*$ spanned by functionals of the form

$$T \longmapsto (Te_i, y_i')$$

for $i = 1, 2, \cdots, n$, and each y_i in the closed span of $\{e_{n+1}, e_{n+2}, \cdots\}$. It is easy to check that V is w^*-closed.

For any state ϕ define a linear functional ϕ^* by

$$\phi^*(T) = \phi(P^{\perp}TP)$$

for all $T \in B(l_p)$. In the particular case of a vector state ω defined by a unit vector $x \in l_p$,

$$\omega^*(T) = (P^{\perp}TPx, x') = (TPx, P^{\perp}x') .$$

From the nature of P it is clear that $\omega^* \in V$. This conclusion applies equally to any combination of vector states, and the w^*-continuity of this operation together with Theorem 4.2 implies that $\phi^* \in V$ for every state ϕ. Hence there exist vectors

$$y_1, y_2, \cdots, y_n \in \overline{\text{span}}\, \{e_{n+1}, e_{n+2}, \cdots\}$$

such that

$$\phi^*(T) = \sum_{i=1}^{n} (Te_i, y_i') .$$

For each i, e_i, and y_i have disjoint supports, and so from these two vectors a unit vector x_i may be constructed so that

$$(Te_i, y_i') = \alpha_i(TPx_i, P^{\perp}x_i') ,$$

where α_i is a constant. If ω_i is the vector state associated with x_i then

$$\phi^* = \sum_{i=1}^{n} \alpha_i \omega_i^* .$$

If H is an hermitian element of $B(l_p)^{**}$ then $w_i^*(H) = 0$, by the preceding proposition, and so

$$\phi^*(H) = \phi(P^{\perp}HP) = 0$$

for all states ϕ. Thus $P^{\perp}HP = 0$.

PROPOSITION 4.7. *Each hermitian element in $B(l_p)^{**}$ commutes with every compact diagonal operator.*

Proof. If P is a finite dimensional projection and H is hermitian then, from above, $P^{\perp}HP = 0$. Similar techniques yield $PHP^{\perp} = 0$ and thus

$$PH = HP .$$

The result is now clear.

Added in proof. The authors have established that $C(l_p)$ is the only nontrivial M-ideal in $B(l_p)$. This result will appear elsewhere.

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