Pacific Journal of Mathematics

EXAMPLES OF LOCALLY COMPACT NONCOMPACT MINIMAL TOPOLOGICAL GROUPS

SUSANNE DIEROLF AND ULRICH SCHWANENGEL

Vol. 82, No. 2 February 1979

PACIFIC JOURNAL OF MATHEMATICS
Vol. 82, No. 2, 1979

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In 1971, R. M. Stephenson, Jr., [4], showed that an abelian locally compact topological group must be compact if it is minimal (i.e., if it does not admit a strictly coarser Hausdorff group topology). He left open the question, whether there exist locally compact noncompact minimal topological groups.

In this note we give an example of a closed noncompact subgroup of GL (2; R) which is minimal. Moreover we prove that every discrete topological group is topologically isomorphic to a subgroup of a locally compact minimal topological group. Another example shows that a minimal topological group can contain a discrete, nonminimal normal subgroup.

Let $(X, \mathfrak{T})$ be a topological group. Then $\mathfrak{U}_e(X, \mathfrak{T})$ denotes the filter of all $\mathfrak{T}$-neighborhoods of the neutral element $e \in X$. If $G \subset X$ is a subgroup, let $\mathfrak{T}|G$ denote the relative topology induced by $\mathfrak{T}$ on G, and let $\mathfrak{T}/G$ denote the quotient topology on the left coset space X/G.

In the sequel we will need the following technical result.

LEMMA 1. *Let X be group and $G \subset X$ be a subgroup. Let $\mathfrak{S}, \mathfrak{T}$ be group topologies on X such that $\mathfrak{S} \subset \mathfrak{T}$, $\mathfrak{S}|G = \mathfrak{T}|G$, and $\mathfrak{S}/G = \mathfrak{T}/G$. Then $\mathfrak{S} = \mathfrak{T}$.*

Proof. Let $U \in \mathfrak{U}_e(X, \mathfrak{T})$. Then there is $V \in \mathfrak{U}_e(X, \mathfrak{S})$ such that $(V^{-1}V) \cap G \subset U$. Because of $U \cap V \in \mathfrak{U}_e(X, \mathfrak{T})$, there exists $W \in \mathfrak{U}_e(X, \mathfrak{S})$, $W \subset V$, such that $W \subset (U \cap V)G$.

Let $w \in W$; then there are $x \in U \cap V$ and $y \in G$ satisfying $w = xy$ whence $y = x^{-1}w \in ((U \cap V)^{-1}W) \cap G \subset (V^{-1}V) \cap G \subset U$. Thus $w = xy \in (U \cap V)U \subset U^2$. This proves $W \subset U^2$, hence $U^2 \in \mathfrak{U}_e(X, \mathfrak{S})$.

Given a group X, let Aut X denote the group of all automorphisms $f: X \to X$.

Let G, H be groups and let $\sigma: H \to \mathrm{Aut}\, G$ be a homomorphism; by $G \times_\sigma H$ we denote the corresponding semi-direct product, i.e., the set $G \times H$ provided with the group structure $(x, y) \cdot (x', y') := (x \cdot \sigma(y)(x'), y \cdot y')$ $(x, x' \in G, y, y' \in H)$ (cf. [1; Ch. III, §2, Prop. 27]).

In this situation we will often identify G with the normal subgroup $G \times \{e\} \subset G \times_\sigma H$ as well as H with the subgroup $\{e\} \times H$.

Let $\mathfrak{S}$ and $\mathfrak{T}$ be group topologies on G and H, respectively; then the product topology $\mathfrak{S} \times \mathfrak{T}$ is a group topology on $G \times_\sigma H$ if and only if the map

$$(G, \mathfrak{S}) \times (H, \mathfrak{T}) \longrightarrow (G, \mathfrak{S}), \quad (x, y) \longmapsto \sigma(y)(x),$$

is continuous (cf. [1; Ch. III, §2, Prop. 28]). If $\mathfrak{S} \times \mathfrak{T}$ is a group topology on $G \times_\sigma H$, we will call the topological group $(G \times_\sigma H, \mathfrak{S} \times \mathfrak{T})$ the topological semi-direct product of the topological groups $(G, \mathfrak{S})$ and $(H, \mathfrak{T})$, and use the notation $(G, \mathfrak{S}) \times_\sigma^{\text{top}} (H, \mathfrak{T})$.

(*) Moreover, if $\mathfrak{Z}$ is any group topology on $G \times_\sigma H$, then the map $(G, \mathfrak{Z}|G) \times (H, \mathfrak{Z}|H) \to (G, \mathfrak{Z}|G)$, $(x, y) \mapsto \sigma(y)(x)$, is continuous (see the passage after Prop. 28 in [1; Ch. III, §2]).

In the following examples topological semi-direct products and (*) will be the main tools.

DEFINITION. A Hausdorff topological group $(X, \mathfrak{T})$ is called minimal, if there does not exist a Hausdorff group topology $\mathfrak{S}$ on X which is strictly coarser than $\mathfrak{T}$.

REMARK. Let $(X, \mathfrak{T})$ be a locally compact topological group, which admits a separating family $(p_i)_{i \in I}$ of continuous irreducible unitary finite-dimensional representations. If $(X, \mathfrak{T})$ is minimal, then $(X, \mathfrak{T})$ is compact. In fact, let $\mathfrak{S}$ denote the initial topology on X with respect to the representations p_i $(i \in I)$. Then $\mathfrak{S} \subset \mathfrak{T}$; moreover $(X, \mathfrak{S})$ is a Hausdorff precompact topological group. $(X, \mathfrak{T})$ being minimal, one obtains $\mathfrak{T} = \mathfrak{S}$, whence $(X, \mathfrak{T})$ is a precompact and complete topological group hence compact (cf. also [4]). It is clear that the above statement remains true, if we only assume $(X, \mathfrak{T})$ to be complete in its two-sided uniformity instead of being locally compact.

EXAMPLE 1. Let X be the group of all matrices $\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$ in the general linear group $\mathrm{GL}(2; \boldsymbol{R})$ such that $a \in \boldsymbol{R}_+ := \{c \in \boldsymbol{R}: c > 0\}$ and $b \in \boldsymbol{R}$. Then X provided with its usual locally compact, noncompact group topology $\mathfrak{T}$, induced by $\boldsymbol{R}^4$, is a minimal topological group.

Proof. It is well-known that $(X, \mathfrak{T})$ may be identified with the topological semi-direct product $\boldsymbol{R} \times_\sigma^{\text{top}} \boldsymbol{R}_+$ with respect to the homomorphism $\sigma: \boldsymbol{R}_+ \to \mathrm{Aut}\, \boldsymbol{R}$, $\sigma(y)(x) := xy$ $(y \in \boldsymbol{R}_+, x \in \boldsymbol{R})$, where the groups $\boldsymbol{R} = (\boldsymbol{R}, +)$ and $\boldsymbol{R}_+ = (\boldsymbol{R}_+, \cdot)$ are given their standard topologies. In fact, the map

$$(X, \mathfrak{T}) \longrightarrow \boldsymbol{R} \times_{\sigma}^{\mathrm{top}} \boldsymbol{R}_{+}\,, \quad \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \longmapsto (b, a)\,,$$

is a topological isomorphism. $G := \boldsymbol{R} \times \{1\}$ is a normal subgroup of X, which we will write additively, $H := \{0\} \times \boldsymbol{R}_{+}$ is a nonnormal subgroup of X, which we will write multiplicatively.

Let $\mathfrak{S}$ be a Hausdorff group topology on X such that $\mathfrak{S} \subset \mathfrak{T}$. We have to prove that $\mathfrak{S} = \mathfrak{T}$.

(a) We first show that $\mathfrak{S}|G = \mathfrak{T}|G$.

Because of (*) and because of $\mathfrak{T} \supset \mathfrak{S}$, we obtain that the map $w: (G, \mathfrak{S}|G) \times \boldsymbol{R}_{+} \to (G, \mathfrak{S}|G)$, $((x, 1), y) \mapsto (xy, 1)$, is continuous. $\mathfrak{S}$ being Hausdorff, there exists $U \in \mathfrak{U}_0(G, \mathfrak{S}|G)$ such that $U - U \neq G$. Choose $V \in \mathfrak{U}_0(G, \mathfrak{S}|G)$ and $\varepsilon > 0$ satisfying $w(V \times [1 - \varepsilon, 1 + \varepsilon]) \subset U$. Hence $\bigcup_{(x,1) \in V} [-\varepsilon x, \varepsilon x] \times \{1\} = \bigcup_{(x,1) \in V} (x \cdot [1 - \varepsilon, 1 + \varepsilon] - x) \times \{1\} \subset w(V \times [1 - \varepsilon, 1 + \varepsilon]) - w(V \times \{1\}) \subset U - U \neq G$; consequently there is $M > 0$ such that $V \subset [-M, M] \times \{1\}$. Thus V is contained in a compact subset of $(G, \mathfrak{T}|G)$, hence $\mathfrak{T}|V = \mathfrak{S}|V$, which implies $\mathfrak{T}|G = \mathfrak{S}|G$.

(b) Next we show that $\mathfrak{S}/G = \mathfrak{T}/G$.

Because of (a), $(G, \mathfrak{S}|G)$ is a complete subgroup of the Hausdorff topological group $(X, \mathfrak{S})$, whence G is closed in $(X, \mathfrak{S})$. Consequently $\mathfrak{S}/G$ is a Hausdorff group topology on the factor group X/G. Let $q: X \to X/G$ denote the quotient map.

Because of (a) there exists $U \in \mathfrak{U}_e(X, \mathfrak{S})$ such that $U \cap G = [-1, 1] \times \{1\}$; choose $V \in \mathfrak{U}_e(X, \mathfrak{S})$ such that $V = V^{-1}$ and $V^3 \subset U$. There exists $\varepsilon \in]0, 1[$ satisfying $[-\varepsilon, \varepsilon] \times \{1\} \subset V$. If $(x, y) \in V$, then $(\varepsilon y, 1) = (x, y) \cdot (\varepsilon, 1) \cdot (x, y)^{-1} \in V^3 \cap G \subset U \cap G$, whence $y \leqq 1/\varepsilon$. V being symmetric, we obtain that $q(V) \subset q(R \times [\varepsilon, 1/\varepsilon]) = q(\{0\} \times [\varepsilon, 1/\varepsilon])$. Thus $q(V)$ is contained in a compact subset of $(X/G, \mathfrak{T}/G)$, hence $(\mathfrak{S}/G)|q(V) = (\mathfrak{T}/G)|q(V)$, which implies $\mathfrak{S}/G = \mathfrak{T}/G$.

Now, from (a) and (b) we obtain $\mathfrak{S} = \mathfrak{T}$ by Lemma 1.

Example 1 shows that a minimal locally compact topological group may have nonminimal normal subgroups and nonminimal factor groups, since clearly $\boldsymbol{R}$ and $\boldsymbol{R}_{+}$ are nonminimal topological groups.

REMARK. We mention without proof that for all $n \in \boldsymbol{N}$, the groups $\mathrm{GL}(n; \boldsymbol{K})$ $(\boldsymbol{K} \in \{\boldsymbol{R}, \boldsymbol{C}\})$ are not minimal.

EXAMPLE 2. Let K be a compact topological group, and let H be a discrete topological group. Let $G := K^H$ be endowed with the product topology.

$$\sigma: H \longrightarrow \mathrm{Aut}\, G\,, \quad \sigma(k)((x_h)_{h \in H}) := (x_{hk})_{h \in H} \ (k \in H, (x_h)_{h \in H} \in G)$$

is a homomorphism. Moreover, the map

$$w\colon G \times H \longrightarrow G\,, \quad ((x_h)_{h\in H}, k) \longmapsto (x_{hk})_{h\in H}\,,$$

is continuous, as can easily be verified.

Thus the topological semi-direct product $(X, \mathfrak{T}) := G \times_\sigma^{\mathrm{top}} H$ is a well-defined locally compact topological group.

From now on we assume that $K \neq \{e\}$. We prove that $(X, \mathfrak{T})$ is a minimal topological group.

Let $\mathfrak{S}$ be a Hausdorff group topology on X such that $\mathfrak{S} \subset \mathfrak{T}$. We have to show that $\mathfrak{S} = \mathfrak{T}$.

(a) $(G, \mathfrak{T}|G)$ being compact, we have $\mathfrak{S}|G = \mathfrak{T}|G$.

(b) We show that $\mathfrak{S}|H = \mathfrak{T}|H$.

K being nontrivial and Hausdorff, there exists $U \in \mathfrak{U}_e(K)$ such that $U \neq K$. Because of $(*)$ and (a), the map

$$w\colon (G, \mathfrak{T}|G) \times (H, \mathfrak{S}|H) \longrightarrow (G, \mathfrak{T}|G), ((x_h)_{h\in H}, k) \longmapsto (x_{hk})_{h\in H}\,,$$

is continuous. Thus there exist $V \in \mathfrak{U}_e(H, \mathfrak{S}|H)$ and a finite subset $E \subset H$ such that the following implication holds:

$$\left.\begin{array}{c}(x_h)_{h\in H} \in G,\ x_h = e \text{ for all } h \in E \\ k \in V\end{array}\right\} \Longrightarrow x_k \in U\,.$$

Now we easily deduce that $k \in E$ for all $k \in V$. Thus V is finite, whence $-\mathfrak{S}|H$ being Hausdorff $-$ $\mathfrak{S}|H$ equals the discrete topology on H.

(c) Next we obtain that $\mathfrak{S}/H = \mathfrak{T}/H$.

In fact, $\mathfrak{S}|H$ being discrete, H is a closed subgroup of $(X, \mathfrak{S})$. Consequently, $\mathfrak{S}/H$ is Hausdorff and coarser than the compact topology $\mathfrak{T}/H$ (mind that $(G, \mathfrak{T}|G)$ and $(X/H, \mathfrak{T}/H)$ are homeomorphic).

Now, from (b) and (c) we obtain $\mathfrak{S} = \mathfrak{T}$ by Lemma 1.

Specializing for instance $K := \boldsymbol{Z}/2\boldsymbol{Z}$ in the above Example 2, we obtain:

PROPOSITION 1. *For every group H there exists a locally compact minimal topological group $(X, \mathfrak{T})$ containing H as a subgroup such that $\mathfrak{T}|H$ equals the discrete topology. Moreover, X can be chosen such that* $\operatorname{card} X \leqq 2^{\operatorname{card} H}$.

The following example shows that minimal topological groups can even contain discrete nonminimal normal subgroups.

EXAMPLE 3. Let $p \in \boldsymbol{Z}$ be a prime number, $K := \boldsymbol{Z}/p\boldsymbol{Z}$, and let X be an infinite-dimensional vector space over K. Let X be provided with its discrete topology. X being algebraically isomorphic to $K^{(I)} := \{(x_i)_{i\in I} \in K^I\colon \{i \in I\colon x_i \neq 0\}$ is finite$\}$ for some index set I, it is clear

that X is not a minimal topological group (consider the relative product topology on $K^{(I)}$). Moreover, every subgroup of $(X, +)$ is a linear subspace of the K-vector space X, and for every finite subset $E \subset X$ the subgroup $\langle E \rangle$ generated by E is finite.

The group $Z(X)$ of all bijections $f: X \to X$ provided with the topology of pointwise convergence, is a topological group which is complete in its two-sided uniformity according to [2; §3, Ex. 19]. Cf. also [3]. — Clearly, $\operatorname{Aut} X$ is a closed subgroup of $Z(X)$. Thus $\operatorname{Aut} X$ provided with the relative topology induced by $Z(X)$ is a Hausdorff topological group, which is complete in its two-sided uniformity. Moreover, $\operatorname{Aut} X$ is metrizable and separable if X is countable.

We mention that $\operatorname{Aut} X$ does not have a group completion. In fact, let $(e_i)_{i \in I}$ be a basis of the K-vector space X, and let $Z(I)$ denote the group of all bijections $\tau: I \to I$ provided with the topology of pointwise convergence. On account of [2; §3, Ex. 19], $Z(I)$ does not have a group completion. The map

$$j: Z(I) \longrightarrow \operatorname{Aut} X,\ j(\tau)\Big(\sum_{i \in I} \alpha_i e_i\Big) := \sum_{i \in I} \alpha_i e_{\tau(i)}\ (\tau \in Z(I),\ (\alpha_i)_{i \in I} \in K^{(I)}),$$

being a topological isomorphism of $Z(I)$ onto a subgroup of $\operatorname{Aut} X$, also $\operatorname{Aut} X$ does not have a group completion (hence $\operatorname{Aut} X$ is not locally compact).

Obviously, the map

$$w: X \times \operatorname{Aut} X \longrightarrow X,\ (x, f) \longmapsto f(x),$$

is continuous; thus the topological semi-direct product $(Y, \mathfrak{T}) := X \rtimes^{\mathrm{top}}_{\mathrm{id}} \operatorname{Aut} X$ is a well-defined Hausdorff topological group without a group completion, which is metrizable and separable if X is countable. We show that $(Y, \mathfrak{T})$ is a minimal topological group.

Let $\mathfrak{S} \subset \mathfrak{T}$ be a Hausdorff group topology on Y.

(a) We first show that $\mathfrak{S}|X = \mathfrak{T}|X$.

There exists $U \in \mathfrak{U}_0(X, \mathfrak{S}|X)$ such that $X \setminus U$ is infinite. Because of $(*)$ and because of $\mathfrak{T} \supset \mathfrak{S}$, the map

$$w: (X, \mathfrak{S}|X) \times (\operatorname{Aut} X, \mathfrak{T}|\operatorname{Aut} X) \longrightarrow (X, \mathfrak{S}|X),\ (x, f) \longmapsto f(x),$$

is continuous. Thus there exist $V \in \mathfrak{U}_0(X, \mathfrak{S}|X)$ and a finite subset $E \subset X$ such that the following implication holds:

$$\left.\begin{array}{c} x \in V \\ f \in \operatorname{Aut} X,\ f(y) = y \text{ for all } y \in E \end{array}\right\} \Longrightarrow f(x) \in U.$$

$\langle E \rangle$ being finite, there exists $y \in X \setminus (\langle E \rangle \cup U)$. — For every $z \in X \setminus \langle E \rangle$ we can construct $f \in \operatorname{Aut} X$ such that $f(z) = y$ and $f(x) = x$ for all

$x \in \langle E \rangle$; because of $f(z) = y \notin U$ we obtain that $z \notin V$. This implies $V \subset \langle E \rangle$, whence V is finite. Consequently, $\mathcal{S}|X$ equals the discrete topology.

(b) Next we show that $\mathcal{S}|\operatorname{Aut} X = \mathfrak{T}|\operatorname{Aut} X$.

$$w\colon (X, \mathcal{S}|X) \times (\operatorname{Aut} X, \mathcal{S}|\operatorname{Aut} X) \longrightarrow (X, \mathcal{S}|X)$$

being continuous, we obtain—using (a)—that for every $x \in X$ the set $\{f \in \operatorname{Aut} X\colon f(x) = x\}$ belongs to $\mathfrak{U}_{id}(\operatorname{Aut} X, \mathcal{S}|\operatorname{Aut} X)$, whence clearly $\mathfrak{U}_{id}(\operatorname{Aut} X, \mathfrak{T}|\operatorname{Aut} X) \subset \mathfrak{U}_{id}(\operatorname{Aut} X, \mathcal{S}|\operatorname{Aut} X)$. This proves $\mathfrak{T}|\operatorname{Aut} X \subset \mathcal{S}|\operatorname{Aut} X$.

(c) Finally we show that $\mathcal{S}/\operatorname{Aut} X = \mathfrak{T}/\operatorname{Aut} X$ (which together with (b) implies $\mathcal{S} = \mathfrak{T}$ by Lemma 1).

Because of (b) and the fact that $(\operatorname{Aut} X, \mathfrak{T}|\operatorname{Aut} X)$ is complete in its two-sided uniformity, $\operatorname{Aut} X$ is a closed subgroup of the Hausdorff topological group $(Y, \mathcal{S})$, as can easily be verified. Consequently, $(Y/\operatorname{Aut} X, \mathcal{S}/\operatorname{Aut} X)$ is Hausdorff.

Let $q\colon Y \to Y/\operatorname{Aut} X$ denote the quotient map. There exists $U \in \mathfrak{U}_e(Y, \mathcal{S})$ such that $(Y/\operatorname{Aut} X) \setminus q(U)$ is infinite. Let $V \in \mathfrak{U}_e(Y, \mathcal{S})$ such that $V^2 \subset U$. Then there is a finite subset $E \subset X$ such that $(0, f) \in V$ for all $f \in \operatorname{Aut} X$ satisfying $f(x) = x$ $(x \in E)$. Fix $y \in X \setminus \langle E \rangle$ such that $q(y, id) \notin q(U)$.

Let $z \in X \setminus \langle E \rangle$ and let $g \in \operatorname{Aut} X$. Then there exists $f \in \operatorname{Aut} X$ such that $f(z) = y$ and $f(x) = x$ for all $x \in \langle E \rangle$. Thus $(0, f) \in V$. Because of $q((0, f) \cdot (z, g)) = q(y, fg) = q(y, id) \notin q(U)$ we obtain $(z, g) \notin V$. — Consequently $V \subset \langle E \rangle \times \operatorname{Aut} X$, whence $q(V) \subset q(\langle E \rangle \times \{id\})$ is finite. $(Y/\operatorname{Aut} X, \mathcal{S}/\operatorname{Aut} X)$ being a Hausdorff topological homogeneous space, we obtain that $\mathcal{S}/\operatorname{Aut} X$ is discrete, whence $\mathcal{S}/\operatorname{Aut} X = \mathfrak{T}/\operatorname{Aut} X$.

Added in proof. From the construction of Example 2 it is clear that H is topologically isomorphic to the factor group X/G. Thus we obtain the following.

PROPOSITION 1′. *For every group H there exists a locally compact minimal topological group $(X, \mathfrak{T})$ containing a normal subgroup N such that $X/N = H$ and such that $\mathfrak{T}/N$ equals the discrete topology on H.*

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Received August 23, 1978.

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The *Pacific Journal of Mathematics* is issued monthly as of January 1966. Regular subscription rate: $84.00 a year (6 Vols., 12 issues). Special rate: $42.00 a year to individual members of supporting institutions.

Subscriptions, orders for numbers issued in the last three calendar years, and changes of address should be sent to Pacific Journal of Mathematics, P.O. Box 969, Carmel Valley, CA 93924, U.S.A. Older back numbers obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION
Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.).
8-8, 3-chome, Takadanobaba, Shinjuku-ku, Tokyo 160, Japan.

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Pacific Journal of Mathematics
Vol. 82, No. 2 February, 1979

Krishnaswami Alladi and Paul Erdős, *On the asymptotic behavior of large prime factors of integers* 295
Alfred David Andrew, *A remark on generalized Haar systems in L_p, $1 < p < \infty$* 317
John M. Baker, *A note on compact operators which attain their norm* 319
Jonathan Borwein, *Weak local supportability and applications to approximation* 323
Tae Ho Choe and Young Soo Park, *Wallman's type order compactification* 339
Susanne Dierolf and Ulrich Schwanengel, *Examples of locally compact noncompact minimal topological groups* 349
Michael Freedman, *A converse to (Milnor-Kervaire theorem) $\times R$ etc...* 357
George Golightly, *Graph-dense linear transformations* 371
H. Groemer, *Space coverings by translates of convex sets* 379
Rolf Wim Henrichs, *Weak Frobenius reciprocity and compactness conditions in topological groups* 387
Horst Herrlich and George Edison Strecker, *Semi-universal maps and universal initial completions* 407
Sigmund Nyrop Hudson, *On the topology and geometry of arcwise connected, finite-dimensional groups* 429
K. John and Václav E. Zizler, *On extension of rotund norms. II* 451
Russell Allan Johnson, *Existence of a strong lifting commuting with a compact group of transformations. II* 457
Bjarni Jónsson and Ivan Rival, *Lattice varieties covering the smallest nonmodular variety* 463
Grigori Abramovich Kolesnik, *On the order of Dirichlet L-functions* 479
Robert Allen Liebler and Jay Edward Yellen, *In search of nonsolvable groups of central type* 485
Wilfrido Martínez T. and Adalberto Garcia-Maynez Cervantes, *Unicoherent plane Peano sets are σ-unicoherent* 493
M. A. McKiernan, *General Pexider equations. I. Existence of injective solutions* 499
M. A. McKiernan, *General Pexider equations. II. An application of the theory of webs* 503
Jan K. Pachl, *Measures as functionals on uniformly continuous functions* 515
Lee Albert Rubel, *Convolution cut-down in some radical convolution algebras* 523
Peter John Slater and William Yslas Vélez, *Permutations of the positive integers with restrictions on the sequence of differences. II* 527
Raymond Earl Smithson, *A common fixed point theorem for nested spaces* 533
Indulata Sukla, *Generalization of a theorem of McFadden* 539
Jun-ichi Tanaka, *A certain class of total variation measures of analytic measures* 547
Kalathoor Varadarajan, *Modules with supplements* 559
Robert Francis Wheeler, *Topological measure theory for completely regular spaces and their projective covers* 565