

Pacific Journal of Mathematics

**MEASURES AS FUNCTIONALS ON UNIFORMLY
CONTINUOUS FUNCTIONS**

JAN K. PACHL

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The space \mathfrak{M}_t of bounded Radon measures on a complete metric space is studied in duality with the space \mathcal{U}_b of bounded uniformly continuous functions. The weak topology has reasonable properties: the space \mathfrak{M}_t is \mathcal{U}_b -weakly sequentially complete, and every \mathcal{U}_b -weakly compact subset of \mathfrak{M}_t is pointwise equicontinuous on the set of 1-Lipschitz functions.

1. **Introduction.** Let (X, d) be a complete metric space and $\mathfrak{M}_t(X)$ the space of (bounded) Radon (=tight) measures on X . This space is usually studied in duality with the space $\mathcal{C}_b(X)$ of bounded continuous functions on X . It is known that the weak topology $w(\mathfrak{M}_t(X), \mathcal{C}_b(X))$ is sequentially complete, and there is a useful criterion (Prohorov's condition) for $w(\mathfrak{M}_t, \mathcal{C}_b)$ -compactness [11].

In this paper we turn to the space $\mathcal{U}_b(X)$ of bounded uniformly continuous functions on X and to the weak topology $w(\mathfrak{M}_t(X), \mathcal{U}_b(X))$. The topologies $w(\mathfrak{M}_t, \mathcal{C}_b)$ and $w(\mathfrak{M}_t, \mathcal{U}_b)$ coincide on the positive cone \mathfrak{M}_t^+ ; thus our results say nothing new about positive measures. Obviously, the two topologies differ (on \mathfrak{M}_t) whenever $\mathcal{U}_b \neq \mathcal{C}_b$.

The main results are: (A) the topology $w(\mathfrak{M}_t, \mathcal{U}_b)$ is sequentially complete, and (B) a norm-bounded subset of \mathfrak{M}_t is relatively $w(\mathfrak{M}_t, \mathcal{U}_b)$ -compact if and only if its restriction to the set

$$\text{Lip}(1) = \{f: X \rightarrow R \mid \|f\| \leq 1 \text{ and } |f(x) - f(y)| \leq d(x, y) \text{ for } x, y \in X\}$$

is equicontinuous in the compact-open topology.

The topology of uniform convergence on $\text{Lip}(1)$ was discussed by Dudley [3]. Here we improve some of Dudley's results. For example, Theorem 6 in [3] says, in the present setup, that $\mu_n \rightarrow \mu$ uniformly on $\text{Lip}(1)$ whenever $\mu \in \mathfrak{M}_t$, $\mu_n \in \mathfrak{M}_t$ for $n = 1, 2, \dots$, and $\mu_n(f) \rightarrow \mu(f)$ for each $f \in \mathcal{C}_b(X)$. Here we obtain the same conclusion, assuming only that $\mu_n(f) \rightarrow \mu(f)$ for each $f \in \mathcal{U}_b(X)$.

A reasonable generalization is to allow X to be an arbitrary uniform space and replace \mathfrak{M}_t by the space $\mathfrak{M}_u(X)$ of uniform measures on X (see [4] and the references therein). The results extend to the space $\mathfrak{M}_u(X)$, as well as to the space $\mathfrak{M}_F(X)$ of free uniform measures. Several previously studied spaces of measures can be described as \mathfrak{M}_u or \mathfrak{M}_F —see [5], [8]. To cover both \mathfrak{M}_u and \mathfrak{M}_F , in § 2 we employ sets of Lipschitz functions more general than $\text{Lip}(1)$.

As in similar situations studied before (e.g., [1], [10]), the goal

of the construction is to pass from $\mathfrak{M}_i(X)$ to the space $l^1 = \mathfrak{M}_i(N)$. It should be noted, however, that the approach through partitions of unity ([10], [12]) seems to be barred, in view of the theorem by Zahradník [13] which says that there are metric spaces without a sufficient supply of l^1 -continuous partitions of unity.

An earlier version of this paper was announced in [9].

2. Construction. The property of Radon measures we are chiefly interested in is their continuity on $\text{Lip}(1)$ (or on more general sets of Lipschitz functions). In $\text{Lip}(1)$, the compact-open topology agrees with the topology of pointwise convergence, and the latter will be easier to deal with.

Throughout this section, (X, d) will be metric space and h a Lipschitz function on X ; that is, h maps X into the field R of real numbers and

$$|h(x) - h(y)| \leq d(x, y)$$

for $x, y \in X$. Put

$$\text{Lip}(h) = \{f: X \rightarrow R \mid |f| \leq h \text{ and } |f(x) - f(y)| \leq d(x, y) \text{ for } x, y \in X\},$$

and denote by U the linear space spanned by $\text{Lip}(h)$. Endow U with the topology of pointwise convergence (i.e., U is a topological subspace of R^X) and denote by \mathfrak{M} the space of the linear forms on U whose restrictions to $\text{Lip}(h) \subset U$ are continuous. Endow \mathfrak{M} with the norm

$$\|\mu\|_{d,h} = \sup \{|\mu(f)| \mid f \in \text{Lip}(h)\}.$$

Needless to say, both U and \mathfrak{M} depend on h .

As $\text{Lip}(h)$ is compact, the Ascoli theorem ([6], Ch. 7, Th. 17) gives the following precompactness criterion.

LEMMA 2.1. *A subset of \mathfrak{M} is $\|\cdot\|_{d,h}$ -precompact if and only if it is equicontinuous on $\text{Lip}(h)$.*

The main idea in the proof of the following lemma is to choose as small functions in $\text{Lip}(h)$ as possible and then use the fact that they cannot be made smaller. This is why it will be convenient to work with (nonnegative) functions in $\text{Lip}(h)$ which are "small far from a finite set": say that $f \in \text{Sm}(h)$ if and only if there is a non-empty finite set $F(f) \subset X$ such that

$$f = \inf \{g \in \text{Lip}(h) \mid g \geq 0 \text{ and } g(y) \geq f(y) \text{ for every } y \in F(f)\}.$$

Obviously $\text{Sm}(h) \subset \text{Lip}(h)$. The set $F(f)$ is not unique (in fact, the

equality remains true when $F(f)$ is replaced by any larger set); we fix arbitrarily, for each $f \in \text{Sm}(h)$, a nonempty finite set $F(f)$ satisfying the above equality.

Notice that each $f \in \text{Sm}(h)$ can be described explicitly in terms of d and $F(f)$:

$$f(x) = \max \{ (f(y) - d(y, x))^+ \mid y \in F(f) \} .$$

Note also that $\text{Sm}(h)$ is pointwise dense in $\text{Lip}^+(h) = \{f \in \text{Lip}(h) \mid f \geq 0\}$; indeed, every nonnegative function in $\text{Lip}(h)$ is the supremum of a subset of $\text{Sm}(h)$.

The system of finite subsets of X is denoted by $\text{Fin}(X)$.

When $Y \subset X$ and f is a function on X , write

$$\|f\|_Y = \sup \{ |f(y)| \mid y \in Y \}$$

and $\|f\| = \|f\|_X$.

LEMMA 2.2. *Let $M \subset \mathfrak{M}$ and suppose that there is a $t > 0$ such that $|\mu(f)| \leq t \|f\|$ for any $\mu \in M$ and any bounded $f \in U$. If M is not $\|\cdot\|_{d,h}$ -precompact then there are: an $\varepsilon > 0$, $g_k \in \text{Sm}(h)$ and $\mu_k \in M$, $k = 1, 2, \dots$, such that for each k we have*

- 1^o. $|\mu_k(g_k)| > 2\varepsilon$,
- 2^o. $|\mu_j(g_k)| \leq \varepsilon$ for $j < k$, and
- 3^o. $g_j \wedge g_k = 0$ for $j < k$.

Proof. By 2.1, M is not equicontinuous on $\text{Lip}(h)$ at 0. Every $f \in \text{Lip}(h)$ may be written as $f = f^+ - f^-$ with $f^+, f^- \in \text{Lip}^+(h)$, and $\text{Sm}(h)$ is dense in $\text{Lip}^+(h)$. Hence M is not equicontinuous on $\text{Sm}(h)$ at 0: there is a $\gamma > 0$ such that

$$\forall \delta > 0 \forall F \in \text{Fin}(X) \exists f \in \text{Sm}(h) \exists \mu \in M: \|f\|_F < \delta \quad \text{and} \quad |\mu(f)| > 3\gamma .$$

Take such a $\gamma > 0$ and keep it fixed through the whole proof. To reduce the number of quantifiers, we drop δ : Put $\delta = \gamma/t$ and $g = (f - \delta)^+$ to get

$$(1) \quad \forall F \in \text{Fin}(X) \exists g \in \text{Sm}(h) \exists \mu \in M: \|g\|_F = 0 \quad \text{and} \quad |\mu(g)| > 2\gamma .$$

Now we distinguish two cases. Case II can arise only when h is unbounded.

Case I. Assume that there is a $r \geq 0$ such that for all $\mu \in M$ and $f \in \text{Sm}(h)$ we have $|\mu(f - f \wedge r)| \leq \gamma$. (This is automatically satisfied when h is bounded.) Substituting this to (1) we get

$$(2) \quad \forall F \in \text{Fin}(X) \exists g \in \text{Sm}(h) \exists \mu \in M: \|g\| \leq r, \|g\|_F = 0 \quad \text{and} \quad |\mu(g)| > \gamma .$$

For $n = 1, 2, \dots$ consider the statement

$$(\mathcal{S}_n) \quad \forall F \in \text{Fin}(X) \exists g \in \text{Sm}(h) \exists \mu \in M: \|g\| \leq r/2^{n-1}, \quad \|g\|_F = 0 \quad \text{and} \\ |\mu(g)| > \left(\frac{1}{2} + \frac{1}{2n}\right)\gamma.$$

Plainly (\mathcal{S}_n) does not hold for $2^n \geq 4rt/\gamma$; on the other hand, (\mathcal{S}_1) does hold by (2). Choose n such that (\mathcal{S}_n) is true and (\mathcal{S}_{n+1}) is not. With $\eta = r/2^n$, $\gamma^* = (1/2 + 1/2n)\gamma$ and $\varepsilon = \gamma/4n(n+1)$ we have

$$(3) \quad \forall F \in \text{Fin}(X) \exists g \in \text{Sm}(h) \exists \mu \in M: \|g\| \leq 2\eta, \quad \|g\|_F = 0 \quad \text{and} \\ |\mu(g)| > \gamma^*,$$

$$(4) \quad \exists F_0 \in \text{Fin}(X) \forall g \in \text{Lip}(h) \forall \mu \in M: [0 \leq g \leq \eta, \|g\|_{F_0} = 0] \\ \Rightarrow |\mu(g)| \leq \gamma^* - 2\varepsilon.$$

(The negation of (\mathcal{S}_{n+1}) gives only $\exists F_0 \forall g \in \text{Sm}(h) \dots$; however, $\{g \in \text{Sm}(h) \mid g \leq \eta\}$ is dense in $\{g \in \text{Lip}(h) \mid 0 \leq g \leq \eta\}$. Hence (4) follows.)

We are going to construct $g_k^* \in \text{Sm}(h)$ and $\mu_k \in M$ for $k = 1, 2, \dots$ such that

- 1⁰⁰. $\|g_k^*\| \leq 2\eta$ and $|\mu_k(g_k^*)| > \gamma^*$,
- 2⁰⁰. $|\mu_j(g_k^* - g_k^* \wedge \eta)| \leq \varepsilon$ for $j < k$, and
- 3⁰⁰. $g_j^* \wedge g_k^* \leq \eta$ for $j < k$.

First use (3) to find $g_1^* \in \text{Sm}(h)$ and $\mu_1 \in M$ such that $\|g_1^*\| \leq 2\eta$ and $|\mu_1(g_1^*)| > \gamma^*$ (conditions 2⁰⁰ and 3⁰⁰ are empty for $k = 1$). For $k \geq 2$, when μ_j and g_j^* have been constructed for $j < k$, take a finite set $F \subset X$ such that $F \supset F_0$, $F \supset F(g_j^*)$ for $j < k$, and $|\mu_j(f)| \leq \varepsilon$ whenever $f \in \text{Lip}(h)$, $\|f\|_F = 0$ and $j < k$. Use (3) to get a $g_k^* \in \text{Sm}(h)$ and a $\mu_k \in M$ such that $\|g_k^*\| \leq 2\eta$, $\|g_k^*\|_F = 0$ and $|\mu_k(g_k^*)| > \gamma^*$. Conditions 1⁰⁰ and 2⁰⁰ are obviously satisfied. As for 3⁰⁰, put $f^* = (2\eta - g_k^*)^+ \wedge h$; then $f^* \in \text{Lip}^+(h)$ and for $y \in F$, $j < k$ we have $f^*(y) = 2\eta \wedge h \geq g_j^*(y)$. This together with $F \supset F(g_j^*)$ gives $f^* \geq g_j^*$. Now, if $g_k^*(x) > \eta$ for some $x \in X$ then $\eta > f^*(x) \geq g_j^*(x)$; hence $g_j^* \wedge g_k^* \leq \eta$.

Finally, put $g_k = g_k^* - g_k^* \wedge \eta$. Conditions 2⁰, 3⁰ follow from 2⁰⁰, 3⁰⁰. As for 1⁰, we have

$$|\mu_k(g_k)| \geq |\mu_k(g_k^*)| - |\mu_k(g_k^* \wedge \eta)| > \gamma^* - (\gamma^* - 2\varepsilon) = 2\varepsilon,$$

by (4).

This concludes the proof when h is bounded. In the general case we have to consider one more possibility:

Case II. Assume that the assumption made in Case I does not hold. Thus for every $r \geq 0$ there are a $\mu \in M$ and an $f \in \text{Sm}(h)$ such that $|\mu(f - f \wedge r)| > \gamma$. Put $\varepsilon = \gamma/2$.

Choose $\mu_1 \in M$ and $g_1 \in \text{Sm}(h)$ such that $|\mu_1(g_1)| > 2\varepsilon$. For $k \geq 2$, when μ_j and g_j have been constructed for $j < k$, take a finite set $F \subset X$ such that $F \supset F(g_j)$ for $j < k$ and $|\mu_j(f)| \leq \varepsilon$ whenever $j < k$, $f \in \text{Lip}(h)$ and $\|f\|_F = 0$. Put $r_k = 2 \max \{h(y) \mid y \in F\}$ and use the assumption to produce a $\mu_k \in M$ and an $f_k \in \text{Sm}(h)$ with $|\mu_k(f_k - f_k \wedge r_k)| > 2\varepsilon$. Put $g_k = f_k - f_k \wedge r_k$; condition 1^o is satisfied. We have $f_k(y) \leq h(y) \leq r_k$ for each $y \in F$, hence $g_k(y) = 0$. Thus $\|g_k\|_F = 0$ and 2^o follows.

Finally, put $f^* = (r_k - f_k)^+ \wedge h$. Then $f^* \in \text{Lip}^+(h)$, and for $y \in F$, $j < k$, we have

$$f^*(y) \geq (r_k - f_k(y)) \wedge h(y) \geq (r_k - h(y)) \wedge h(y) \geq h(y) \geq g_j(y).$$

This along with $F \supset F(g_j)$ implies $f^* \geq g_j$. If $x \in X$ and $g_k(x) > 0$ then $f_k(x) > r_k$, hence $f^*(x) = 0$; this proves 3^o, for $g_k \wedge g_j \leq g_k \wedge f^* = 0$.

COROLLARY 2.3 *Let $M \subset \mathfrak{M}$ and suppose that there is a $t > 0$ such that $|\mu(f)| \leq t\|f\|$ for any $\mu \in M$ and any bounded $f \in U$. If M is not $\|\cdot\|_{d,h}$ -precompact then there is a continuous linear map $p: \mathfrak{M} \rightarrow l^1$ such that $p(M) \subset l^1$ is not norm-precompact.*

Proof. Produce μ_k and g_k as in 2.2, satisfying 1^o, 2^o and 3^o. Define a linear map $q: l^\infty \rightarrow U$ by

$$q(\{z_k\}_{k=1}^\infty) = \sum_{k=1}^\infty z_k g_k$$

for every bounded real sequence $\{z_k\}_{k=1}^\infty$. Since the functions g_k are pairwise disjoint, the sum is well defined and, moreover, $q(z) \in 2 \text{Lip}(h)$ whenever z is in the unit ball of l^∞ . It follows that the transposed map $p = {}^t q$ maps \mathfrak{M} into l^1 and is continuous, with $\|p\| \leq 2$. In order to show that $p(M)$ is not precompact in l^1 , we prove that the infinite set $\{p(\mu_k) \mid k = 1, 2, \dots\}$ is norm-discrete:

$$\begin{aligned} \|p(\mu_j) - p(\mu_k)\| &= \sup \{ |\langle p(\mu_j) - p(\mu_k), z \rangle| \mid z \in l^\infty, \|z\| \leq 1 \} \\ &= \sup \{ |\langle \mu_j - \mu_k, q(z) \rangle| \mid z \in l^\infty, \|z\| \leq 1 \} \\ &\geq |\mu_j(g_k) - \mu_k(g_k)| > \varepsilon \end{aligned}$$

for $j < k$.

3. Results. Corollary 2.3 allows us to deduce the properties of $\mathfrak{M}_i(X)$ from those of l^1 . Let us recall the relevant facts about l^1 :

- THEOREM 3.1.** (a) *The space l^1 is weakly sequentially complete.*
 (b) *Every weakly convergent sequence in l^1 is norm convergent.*

Hence every weakly countably compact set in l^1 is norm-compact.

Proof is in ([2], II-§ 2). The second assertion in (b) uses the theorem of Eberlein ([2], III-§ 2).

Let X be a complete metric space and h a Lipschitz function on X . The compact-open topology and the topology of pointwise convergence agree on $\text{Lip}(h)$; this is the only topology on $\text{Lip}(h)$ we consider. It is well known (see e.g., [4], [7]) that a bounded Radon measure on X can be characterized as a linear form on $\mathcal{U}_b(X)$ which is $\|\cdot\|$ -continuous and whose restriction to $\text{Lip}(1)$ is continuous.

Define again the norm $\|\cdot\|_a = \|\cdot\|_{a,1}$ on $\mathfrak{M}_t(X)$ by

$$\|\mu\|_a = \sup \{ |\mu(f)| \mid f \in \text{Lip}(1) \}.$$

THEOREM 3.2. *Let X be a complete metric space. (a) The space $\mathfrak{M}_t(X)$ is $w(\mathfrak{M}_t, \mathcal{U}_b)$ sequentially complete.*

(b) *Let a set $M \subset \mathfrak{M}_t(X)$ be bounded on the unit $\|\cdot\|$ -ball in $\mathcal{U}_b(X)$. The following conditions are equivalent:*

- (i) *M is relatively $\|\cdot\|_a$ -compact;*
- (ii) *M is relatively $w(\mathfrak{M}_t, \mathcal{U}_b)$ countably compact;*
- (iii) *The restriction of M to $\text{Lip}(1)$ is equicontinuous.*

Proof. (a) Suppose that $\{\mu_n\}_{n=1}^\infty$ is a $w(\mathfrak{M}_t, \mathcal{U}_b)$ Cauchy sequence and $\{\mu_n \mid n = 1, 2, \dots\}$ is not $\|\cdot\|_a$ -precompact. The sequence is bounded on the unit $\|\cdot\|$ -ball in $\mathcal{U}_b(X)$ by the Banach-Steinhaus theorem, and 2.3 produces a $p: \mathfrak{M}_t \rightarrow l^1$ such that $\{p(\mu_n) \mid n = 1, 2, \dots\} \subset l^1$ is not precompact. As the sequence $\{p(\mu_n)\}_{n=1}^\infty$ is $w(l^1, l^\infty)$ Cauchy, this contradicts 3.1. Hence $\{\mu_n \mid n = 1, 2, \dots\}$ is $\|\cdot\|_a$ -precompact. It follows that the $w(\mathcal{U}_b^*, \mathcal{U}_b)$ limit of the sequence (in the algebraic dual \mathcal{U}_b^* of \mathcal{U}_b) is both $\|\cdot\|_X$ -continuous on \mathcal{U}_b and continuous on $\text{Lip}(1)$, i.e., belongs to \mathfrak{M}_t .

(b) Obviously (i) \Leftrightarrow (iii) and (i) \Rightarrow (ii). If M is relatively $w(\mathfrak{M}_t, \mathcal{U}_b)$ countably compact but not $\|\cdot\|_a$ -precompact, then there is, again by 2.3, a $p: \mathfrak{M}_t \rightarrow l^1$ such that $p(M)$ is relatively $w(l^1, l^\infty)$ countably compact but not norm-precompact. This contradiction proves the implication (ii) \Rightarrow (i).

Now let X be a uniform space. The uniform structure of X is projectively generated by uniformly continuous maps into complete metric spaces; the *UEB*-topology in the space $\mathfrak{M}_u(X)$ is generated by the corresponding maps into the spaces of Radon measures ([4], [5]).

COROLLARY 3.3. *Let X be a uniform space. (a) The space $\mathfrak{M}_u(X)$ is $w(\mathfrak{M}_u, \mathcal{U}_b)$ sequentially complete.*

(b) *The following properties of a set $M \subset \mathfrak{M}_u(X)$ are equivalent:*

- (i) M is relatively UEB -compact;
- (ii) M is relatively $w(\mathfrak{M}_u, \mathcal{U}_b)$ countably compact;
- (iii) The restriction of M to any UEB set is equicontinuous.

Proof. (a) follows immediately from 3.2(a). In order to deduce (b) from 3.2(b), it is enough to realize that every $w(\mathfrak{M}_u, \mathcal{U}_b)$ bounded set is UEB -bounded and also bounded on the unit $\|\cdot\|$ -ball in $\mathcal{U}_b(X)$.

Thus the UEB -topology agrees with $w(\mathfrak{M}_u, \mathcal{U}_b)$ on every relatively $w(\mathfrak{M}_u, \mathcal{U}_b)$ countably compact subset of $\mathfrak{M}_u(X)$. LeCam [7] proved that the two topologies agree on the positive cone $\mathfrak{M}_u^+(X)$.

In the same way as the sets $\text{Lip}(1)$ generate the UEB -topology in $\mathfrak{M}_u(X)$, the general sets $\text{Lip}(h)$ generate the UE -topology in the space $\mathfrak{M}_F(X)$ of free uniform measures [8]. Thus 2.3 yields the following analogue to 3.3.

PROPOSITION 3.4. *Let X be a uniform space. (a) The space $\mathfrak{M}_F(X)$ is $w(\mathfrak{M}_F, \mathcal{U})$ sequentially complete.*

- (b) *The following properties of a set $M \subset \mathfrak{M}_F(X)$ are equivalent:*
 - (i) M is relatively UE -compact;
 - (ii) M is relatively $w(\mathfrak{M}_F, \mathcal{U})$ countably compact;
 - (iii) The restriction of M to any UE set is equicontinuous.

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Received April 12, 1978. Research supported in part by National Research Council of Canada.

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PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.).

8-8, 3-chome, Takadanobaba, Shinjuku-ku, Tokyo 160, Japan.

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