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## **ISOMORPHISMS OF THE FOURIER ALGEBRAS IN CROSSED PRODUCTS**

YOSHIKAZU KATAYAMA

# ISOMORPHISMS OF THE FOURIER ALGEBRAS IN CROSSED PRODUCTS

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Let  $(M, G, \alpha)$ ,  $(N, H, \beta)$  be  $W^*$ -systems,  $F_\alpha(G; M_*)$  and  $F_\beta(H; N_*)$ , their Fourier algebras. The main result is that  $F_\alpha(G; M_*)$  and  $F_\beta(H; N_*)$  are isometrically isomorphic as Banach algebras if and only if  $M$  (resp.  $G$ ) is isomorphic to  $N$  (resp.  $H$ ) by  $\theta$  (resp.  $I$ ) such that  $\beta_{I(g)} \circ \theta = \theta \circ \alpha_g$  for all  $g \in G$ , or  $M$  (resp.  $G$ ) is anti-isomorphic to  $N$  (resp.  $H$ ) such that  $\beta_{I(g^{-1})} \circ \theta = \theta \circ \alpha_g$  for all  $g \in G$ .

1. Introduction. For locally compact abelian groups  $G$  and  $H$ , Pontryagin's duality theorem mentions that  $L^1(G)$  is isomorphic to  $L^1(H)$  if and only if  $G$  is isomorphic to  $H$ . Y. Kawada [4] and J. G. Wendel [11] proved the same statement for arbitrary locally compact groups.

When  $G$  is a locally compact abelian group,  $L^1(G)$  is isometrically isomorphic to the Fourier algebra  $A(G)$  in [7]. Therefore  $A(G)$  is isomorphic to  $A(H)$  as Banach algebras if and only if  $G$  is isomorphic to  $H$ .

P. Eymard [1], on the other hand, defined the Fourier algebra  $A(G)$  of a locally compact group  $G$  and showed that it is isomorphic to the predual  $m(G)_*$  of the von Neumann algebra  $m(G)$  generated by the left regular representation of  $G$ .

M. E. Walter [10] showed that  $A(G)$  and  $A(H)$  are isometrically isomorphic as Banach algebras if and only if  $G$  and  $H$  are isomorphic.

Recently for  $W^*$ -system  $(M, G, \alpha)$ , the Fourier space  $F_\alpha(G; M_*)$  was defined in [8] such that  $F_\alpha(G; M_*)$  is isometrically isomorphic to the predual of the crossed product  $G \otimes_\alpha M$  as Banach spaces.

M. Fugita [2] quite recently defined the Banach algebra structure in the Fourier space  $F_\alpha(G; M_*)$ . Then he showed that the group of all characters  $\widehat{F_\alpha(G; M_*)}$  of  $F_\alpha(G; M_*)$  is isomorphic to  $G$  and studied the support of the operators in  $G \otimes_\alpha M$ .

In this paper we generalize the Walter's result for  $W^*$ -system  $(M, G, \alpha)$ .

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2. Notations and preliminaries. Let  $M$  be a von Neumann

algebra on a Hilbert space  $\mathfrak{H}$  and  $G$  be a locally compact group. The triple  $(M, G, \alpha)$  is said to be a  $W^*$ -system if the mapping  $\alpha$  of  $G$  into the group  $\text{Aut}(M)$  of all automorphisms of  $M$  is a homomorphism and the function  $g \mapsto \omega \circ \alpha_g(x)$  is continuous on  $G$  for all  $x \in M$  and  $\omega \in M_*$  where  $M_*$  is the predual of  $M$ .

The crossed product  $G \otimes_{\alpha} M$  of  $M$  by  $\alpha$  is the von Neumann algebra generated by the family of operators  $\{\pi_{\alpha}(x), \lambda_G(g); x \in M, g \in G\}$ ;

$$(2.1) \quad \begin{aligned} (\pi_{\alpha}(x)\xi)(h) &= \alpha_{h^{-1}}(x)\xi(h) \\ (\lambda_G(g)\xi)(h) &= \xi(g^{-1}h) \end{aligned}$$

for  $\xi \in L^2(G; \mathfrak{H})$ .

Each element  $\omega$  in the predual  $(G \otimes_{\alpha} M)_*$  of  $G \otimes_{\alpha} M$  may be regarded as an element  $u_{\omega}$  of  $C^b(G; M_*)$ ;

$$(2.2) \quad u_{\omega}[g](x) = \langle \pi_{\alpha}(x)\lambda_G(g), \omega \rangle$$

for all  $x \in M, g \in G$  where  $C^b(G; M_*)$  is the space of all bounded continuous  $M_*$ -valued functions on  $G$ . We denote  $F_{\alpha}(G; M_*) = \{u_{\omega}; \omega \in (G \otimes_{\alpha} M)_*\} \subset C^b(G; M_*)$ . A norm  $\| \cdot \|$  is defined on  $F_{\alpha}(G; M_*)$  by

$$\|u_{\omega}\| = \|\omega\|.$$

Then  $\|u\|_{\infty} \leq \|u\|$  for all  $u \in F_{\alpha}(G; M_*)$  where  $\| \cdot \|_{\infty}$  is the sup-norm on  $C^b(G; M_*)$ . We define a product on  $F_{\alpha}(G; M_*)$  by

$$(2.3) \quad (u*v)[g](x) = u[g](x)v[g](1)$$

for all  $u, v \in F_{\alpha}(G; M_*)$ ,  $x \in M$  and  $g \in G$ . Then  $F_{\alpha}(G; M_*)$  is a Banach algebra ([2] Theorem 3.5). So that we can define products between  $G \otimes_{\alpha} M$  and  $F_{\alpha}(G; M_*)$ ;

$$\begin{aligned} \langle uT, v \rangle &= \langle T, v*u \rangle \\ \langle Tu, v \rangle &= \langle T, u*v \rangle \end{aligned}$$

for  $T \in G \otimes_{\alpha} M, u, v \in F_{\alpha}(G; M_*)$  ((3.7), (3.9) in [2]).

Let  $T$  be an operator in  $G \otimes_{\alpha} M$ . Then the  $\text{supp}(T)$  of  $T$  is the set of all  $g \in G$  satisfying the condition that  $\lambda_G(g)$  belongs to the  $\sigma$ -weak closure of  $TF_{\alpha}(G; M_*)$  [See [2] Proposition 4.1].

**THEOREM 1.** *Let  $(M, G, \alpha), (N, H, \beta)$  be  $W^*$ -systems and  $F_{\alpha}(G; M_*), F_{\beta}(H; N_*)$  their associated Fourier algebras. Let  $\phi$  be an isometric isomorphism of  $F_{\alpha}(G; M_*)$  onto  $F_{\beta}(H; N_*)$  as Banach algebras.*

Then we have five elements  $(k, p, q, I, \theta)$  with the following properties:

(1)  $k \in G$  such that  $\lambda_G(k) = {}^t\phi(\lambda_H(e))$  where  ${}^t\phi$  is the transposed map of  $\phi$  and  $e$  is the identity of  $H$ .

(2)  $I$  is an isomorphism or anti-isomorphism of  $H$  onto  $G$ .

(3)  $p$  (resp.  $q$ ) is a projection of  $Z_M \cap M^G$  (resp.  $Z_N \cap N^H$ ) where  $Z_M$  (resp.  $Z_N$ ) is the center of  $M$  (resp.  $N$ ) and  $M^G = \{x \in M: \alpha_g(x) = x \text{ for all } g \in G\}$ ,  $N^H = \{x \in N: \beta_h(x) = x \text{ for all } h \in H\}$ .

(4)  $\theta$  is an isometric linear map of  $N$  onto  $M$  such that

$\theta$  is an isomorphism of  $N_q$  onto  $M_p$ ,

$\theta$  is an anti-isomorphism of  $N_{l-q}$  onto  $M_{l-p}$ .

(5)  $\phi(u)[h](y) = ({}_k u)[I(h)](\theta(y)p) + ({}_k u)[I(h)](\alpha_{I(h)}(\theta(y)(l-p)))$  for all  $y \in N, h \in H$  and  $u \in F_\alpha(G; M_*)$ , where  $({}_k u)[g](y) = u[kg](\alpha_k(y))$ .

(6)  $\theta[\beta_k(y)] = [\alpha_{I(h)}\theta(y)]p + [\alpha_{I(h)}^{-1}\theta(y)](l-p)$  for all  $y \in N, h \in H$ .

*Proof.* The transposed map  ${}^t\phi$  of  $\phi$  is an isometric linear map of  $H \otimes_\beta N$  onto  $G \otimes_\alpha M$ . Using [3] Theorem 7, 10, we get;

$${}^t\phi = {}^t\phi(\lambda_H(e))(\gamma_I + \gamma_A)$$

where  $\gamma_I$  is an isomorphism of  $(H \otimes_\beta N)_{z'}$  onto  $(G \otimes_\alpha M)_z$ ,  $\gamma_A$  is an anti-isomorphism of  $(H \otimes_\beta N)_{(l-z')}$  onto  $(G \otimes_\alpha M)_{(l-z)}$ ,  $z$  (resp.  $z'$ ) being a central projection of  $G \otimes_\alpha M$  (resp.  $H \otimes_\beta N$ ). (2.4)

It follows from (2.3) that for all  $u, v \in F_\alpha(G; M_*)$ ,

$$\begin{aligned} \langle {}^t\phi(\lambda_H(h)), u*v \rangle &= \langle \lambda_H(h), \phi(u*v) \rangle \\ &= \langle \lambda_H(h), \phi(u)*\phi(v) \rangle \\ &= \langle \lambda_H(h) \otimes \lambda_H(h), \phi(u) \otimes \phi(v) \rangle \\ &= \langle {}^t\phi(\lambda_H(h)), u \rangle \langle {}^t\phi(\lambda_H(h)), v \rangle . \end{aligned}$$

Therefore  ${}^t\phi(\lambda_H(h))$  is a character of  $F_\alpha(G; M_*)$  for all  $h \in H$ , which implies that  ${}^t\phi(\lambda_H(H)) \subseteq \lambda_G(G)$  because the group of all characters  $\widehat{F_\alpha(G; M_*)}$  is isomorphic to  $G$  ([2] Theorem 3.14), moreover since  $\phi$  is an isomorphism,

$${}^t\phi(\lambda_H(H)) = \lambda_G(G) .$$

We denote  $\lambda_G(k) = {}^t\phi(\lambda_H(e))$ .

By the same argument in [10] Theorem 2, we get that

$$(2.5) \quad \gamma \equiv ({}^t\phi(\lambda_H(e)))^{-1}{}^t\phi = \gamma_I + \gamma_A$$

is a  $C^*$ -isomorphism in Kadison's sense [3] and  $\gamma(\lambda_H(h_1)\lambda_H(h_2))$  is either  $\gamma(\lambda_H(h_1))\gamma(\lambda_H(h_2))$  or  $\gamma(\lambda_H(h_2))\gamma(\lambda_H(h_1))$ , moreover if we put  $\lambda_G(I(h)) = \gamma(\lambda_H(h))$ ,

(2.6) then  $I$  is either an isomorphism or an antiisomorphism of  $H$  onto  $G$ .

The transposed map  $\psi$  of  $\gamma$  is also an isometric isomorphism of  $F_\alpha(G; M_*)$  onto  $F_\beta(H; N_*)$ . Then we get;

$$\begin{aligned}
\langle \gamma(\pi_\beta(y)), u*v \rangle &= \langle \pi_\beta(y), \psi(u*v) \rangle \\
&= \langle \pi_\beta(y), \psi(u)*\psi(v) \rangle \\
&= \langle \pi_\beta(y) \otimes \mathbf{1}, \psi(u) \otimes \psi(v) \rangle \\
&= \langle \gamma(\pi_\beta(y)), u*v \rangle
\end{aligned}$$

for all  $y \in N$ ,  $u, v \in F_\alpha(G; M_*)$ .

By [5] Proposition 2.3, we obtain  $\gamma(\pi_\beta(y))$  is an element of  $\pi_\alpha(M)$ , so that we can define an isometric surjective linear map  $\theta$  of  $N$  onto  $M$  by  $\theta = \pi_\alpha^{-1} \circ \gamma \circ \pi_\beta$ .

Since  $\gamma$  is a Jordan isomorphism,

$$\gamma(T)\gamma(z') + \gamma(z')\gamma(T) = \gamma([T, z']) = 2\gamma(Tz')$$

for all  $T \in H \otimes_\beta N$ , therefore we get  $\gamma(Tz') = \gamma(T)z$ .

Hence  $\gamma(\pi_\beta(xy))z = \gamma(\pi_\beta(x))\gamma(\pi_\beta(y))z$  for all  $x, y \in N$ .

Since  $z$  is a central projection of  $G \otimes_\alpha M$ ,  $z$  is also a projection of  $\pi_\alpha(M)'$ , then we get;

$$(2.7) \quad \gamma(\pi_\beta(xy))p = \gamma(\pi_\beta(x))\gamma(\pi_\beta(y))p$$

for all  $x, y \in N$  where  $p$  is the central support of  $z$  in  $\pi_\alpha(M)'$ .

We denote by  $q$  the central support of  $z'$  in  $\pi_\beta(N)'$ , then the equations  $\gamma(q)z = \gamma(qz') = \gamma(z') = z$  imply that  $\gamma(q)p = p$ , similarly we obtain  $\gamma^{-1}(p)q = q$  so that  $\gamma(q) = \gamma(\gamma^{-1}(p)q) = \gamma(\gamma^{-1}(p))\gamma(q)p = p\gamma(q) = p$ .

Hence  $\theta$  is an isomorphism of  $N_q$  onto  $M_p$  and  $\theta$  is an anti-isomorphism of  $N_{(1-q)}$  onto  $M_{(1-p)}$ .

The projection  $p$  (resp.  $q$ ) is  $G$ -invariant (resp.  $H$ -invariant) since  $\pi_\alpha(M)' = \lambda_G(g)\pi_\alpha(M)'\lambda_G(g)^*$  and  $\lambda_G(g)z\lambda_G(g)^* = z$ .

Now we have already proved (1)  $\sim$  (4) and the statements (5), (6) still remain to prove.

For all  $y \in N$ ,  $h \in H$  we get,

$$\begin{aligned}
\{\pi_\alpha \circ \theta(\beta_h(y))\}z &= \gamma(\lambda_H(h)\pi_\beta(y)\lambda_H(h)^*z') \\
&= \lambda_G(I(h))\pi_\alpha \circ \theta(y)\lambda_G(I(h)^{-1})z \\
&= \{\pi_\alpha \circ \alpha_{I(h)} \circ \theta\}(y)z,
\end{aligned}$$

hence

$$\theta \circ \beta_h = \alpha_{I(h)} \circ \theta \text{ on } N_q,$$

and similarly

$$\theta \circ \beta_h = \alpha_{I(h^{-1})} \circ \theta \text{ on } N_{(1-q)}.$$

Therefore  $\theta \circ \beta_h(y) = \alpha_{I(h)} \circ \theta(y)p + \alpha_{I(h^{-1})} \circ \theta(y)(1-p)$  for all  $y \in N$  and  $h \in H$ . To prove the statement (5), we shall show first,

$$\text{supp } \gamma(\pi_\beta(y)\lambda_H(h)) = \{I(h)\}.$$

For since  $\gamma(\pi_\beta(y)\lambda_H(h))u = \gamma(\pi_\beta(y)\lambda_H(h))\psi(u)$  for all  $u \in F_\alpha(G; M_*)$  and  $\psi$  is surjective,

$$\begin{aligned} & [\gamma(\pi_\beta(y)\lambda_H(h))F_\alpha(G; M_*)]^{-\sigma-w} \\ & = \gamma[\pi_\beta(y)\lambda_H(h)F_\beta(H; N_*)]^{-\sigma-w} \end{aligned}$$

where  $[\dots]^{-\sigma-w}$  means a  $\sigma$ -weak closure, on the other hand,

$$[\pi_\beta(y)\lambda_H(h)F_\beta(H; N_*)]^{-\sigma-w} \cap \lambda_H(H) = C\lambda_H(h)$$

because of  $\text{supp } \pi_\beta(y)\lambda_H(h) = \{h\}$ , so that we obtain;

$$\begin{aligned} & [\gamma(\pi_\beta(y)\lambda_H(h))F_\alpha(G; M_*)]^{-\sigma-w} \cap \lambda_G(G) = C\lambda_G(I(h)) \\ & \text{supp } \gamma(\pi_\beta(y)\lambda_H(h)) = \{I(h)\} . \end{aligned}$$

By [2] Theorem 4.4 or [6] Proposition 6.1, there exists an element  $x$  of  $M$  such that  $\gamma(\pi_\beta(y)\lambda_H(h)) = \pi_\alpha(x)\lambda_G(I(h))$ .

$$\begin{aligned} & \pi_\alpha(x)\lambda_G(I(h))z \\ & = \gamma(\pi_\beta(y)\lambda_H(h))z \\ & = \gamma(\pi_\beta(y))\gamma(\lambda_H(h))z \\ & = \pi_\alpha(\theta(y))\lambda_G(I(h))z \end{aligned}$$

then

$$xp = \theta(y)p, \text{ and similarly } x(1 - p) = \alpha_{I(h)}\theta(y)(1 - p) .$$

We get;

$$x = \theta(y)p + \alpha_{I(h)}\theta(y)(1 - p) ,$$

$$\gamma(\pi_\beta(y)\lambda_H(h)) = \pi_\alpha(\theta(y)p)\lambda_G(I(h)) + \pi_\alpha(\alpha_{I(h)}\theta(y)(1 - p))\lambda_G(I(h)) .$$

By (2.2),  $\phi(u) = \psi_k(u)$  for  $u \in F_\alpha(G; M_*)$  and the above equation, we can get the statement (5).

REMARK 2. Theorem 1 is a generalization of [10] Theorem 2.

COROLLARY 3. Let  $(M, G, \alpha)$ ,  $(N, H, \beta)$  be  $W^*$ -systems and the two actions  $\alpha$  and  $\beta$  are ergodic on their centers (that is  $Z_M \cap M^G = Z_N \cap N^H = C$ ).

The following statements are equivalent;

(1)  $F_\alpha(G; M_*)$  is isomorphic to  $F_\beta(H; N_*)$  in the sense of Banach algebra

(2) there exists either an isomorphism  $I$  of  $H$  onto  $G$ , an isomorphism  $\theta$  of  $N$  onto  $M$  such that  $\theta \circ \beta_h = \alpha_{I(h)} \circ \theta$  for all  $h \in H$ , or an anti-isomorphism  $I$  of  $H$  onto  $G$ , an anti-isomorphism  $\theta$  of  $N$  onto  $M$  such that  $\theta \circ \beta_h = \alpha_{I(h^{-1})} \circ \theta$  for all  $h \in H$ .

*Proof.* Suppose  $\phi$  is an isometric isomorphism of  $F_\alpha(G; M_*)$  onto  $F_\beta(H; N_*)$  and we use the same notations in Theorem 1. The projection  $p$  in (3) of Theorem 1 must be zero or 1 by the ergodicity of the action  $\alpha$ , then  $\theta$  is either an isomorphism or an anti-isomorphism of  $N$  onto  $M$ .

When  $G$  is a locally compact abelian group (it follows from (2.6) that  $H$  is a locally compact abelian group),  $I$  in (2.6) can be regarded as both an isomorphism and an anti-isomorphism, therefore the statement (2) follows from Theorem 1 when  $G$  is abelian. Hence we may assume that  $G$  is non-abelian.

When  $I$  is an anti-isomorphism of  $H$  onto  $G$ , the projection  $(1 - z)$  in (2.4) must be nonzero. For if the projection  $z$  is the identity in  $G \otimes_\alpha M$ , then  $\gamma$  in (2.5) is an isomorphism of  $H \otimes_\beta N$  onto  $G \otimes_\alpha M$ , so  $I$  is an isomorphism, which is a contradiction. Taking the central support of  $(1 - z)$  in  $\pi_\alpha(M)'$  as (2.7),  $\theta$  is an anti-isomorphism of  $H$  onto  $G$  such that  $\alpha_{I(h^{-1})} \circ \theta = \theta \circ \beta_h$  for all  $h \in H$ . If  $I$  is an isomorphism,  $\theta$  is an isomorphism such that  $\alpha_{I(h)} \circ \theta = \theta \circ \beta_h$  for all  $h \in H$ .

Conversely suppose  $I$  is an isomorphism of  $H$  onto  $G$  such that  $\theta \circ \beta_h = \alpha_{I(h)} \circ \beta_h$  for all  $h \in H$ . Then there exists an isomorphism  $\Gamma$  of  $H \otimes_\beta N$  onto  $G \otimes_\alpha M$  such that  $\Gamma(\pi_\beta(y)) = \pi_\alpha(\theta(y))$  for all  $y \in N$  and  $\Gamma(\lambda_H(h)) = \lambda_G(I(h))$  for all  $h \in H$  (cf. [9] Proposition 3.4). Then the transposed map  $\phi$  of  $\Gamma$  is an isometric isomorphism of  $F_\alpha(G; M_*)$  onto  $F_\beta(H; N_*)$ .

Suppose  $I$  is an anti-isomorphism of  $H$  onto  $G$  such that  $\theta \circ \beta_h = \alpha_{I(h^{-1})} \circ \theta$  for all  $h \in H$ . Considering the opposite von Neumann algebra  $M^\circ$  of  $M$  and the isomorphism  $J$  of  $H$  onto  $G$  by  $J(h) = I(h^{-1})$  for all  $h \in H$ , there exists an isomorphism  $\Gamma$  of  $H \otimes_\beta N$  onto  $G \otimes_\alpha M^\circ$  such that  $\Gamma(\pi_\beta(y)) = \pi_\alpha(\theta(y))$  for all  $y \in N$ ,  $\Gamma(\lambda_H(h)) = \lambda_G(J(h))$  for all  $h \in H$ . On the other hand,  $G \otimes_\alpha M^\circ$  is isometrically isomorphic to  $G \otimes_\alpha M$  as Banach spaces, therefore  $\Gamma$  is a  $\sigma$ -weakly continuous isometric linear map of  $H \otimes_\beta N$  onto  $G \otimes_\alpha M$ . Then the transposed map  $\phi$  of  $\Gamma$  is an isometric isomorphism of  $F_\alpha(G; M_*)$  onto  $F_\beta(H; N_*)$ .

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