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David P. Bellamy has shown that there exist indecomposable Hausdorff continua with only one or only two composants. The continua that he constructs are small in the sense that they do not have more than 2^c points. In this paper his results are generalized; in particular it will be shown that if X is a Hausdorff continuum then X is a retract of an indecomposable continuum with exactly one composant and of an indecomposable continuum with exactly two composants.

Definitions and Notations. A continuum is a compact connected ıusdorff space. Suppose λ is an ordinal, I_a is a topological space for each $a < \lambda$, and if $a < b$ then r_a^b is a mapping from I_b onto I_a so that if $a < b < c < \lambda$ then $r_a^b \circ r_b^c = r_a^c$. Then the space $I = \varprojlim \{I_a, r\}_{a<\lambda}$ denotes the space which is the inverse limit of the inverse system $\{I_a, r_a^b\}_{a<b<\lambda}$. Each point P of I is a function from λ into $\bigcup_{a<\lambda} I_a$ such that $P_a \in I_a$. $\prod_a$ denotes the function from I into I_a such that $\prod_a(P) = P_a$. If $R \subset I_a$ then $\overleftarrow{R} = \{x \mid x_a \in R\}$. If $S = \prod_{a\in A} S_a$ is a product space then $x = \{x_a\}_{a\in A}$ denotes a point of S so that $x_a \in S_a$ and π_a denotes the function from S into S_a so that $\pi_a(x) = x_a$. The composant of the continuum M containing the point P of M is the set of points Q of M such that there is a proper subcontinuum of M containing P and Q, it is denoted by Cmps (M, P).

Construction. The following construction employs techniques used in [1] and [4]. The continuum will be constructed as an inverse limit $\varprojlim\{I_a, r\}_{a<\omega_1}$ such that for each $a < \omega_1$ I_a is a subset of the cartesian product of I_0 and ω_1 copies of $[0, 1]$ so that if b is an ordinal with $a < b < \omega_1$ then I_a will be homeomorphic to a subset of I_b; in fact it will be convenient to identify I_a with this subset so that $\{I_a\}_{a<\omega_1}$ will be a monotonic collection of continua, I_a may be considered to be a subset of $I_0 \times \prod_{i\leq a} [0, 1] \times \prod_{a<j<\omega_1} \{0\}$, and if $x \in I_{a+1}$ then $\pi_{a+1}(x) \in [0, 1]$, $\pi_j(x) = 0$ if $j > a + 1$, and $\prod_{i<a+1} \{\pi_i(x)\} \times \prod_{j>a+1} \{0\}$ is a point of I_a. In general the space $\prod_{j<a} [0, 1]$ may be considered to be the space $\prod_{j<a} [0, 1] \times \prod_{a<i<\omega_1}\{0\}$.

Construction of I_0: If X is a continuum then there exists a continuum I_0 containing X as a retract which is irreducible from some point 1_0 to X so that: there exists a sequence of points $\{a_i^0\}_{i=1}^\infty$ and a monotonic sequence of proper subcontinua of I_0, $\{A_i^0\}_{i=1}^\infty$ such that

(1) $\{a_i^0\}_{i=1}^{\infty}$ converges to a point a in X, (2) A_i^0 is irreducible from 1_0 to a_i^0 and $A_i^0 \subset A_{i+1}^0$ for each positive integer i, and (3) Cmps $(I_0, 1_0) = \bigcup_{i=1}^{\infty} A_i^0$. (The existence of I_0 follows from [4] or from [2] and the construction for I_1 used below.)

Construction of I_1: Let I_1 be the subcontinuum of $I_0 \times [0, 1]$ defined as follows: for each positive integer n let $a_n^1 = (a_n^0, 1/(2n-1))$,

$$\begin{aligned} A_1^1 &= A_1^0 \times \{1\}, \\ A_2^1 &= A_1^1 \cup (\{a_1^0\} \times [1/2, 1]) \cup (A_1^0 \times \{1/2\}) \\ &\quad \cup \left(\{1_0\} \times \left[\frac{1}{3}, 1/2\right]\right) \cup \left(A_2^0 \times \left\{\frac{1}{3}\right\}\right), \\ &\vdots \\ A_n^1 &= A_{n-1}^1 \cup \left(\{a_{n-1}^0\} \times \left[\frac{1}{2n-2}, \frac{1}{2n-3}\right]\right) \\ &\quad \cup \left(\{A_{n-1}^0\} \times \left\{\frac{1}{2n-2}\right\}\right) \cup \left(\{1_0\} \times \left[\frac{1}{2n-1}, \frac{1}{2n-2}\right]\right) \\ &\quad \cup \left(A_n^0 \times \left\{\frac{1}{2n-1}\right\}\right), \\ &\vdots \end{aligned}$$

and let $I_1 = (I_0 \times \{0\}) \cup \bigcup_{n=1}^{\infty} A_n^1$. Let $1_1 = (1_0, 1)$ and identify I_0 with $I_0 \times \{0\}$ using the natural mapping. Thus $\{a_n^1\}_{n=1}^{\infty}$ converges to $(a, 0)$ which has been identified with a, A_n^1 is irreducible from a_n^1 to 1_1, and $A_n^1 \subset A_{n+1}^1$. Let r_0^1 be the projection π_1 of I_1 onto I_0, thus $r_0^1(A_n^1) = A_n^0$, $r_0^1(a_n^1) = a_n^0$, and $\mathrm{Cmps}(I_1, 1_1) = \bigcup_{n=1}^{\infty} A_n^1$.

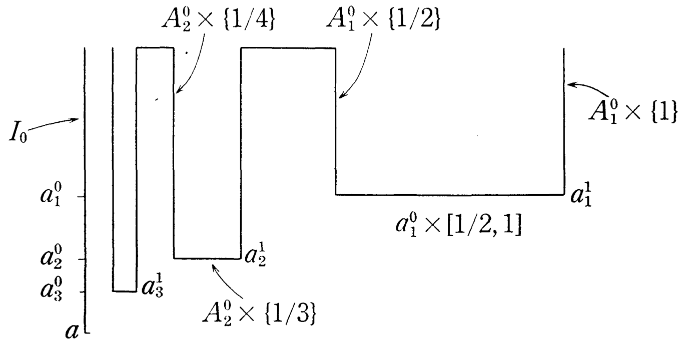
Construction of I_k for each positive integer $k > 1$: Let I_k be a subcontinuum of $I_{k-1} \times [0, 1]$ defined as follows: for each positive integer n let

$$\begin{aligned} a_n^k &= (a_n^{k-1}, 1) \quad \text{if} \quad n \leqq k \\ &= \left(a_n^{k-1}, \frac{1}{2n-1}\right) \quad \text{if} \quad n > k, \\ A_n^k &= A_n^{k-1} \times \{1\} \quad \text{if} \quad n \leqq k, \end{aligned}$$

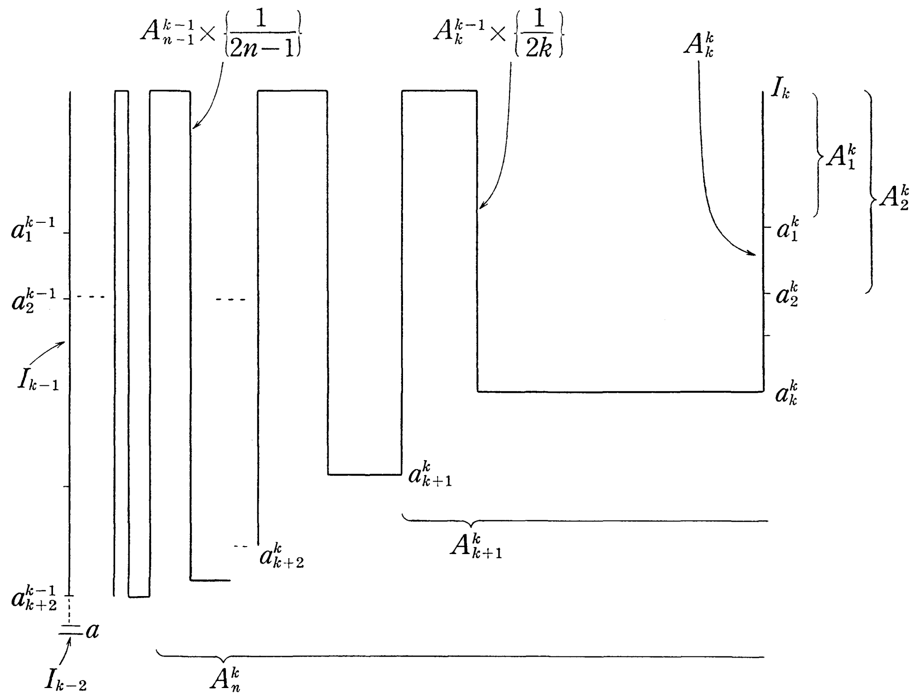
and if $n > k$ A_n^k is defined by recursion,

$$\begin{aligned} A_{k+1}^k &= A_k^k \cup \left(\{a_k^{k-1}\} \times \left[\frac{1}{2k}, 1\right]\right) \cup \left(A_k^{k-1} \times \left\{\frac{1}{2k}\right\}\right) \\ &\quad \cup \left(\{1_{k-1}\} \times \left[\frac{1}{2k+1}, \frac{1}{2k}\right]\right) \cup \left(A_{k+1}^{k-1} \times \left\{\frac{1}{2k+1}\right\}\right), \\ &\vdots \\ A_n^k &= A_{n-1}^k \cup \left(\{a_{n-1}^{k-1}\} \times \left[\frac{1}{2n-2}, \frac{1}{2n-3}\right]\right) \end{aligned}$$

$$\cup \left(A_{n-1}^{k-1} \times \left\{\frac{1}{2n-2}\right\}\right) \cup \left(\{1_{k-1}\} \times \left[\frac{1}{2n-1}, \frac{1}{2n-2}\right]\right)$$
$$\cup \left(A_n^{k-1} \times \left\{\frac{1}{2n-1}\right\}\right), \cdots .$$



I_1



I_k

Then let $I_k = (I_{k-1} \times \{0\}) \cup \bigcup_{n=1}^{\infty} A_n^k$, $1_k = (1_{k-1}, 1)$ and identify I_{k-1} with $I_{k-1} \times \{0\}$ using the natural mapping; let r_{k-1}^k be the projection of I_k onto I_{k-1}. Thus $\{a_n^k\}_{n=1}^{\infty}$ converges to $a = (a, 0)$, $A_n^k \subset A_{n+1}^k$, and $\mathrm{Cmps}(I_k, 1_k) = \bigcup_{n=1}^{\infty} A_n^k$.

The following properties of the construction will be used in the proofs:

(P1) I_k is irreducible from 1_k to I_{k-1};

(P2) no point of $I_k - I_{k-1}$ is mapped by r^k_{k-1} into I_{k-2} and each point of $I_k - I_{k-1}$ is mapped into $\mathrm{Cmps}(I_{k-1}, 1_{k-1})$;

(P3) for each n and $\beta < \alpha$ $r^\alpha_\beta(a^\alpha_n) = a^\beta_n$ and $r^\alpha_\beta(A^\alpha_n) = A^\beta_n$;

(P4) if $k \leqq n$ then $\pi_n(A^n_k) = \{1\}$, if $k > n$ then $\pi_n(A^n_k) = [1/(2k-1), 1]$, and $\pi^{-1}_n(1) = A^n_n$;

and

(P5) every point of $\{a^{k-1}_{k+1}\} \times [1/(2k-1), 1]$ separates I_k. Let $I_{\omega_0} = \underleftarrow{\lim}\{I_n, r\}_{n<\omega_0}$, and let 1_{ω_0} be the point x such that $x_n = 1_n$. Then for each integer n, I_n can be identified with $\underleftarrow{\lim}\{I_n, r^{k+1}_k\}_{n<k<\omega_0}$ since r^{k+1}_k is the identity on I_k for $k > n$. Let I_n be so identified using the natural mapping. So $I_n \subset I_0 \times \prod^n_{i=1}[0, 1]$ and I_{ω_0} is identified with the subset $\overline{\bigcup^\infty_{n=1} I_n}$ of $I_0 \times \prod^{\omega_0}_{i=1}[0, 1]$. Further define $A^{\omega_0}_i$ for each positive integer i by $A^{\omega_0}_i = \underleftarrow{\lim}\{A^n_i, r\}^\infty_{n=1}$, property P3 insures that $A^{\omega_0}_i$ is well defined. Note that it follows from the construction that if $x \in I_{\omega_0}$ and $a < b$ then $\pi_i(x_a) = \pi_i(x_b)$ for all $i \leqq a$.

Claim 1. I_{ω_0} is indecomposable.

Proof. Suppose not and that H and K are two proper subcontinua of I_{ω_0} whose union is I_{ω_0}. Then there exist open sets R and S such that $R \subset H \backslash K$ and $S \subset K \backslash H$ and hence are mutually exclusive. There exists an integer j and two open sets R_j and S_j in I_j such that $\overleftarrow{R}_j \subset R$ and $\overleftarrow{S}_j \subset S$. Since $I_j = \overline{\bigcup^\infty_{n=1} A^j_n}$ there is an integer i so that both R_j and S_j intersect A^j_i. Therefore $R_j \times [0, 1]$ and $S_j \times [0, 1]$ both intersect $A^j_i \times \{1/(2i-1)\}$. So each of $\prod_{j+1}(R)$ and $\prod_{j+1}(S)$ intersect both I_j and A^{j+1}_i, hence each of $\prod(H)$ and $\prod(K)$ intersect both I_j and A^{j+1}_i. By the irreducibility of I_{j+1} from 1_{j+1} to I_j it follows that I_j is a subset of both $\prod_{j+1}(H)$ and $\prod_{j+1}(K)$ (recall that $I_j = I_j \times \{0\}$) and hence $I_j = \prod_j(H) = \prod_j(K)$ which contradicts the fact that $\vec{R}_j$ and $\vec{S}_j$ must be mutually exclusive. Thus I_{ω_0} is indecomposable.

Claim 2. If $x \in I_{\omega_0}$ and there is a positive integer j such that $\pi_j(x_j) = 0$, then $\pi_i(x_i) = 0$ for all $i > j$.

Proof. Suppose $x \in I_{\omega_0}$, $x_\alpha \in I_\alpha$ and $\pi_\alpha(x_\alpha) \neq 0$. Then there exists an integer n such that $x_\alpha \in A^\alpha_n$. But $r^\alpha_{\alpha-1}(A^\alpha_n) = A^{\alpha-1}_n$ and either $\pi_{\alpha-1}(A^{\alpha-1}_n) = [1/(2n-1), 1]$ or $\pi_{\alpha-1}(A^{\alpha-1}_n) = 1$, and in either case $\pi_{\alpha-1}(x_{\alpha-1}) \neq 0$ so $\pi_{\alpha-1}(x_\alpha) \neq 0$. So if $\pi_j(x_j) = 0$ then $\pi_{j+1}(x_{j+1}) = 0$ and the claim follows by induction.

Claim 3. If K is a proper subcontinuum of I_{ω_0} containing 1_{ω_0} then there exists an integer β so that if $\gamma > \beta$ then $\pi_\alpha(\prod_\gamma(K)) = 1$ for all α so that $\beta < \alpha \leqq \gamma$.

Proof. Suppose that there is a proper subcontinuum K of I_{ω_0} for which the claim is not true. Then if β is an integer there exists an integer $\gamma > \beta$ so that $\pi_\gamma(\prod_\gamma(K))$ is nondegenerate. Suppose in addition that for each β there is a $\gamma > \beta$ so that $\pi_\gamma(\prod_\gamma(K)) = 0$. Then by Claim 2 since $1 \in \pi_\alpha(\prod_\gamma(K))$ for all $\alpha < \gamma$ it follows that $I_{\gamma-1} \subset \prod_\gamma(K)$. But then $K = I_{\omega_0}$ which is a contradiction. So the supposition is false and there exists an integer b so that if $\gamma > b$ then $0 \notin \pi_\alpha(\prod_\alpha(K))$ for all α such that $b < \alpha < \gamma$.

Suppose $\beta > b$, where b is defined above. Then from the negation of the claim, for each positive integer n there is an integer γ_n with $\beta + n < \gamma_n$ so that $\pi_{\gamma_n}(\prod_{\gamma_n}(K))$ is nondegenerate. But then $(a_{\gamma_n}^{\gamma_n - 1}, 1) \in \prod_{\gamma_n}(K)$. So $\alpha_{\gamma_n}^\beta \in \prod_\beta(K)$ (by P3), thus if $\gamma_n = \beta + k_n$ for some positive integer $k_n > n$ then $a_{\beta+k_n}^\beta \in \prod_{\gamma_n}(K)$ and thus $a_{\beta+k_n}^\beta \in \prod_\beta(K)$ (by P3). So there is unbounded sequence in $\{k_n\}_{n=1}^\infty$ so that $a_{\beta+k_n}^\beta \in \prod_\beta(K)$, but a is the sequential limit of $\{a_i^\beta\}_{i=1}^\infty$ and hence is a limit point of the set $\{a_{\beta+k_n}^\beta \mid n \text{ is a positive integer}\}$, so $a \in \prod_\beta(K)$. Now $\pi_\alpha(a) = 0$ for all $\alpha > 1$ so $0 \in \pi_\alpha(\prod_\beta(K))$ for all $0 < \alpha \leqq \beta$ which contradicts the choice of $\beta > b$. So the claim has been established.

Claim 4. $\mathrm{Cmps}(I_{\omega_0}, 1_{\omega_0}) = \bigcup_{i=1}^\infty A_i^{\omega_0}$.

Proof. Suppose $x \in \mathrm{Cmps}(I_{\omega_0}, 1_{\omega_0})$. By Claim 3 there exists an integer β so that if $\gamma > \beta$ and α is an integer so that $\beta < \alpha \leqq \gamma$ then $\pi_\alpha(x_\gamma) = 1$. Let $\gamma > \beta$, then $x_\gamma \in A_\gamma^\gamma$ (by P4). Thus $x_\alpha \in r_\alpha^\gamma(A_\gamma^\gamma)$ and $r_\alpha^\gamma(A_\gamma^\gamma) = \pi_\alpha(A_\gamma^{\omega_0})$. So $x \in A_\gamma^{\omega_0}$. So Claim 4 has been established.

The construction of I_μ for μ an ordinal greater than ω_0 follows. Suppose δ is a limit ordinal and that $\{A_i\}_{i=1}^\infty$, $\{a_i\}_{i=1}^\infty$, C_λ, r_β^λ, and I_λ have feen defined for all $\lambda \leqq \delta$ so that:

(1) For each positive integer i the continuum A_i^λ is irreducible from a_i^λ to 1_λ.

(2) $C_\lambda = \bigcup_{i=1}^\infty A_i^\lambda$.

(3) If $\beta < \lambda$ then $r_\beta^\lambda(A_i^\lambda) = A_i^\beta$, $r_\beta^\lambda(a_i^\lambda) = a_i^\beta$, and $\{a_i^\lambda\}_{i=1}^\infty$ converges to a.

(4) If $\beta < \lambda$ then $r_\beta^\lambda(I_\lambda - I_\beta) = C_\beta$.

(5) $C_\lambda = \mathrm{Cmps}(I_\lambda, 1_\lambda) = \{P \mid \text{there exists a } \beta < \delta \text{ such that } \pi_\lambda(P_\lambda) = 1 \text{ for all } \gamma > \beta\}$.

Then construct $I_{\delta+n}$ for all positive integers n by substituting I_δ for I_0, A_i^δ for A_i^0, a_i^δ for a_i^0, and 1_δ for 1_0 in the construction of I_n above. Compare condition 4 with a similar condition in Bellamy [1].

Suppose that μ is a limit ordinal and I_γ has been defined for all $\gamma < \mu$. Let $I_\mu = \varprojlim \{I_\gamma, r\}_{\gamma<\mu}$, $A_i^\mu = \varprojlim \{A_i^\gamma, r\}_{\gamma<\mu}$, $a_i^\mu = \varprojlim \{a_i, r\}_{\gamma<\mu}$, and for each $\beta < \mu$ let r_β^μ be the projection of I_μ onto I_β. As above identify I_γ with $\varprojlim \{I_\gamma, r\}_{\gamma<\alpha<\mu}$ and a with $\vec{a}$. The argument of Claim 1 can be used to prove that I_μ is indecomposable. Claim 2 also generalizes for I_μ as follows:

Claim 5. If $x \in I_\mu$ and there is an ordinal $j < \mu$ which is not a limit ordinal such that $\pi_j(x_j) = 0$ then $\pi_i(x_i) = 0$ for all ordinals i, $j < i < \mu$, which are not limit ordinals; and hence $x \in I_j$.

Proof. Suppose $x \in I_\mu$ and $j = \lambda + q$ for some limit ordinal λ and positive integer q. If $\alpha = \lambda' + r$ for some limit ordinal $\lambda' \geqq \lambda$ with $\lambda' + r > \lambda$ and $r > 0$ and it is true that $\pi_\alpha(x_\alpha) \neq 0$, then there exists an integer n so that $x_\alpha \in A_n^\alpha$. But $r_j^\alpha(A_n^\alpha) = A_n^j$ and either $\pi_j(A_n^j) = [1/(2n-1), 1]$ or $\pi_j(A_n^j) = 1$ (by P4). In either case $\pi_j(x_\alpha) \neq 0$. But $\pi_j(x_j) = \pi_j(x_\alpha)$, so that $\pi_j(x_j) \neq 0$, which is a contradiction.

Claims 6, 7, and 8 are concerned with the continuum I_μ.

Claim 6. If K is a subcontinuum of I_μ and $a \in \prod_1(K)$ then $a \in K$.

Proof. If $a \in \prod_1(K)$ then $(a, 0) \in \prod_2(K)$ so $a \in \prod_2(K)$. From Claim 5 it follows that $a \in \prod_\gamma(K)$ for all $\gamma \in \mu$ since a is identified with $a \times \{0\}^\gamma$. Thus a must belong to K.

Claim 7.

$$\mathrm{Cmps}(I_\mu, 1_\mu) = \bigcup_{i=1}^{\infty} A_i^\mu .$$

Proof. Suppose that K is a proper subcontinuum of I_μ containing 1_μ. If it is true that there is an integer n so that if $\gamma < \mu$ then $a_n^\gamma \notin \prod_\gamma(K)$, then it would follow that $\prod_\gamma(K) \subset A_n^\gamma$ for all $\gamma < \mu$, and so $K \subset A_n^\mu$. So suppose that this is not true. Thus for each integer n there exists an ordinal $\gamma_n < \mu$ such that $a_n^{\gamma_n} \in \prod_{\gamma_n}(K)$. But then $a_n^1 \in \prod_1(K)$ for all n, since $r_1^{\gamma_n}(a_n^{\gamma_n}) = a_n^1$. So $a \in \prod_1(K)$ and $a \in K$ by Claim 6. But then $K = I_\mu$ since I_μ is irreducible from a to 1_μ. So the claim is true.

Claim 8. I_μ satisfies the following for each ordinal β, $\beta < \mu$, and each positive integer i:

(1) A_i^μ is irreducible from a_i^μ to 1_μ.

(2) $C = \mathrm{Cmps}(I_\mu, 1_\mu) = \bigcup_{i=1}^\infty A_i^\mu$.

(3) $r_\beta^\mu(A_i^\mu) = A_i^\beta$, $r_\beta^\mu(a_i^\mu) = a_i^\beta$, and $\{a_i^\mu\}_{i=1}^\infty$ converges to a.

(4) $r^{\mu}_{\beta}(I_{\mu} - I_{\beta}) = C_{\beta}$.

Proof. Part (1) follows from the irreducibility of $\prod_{\gamma}(A^{\mu}_{i})$ for each $\gamma<\mu$, and part (2) follows from Claim 7. Since for each ordinal $\gamma < \mu$ the sequence $\{\prod_{\gamma}(a^{\mu}_{i})\}^{\infty}_{i=1}$ converges to a, it follows that $\{a^{\mu}_{i}\}^{\infty}_{i=1}$ converges to $\overleftarrow{a}$ which is identified with a. The rest of (3) follows from the definitions of r^{μ}_{β}, A^{β}_{i}, and a^{β}_{i}. To prove (4) suppose that $x \in I_{\mu} - I_{\beta}$. Then by Claim 5, $\pi_{\beta+1}(x_{\beta+1}) \neq 0$ so $x_{\beta+1} \in A^{\beta+1}_{n}$ for some integer n, but $r^{\beta+1}_{\beta}(A^{\beta+1}_{n}) \subset C_{\beta}$, thus $r^{\beta+1}_{\beta}(x_{\beta+1}) \in C_{\beta}$ so $r^{\mu}_{\beta}(x) \in C_{\beta}$; equality follows from parts (2) and (3).

Claim 9. The continuum $I_{\omega_1} = \varprojlim \{I_{\lambda}, r\}_{\lambda<\omega_1}$ has exactly two composants.

Proof. From the construction, $\{I_{\gamma}\}_{\gamma<\omega_1}$ is a monototic collection of continua. (a) If $\beta > \gamma$ then I_{γ} does not intersect C_{β} because I_{γ} does not intersect $C_{\gamma+1}$ and if $\beta > \gamma$, $C_{\gamma+1} = r^{\beta}_{\gamma+1}(C_{\beta})$. (b) From (4) of Claim 8 it follows that $r^{\alpha}_{\beta}(I_{\alpha} - I_{\beta}) = C_{\beta}$ for $\alpha > \beta$. Let $W = \{x \mid$ there is a γ so that if $\alpha > \gamma$ then $\pi_{\alpha}(x_{\alpha}) = 0\}$. If $x \in W$ and γ is the ordinal specified in the definition of W then $x \in I_{\gamma}$. So x lies in the same composant as a.

Now I_{ω_1} is irreducible from a to 1_{ω_1}, it will now be shown that if y is a point of I_{ω_1} not in W then y lies in $\mathrm{Cmps}(I_{\omega_1}, 1_{\omega_1})$. Suppose $y \notin W$. The following two conditions need to be established: (i) if $\alpha > \beta$ then $y_{\alpha} \notin I_{\beta}$, and (ii) $y_{\alpha} \in C_{\alpha}$. If $\alpha > \beta$ there exists an ordinal $\delta > \alpha$ such that $y_{\alpha} \neq y_{\delta}$ or else $y \in W$ (in particular $y \in I_{\alpha}$). Suppose that $y_{\alpha} \in I_{\beta}$, then $y_{\alpha} \notin C_{\alpha}$ by (a) above. But $r^{\delta}_{\alpha}(I_{\delta} - I_{\alpha}) \subset C_{\alpha}$ so $y_{\delta} \notin I_{\delta} - I_{\alpha}$, so $y_{\delta} \in I_{\alpha}$. But $r^{\delta}_{\alpha}|_{I_{\alpha}}$ is the identity which contradicts the fact that $y_{\delta} \neq y_{\alpha}$. Thus (i) has been shown, also it has been shown that if $\alpha > \beta$ then there exists a $\delta > \alpha$ such that $y_{\delta} \notin I_{\alpha}$. So $y_{\delta} \in I_{\delta} - I_{\alpha}$, $r^{\delta}_{\alpha}(I_{\delta} - I_{\alpha}) \subset C_{\alpha}$, and so (ii) has been shown.

Suppose that $y \notin W$. By (i) if $\alpha > 1$ then $y_{\alpha} \notin I_1$, and by (ii) $y_{\alpha} \in C_{\alpha}$. Thus by (2) of Claim 8 there exists an integer n_{α} so that $y_{\alpha} \in A^{\alpha}_{n_{\alpha}}$. There exists an uncountable subset J of ω_1 and an integer n so that $n_{\alpha} = n$ for all $\alpha \in J$. But since $r^{\alpha}_{\beta}(A^{\alpha}_{n}) = A^{\beta}_{n}$ it follows that $y \in \varprojlim \{A^{\beta}_{n}, r\}_{\beta<\omega_1}$ which is a proper subcontinuum of I containing 1_{ω_1}. Thus it has been shown that if $y \notin W$ then $y \in \mathrm{Cmps}(I_{\omega_1}, 1_{\omega_1}) = C_{\omega_1}$. So I_{ω_1} has exactly two composants W and C_{ω_1}.

One can see that X is a retract of each I_n and hence of I_{ω_1}. In order to construct a continuum with only one composant which has X as a retract it is only necessary to construct I_0 and a retraction r from I_0 onto X that maps 1_0 onto a, then by identifying a and the point 1_{ω_1} the continuum I_{ω_1} satisfies the desired condition.

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