LARGE INDECOMPOSABLE CONTINUA WITH ONLY ONE COMPOSANT

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David P. Bellamy has shown that there exist indecomposable Hausdorff continua with only one or only two composants. The continua that he constructs are small in the sense that they do not have more than $2^c$ points. In this paper his results are generalized; in particular it will be shown that if $X$ is a Hausdorff continuum then $X$ is a retract of an indecomposable continuum with exactly one composant and of an indecomposable continuum with exactly two composants.

Definitions and Notations. A continuum is a compact connected Hausdorff space. Suppose $\lambda$ is an ordinal, $I_a$ is a topological space for each $a < \lambda$, and if $a < b$ then $r^b_a$ is a mapping from $I_b$ onto $I_a$ so that if $a < b < c < \lambda$ then $r^b_a \circ r^c_b = r^c_a$. Then the space $I = \lim_{a < \lambda} \{I_a, r^b_a\}$ denotes the space which is the inverse limit of the inverse system $\{I_a, r^b_a\}_{a < b < \lambda}$. Each point $P$ of $I$ is a function from $\lambda$ into $\bigcup_{a < \lambda} I_a$ such that $P_a \in I_a$. $\Pi_a$ denotes the function from $I$ into $I_a$ such that $\Pi_a(P) = P_a$. If $R \subseteq I_a$ then $\tilde{R} = \{x | x_a \in R\}$. If $S = \prod_{a \in A} S_a$ is a product space then $x = \{x_a\}_{a \in A}$ denotes a point of $S$ so that $x_a \in S_a$ and $\pi_a$ denotes the function from $S$ into $S_a$ so that $\pi_a(x) = x_a$. The composant of the continuum $M$ containing the point $P$ of $M$ is the set of points $Q$ of $M$ such that there is a proper subcontinuum of $M$ containing $P$ and $Q$, it is denoted by $\text{Cmps}(M, P)$.

Construction. The following construction employs techniques used in [1] and [4]. The continuum will be constructed as an inverse limit $\lim_{a < \omega_1} \{I_a, r^b_a\}$ such that for each $a < \omega_1$, $I_a$ is a subset of the cartesian product of $I_0$ and $\omega_1$ copies of $[0, 1]$ so that if $b$ is an ordinal with $a < b < \omega_1$ then $I_a$ will be homeomorphic to a subset of $I_b$; in fact it will be convenient to identify $I_a$ with this subset so that $\{I_a\}_{a < \omega_1}$ will be a monotonic collection of continua, $I_a$ may be considered to be a subset of $I_0 \times \prod_{a \leq a} [0, 1] \times \prod_{a < j < \omega_1} \{0\}$, and if $x \in I_{a+1}$ then $\pi_{a+1}(x) \in [0, 1]$, $\pi_j(x) = 0$ if $j > a + 1$, and $\prod_{\xi < \omega_1} \{\pi_\xi(x)\} \times \prod_{j > a + 1} \{0\}$ is a point of $I_a$. In general the space $\prod_{j < a} [0, 1] \times \prod_{a < i < \omega_1} \{0\}$ may be considered to be the space $\prod_{i < a} [0, 1] \times \prod_{a < i < \omega_1} \{0\}$.

Construction of $I_0$: If $X$ is a continuum then there exists a continuum $I_0$ containing $X$ as a retract which is irreducible from some point $1_0$ to $X$ so that: there exists a sequence of points $\{a_\xi\}_{\xi = 1}^{\infty}$ and a monotonic sequence of proper subcontinua of $I_0$, $\{A_\xi\}_{\xi = 1}^{\infty}$ such that
(1) \( \{a^0_i\}_{i=1}^\infty \) converges to a point \( a \) in \( X \), (2) \( A_i^0 \) is irreducible from \( I_0 \) to \( a^0_i \) and \( A_i^0 \subset A_{i+1}^i \) for each positive integer \( i \), and (3) \( \mathrm{Cmps}(I_0, I_0) = \bigcup_{i=1}^\infty A_i^0 \). (The existence of \( I_0 \) follows from \([4]\) or from \([2]\) and the construction for \( I_1 \) used below.)

**Construction of \( I_i \):** Let \( I_i \) be the subcontinuum of \( I_o \times [0, 1] \) defined as follows: for each positive integer \( n \) let \( a^0_i = (a^0_n, 1/(2n-1)) \),

\[
A_1^i = A_1^0 \times \{1\}, \quad A_2^i = A_1^i \cup (\{a_1^i\} \times [1/2, 1]) \cup (A_0^i \times \{1/2\}) \cup \left( \{1_0\} \times \left[ \frac{1}{3}, 1/2 \right] \right) \cup \left( A_1^i \times \left\{ \frac{1}{3} \right\} \right),
\]

\[
\vdots
\]

\[
A_n^i = A_{n-1}^i \cup (\{a_{n-1}^i\} \times \left[ \frac{1}{2n-2}, \frac{1}{2n-3} \right]) \cup \left( \{A_{n-1}^i\} \times \left\{ \frac{1}{2n-2} \right\} \right) \cup \left( \{1_0\} \times \left[ \frac{1}{2n-1}, \frac{1}{2n-2} \right] \right) \cup \left( A_{n-1}^i \times \left\{ \frac{1}{2n-1} \right\} \right).
\]

and let \( I_1 = (I_0 \times \{0\}) \cup \bigcup_{i=1}^\infty A_i^1 \). Let \( I_1 = (I_0, 1) \) and identify \( I_0 \) with \( I_0 \times \{0\} \) using the natural mapping. Thus \( \{a_1^0\}_{n=1}^\infty \) converges to \((a, 0)\) which has been identified with \( a \), \( A_1^0 \) is irreducible from \( a_2^0 \) to \( 1_1 \), and \( A_2^1 \subset A_{2+1}^1 \). Let \( r^0_i \) be the projection \( \pi_i \) of \( I_i \) onto \( I_0 \), thus \( r^0_i(A_1^i) = A_1^0, r^0_i(a_1^1) = a_2^0 \), and \( \mathrm{Cmps}(I, I_i) = \bigcup_{i=1}^\infty A_i^1 \).

Construction of \( I_k \) for each positive integer \( k > 1 \): Let \( I_k \) be a subcontinuum of \( I_{k-1} \times [0, 1] \) defined as follows: for each positive integer \( n \) let

\[
a_k^k = (a_k^{k-1}, 1) \text{ if } n \leq k
\]

\[
= \left( a_k^{k-1}, \frac{1}{2n-1} \right) \text{ if } n > k,
\]

\[
A_k^k = A_{k-1}^0 \times \{1\} \text{ if } n \leq k,
\]

and if \( n > k \) \( A_k^k \) is defined by recursion,

\[
A_{k+1}^k = A_k^k \cup \left( \{a_{k-1}^k\} \times \left[ \frac{1}{2k}, 1 \right] \right) \cup \left( A_{k-1}^k \times \left\{ \frac{1}{2k} \right\} \right) \cup \left( \{1_{k-1}\} \times \left[ \frac{1}{2k+1}, \frac{1}{2k} \right] \right) \cup \left( A_{k+1}^{k-1} \times \left\{ \frac{1}{2k+1} \right\} \right),
\]

\[
\vdots
\]

\[
A_n^k = A_{n-1}^k \cup \left( \{a_{n-1}^{k-1}\} \times \left[ \frac{1}{2n-2}, \frac{1}{2n-3} \right] \right).
\]
Then let $I_k = (I_{k-1} \times \{0\}) \cup \bigcup_{n=1}^{\infty} A_n^k$, $1_k = (1_{k-1}, 1)$ and identify $I_{k-1}$ with $I_{k-1} \times \{0\}$ using the natural mapping; let $r_{k-1}^k$ be the projection of $I_k$ onto $I_{k-1}$. Thus $(a_n^k)_{n=1}^\infty$ converges to $a = (a, 0)$, $A_n^k \subset A_{n+1}^k$, and $\text{Cmps}(I_k, 1_k) = \bigcup_{n=1}^{\infty} A_n^k$. 
The following properties of the construction will be used in the proofs:

(P1) $I_k$ is irreducible from $1_k$ to $I_{k-1}$;

(P2) no point of $I_k - I_{k-1}$ is mapped by $r_{k-1}$ into $I_{k-2}$ and each point of $I_k - I_{k-1}$ is mapped into $\text{Cmp}(I_{k-1}, 1_k)$;

(P3) for each $n$ and $\beta < \alpha r_n(a_\alpha^n) = a_\alpha^n$ and $r_\alpha^n(A_\alpha^n) = A_\alpha^n$;

(P4) if $k \leq n$ then $\pi_n(A_\alpha^n) = \{1\}$, if $k > n$ then $\pi_n(A_\alpha^n) = [1/(2k-1), 1]$, and $\pi_1^{-1}(1) = A_\alpha^n$;

(P5) every point of $\{a_\alpha^{k+1}\} \times [1/(2k-1), 1]$ separates $I_k$. Let $\gamma = \lim \{I_n, r\}_{n<\omega}$ and let $1_\omega$ be the point $x$ such that $x_n = 1_n$. Then for each integer $n$, $I_n$ can be identified with $\lim \{I_n, r\}_{n<k<\omega}$ since $r_{k+1}$ is the identity on $I_k$ for $k > n$. Let $I_n$ be so identified using the natural mapping. So $I_n \subset I_0 \times \prod_{n=1}^\infty [0,1]$ and $I_{\omega}$ is identified with the subset $\bigcup_{n=1}^\infty I_n$ of $I_0 \times \prod_{n=1}^\omega [0,1]$. Further define $A_\omega$ for each positive integer $i$ by $A_\omega^i = \lim \{A_\omega^n, r\}_{n=i}$, property P3 insures that $A_\omega$ is well defined. Note that it follows from the construction that if $x \in I_\omega$ and $a < b$ then $\pi_i(x_a) = \pi_i(x_b)$ for all $i \leq a$.

Claim 1. $I_\omega$ is indecomposable.

Proof. Suppose not and that $H$ and $K$ are two proper subcontinua of $I_\omega$ whose union is $I_\omega$. Then there exist open sets $R$ and $S$ such that $R \subset H \setminus K$ and $S \subset K \setminus H$ and hence are mutually exclusive. There exists an integer $j$ and two open sets $R_j$ and $S_j$ in $I_j$ such that $R_j \subset R$ and $S_j \subset S$. Since $I_j = \bigcup_{n=1}^\infty A_n^j$ there is an integer $i$ so that both $R_j$ and $S_j$ intersect $A_n^j$. Therefore $R_j \times [0,1]$ and $S_j \times [0,1]$ both intersect $A_n^j \times (1/(2i-1))$. So each of $\prod_{j+1}(R)$ and $\prod_{j+1}(S)$ intersect both $I_j$ and $A_i^{j+1}$, hence each of $\prod(H)$ and $\prod(K)$ intersect both $I_j$ and $A_i^{j+1}$. By the irreducibility of $I_{j+1}$ from $I_j$ to $I_{j+1}$ it follows that $I_j$ is a subset of both $\prod_{j+1}(H)$ and $\prod_{j+1}(K)$ (recall that $I_j = I_j \times \{0\}$) and hence $I_j = \prod_j(H) = \prod_j(K)$ which contradicts the fact that $R_j$ and $S_j$ must be mutually exclusive. Thus $I_\omega$ is indecomposable.

Claim 2. If $x \in I_\omega$ and there is a positive integer $j$ such that $\pi_j(x_i) = 0$, then $\pi_i(x_i) = 0$ for all $i > j$.

Proof. Suppose $x \in I_\omega$, $x_\alpha \in I_\alpha$ and $\pi_\alpha(x_\alpha) \neq 0$. Then there exists an integer $n$ such that $x_\alpha \in A_\alpha^n$. But $r_{\alpha-1}(A_\alpha^n) = A_\alpha^{n-1}$ and either $\pi_{\alpha-1}(A_\alpha^{n-1}) = [1/(2n-1), 1]$ or $\pi_{\alpha-1}(A_\alpha^{n-1}) = 1$, and in either case $\pi_{\alpha-1}(x_{\alpha-1}) \neq 0$ so $\pi_{\alpha-1}(x_\alpha) \neq 0$. So if $\pi_j(x_j) = 0$ then $\pi_{j+1}(x_{j+1}) = 0$ and the claim follows by induction.
Claim 3. If $K$ is a proper subcontinuum of $I_{\omega_0}$ containing $1_{\omega_0}$ then there exists an integer $\beta$ so that if $\gamma > \beta$ then $\pi_\alpha(\Pi_\gamma(K)) = 1$ for all $\alpha$ so that $\beta < \alpha \leq \gamma$.

Proof. Suppose that there is a proper subcontinuum $K$ of $I_{\omega_0}$ for which the claim is not true. Then if $\beta$ is an integer there exists an integer $\gamma > \beta$ so that $\pi_\gamma(\Pi_\gamma(K))$ is nondegenerate. Suppose in addition that for each $\beta$ there is a $\gamma > \beta$ so that $\pi_\gamma(\Pi_\gamma(K)) = 0$. Then by Claim 2 since $1 \in \pi_\alpha(\Pi_\gamma(K))$ for all $\alpha < \gamma$ it follows that $I_{\gamma-1} \subset \Pi_\gamma(K)$. But then $K = I_{\omega_0}$ which is a contradiction. So the supposition is false and there exists an integer $\beta$ so that if $\gamma > \beta$ then $0 \in \pi_\alpha(\Pi_\alpha(K))$ for all $\alpha$ such that $\beta < \alpha < \gamma$.

Suppose $\beta > b$, where $b$ is defined above. Then from the negation of the claim, for each positive integer $n$ there is an integer $\gamma_n$ with $\beta + n < \gamma_n$ so that $\pi_{\gamma_n}(\Pi_{\gamma_n}(K))$ is nondegenerate. But then $(a_{\gamma_n-n-1}, 1) \in \Pi_\gamma(K)$. So $a_{\gamma_n} \in \Pi_{\gamma_n}(K)$ (by P3), thus if $\gamma_n = \beta + k_n$ for some positive integer $k_n > n$ then $a_{\beta + k_n} \in \Pi_\gamma(K)$ and thus $a_{\beta + k_n}^\beta \in \Pi_{\gamma_n}(K)$ (by P3). So there is an unbounded sequence in $(k_n)_{n=1}^\infty$ so that $a_{\beta + k_n}^\beta \in \Pi_{\beta}(K)$, but $\alpha$ is the sequential limit of $(a_n)_{n=1}^\infty$ and hence is a limit point of the set $(a_{\beta + k_n} | n$ is a positive integer), so $\alpha \in \Pi_{\beta}(K)$. Now $\pi_\alpha(\alpha) = 0$ for all $\alpha > 1$ so $0 \in \pi_\alpha(\Pi_{\beta}(K))$ for all $0 < \alpha < \beta$ which contradicts the choice of $\beta > b$. So the claim has been established.

Claim 4. $\text{Cmps}(I_{\omega_0}, 1_{\omega_0}) = \bigcup_{i=1}^{\infty} A_i^{\omega_0}$.

Proof. Suppose $x \in \text{Cmps}(I_{\omega_0}, 1_{\omega_0})$. By Claim 3 there exists an integer $\beta$ so that if $\gamma > \beta$ and $\alpha$ is an integer so that $\beta < \alpha \leq \gamma$ then $\pi_\alpha(x) = 1$. Let $\gamma > \beta$, then $x \in A_\gamma^\beta$ (by P4). Thus $x \in r_\alpha^\gamma(A_\gamma^\beta)$ and $r_\alpha^\gamma(A_\gamma^\beta) = \pi_\alpha(A_\gamma^\beta)$. So $x \in A_\gamma^\beta$. So Claim 4 has been established.

The construction of $I_\mu$ for $\mu$ an ordinal greater than $\omega_0$ follows. Suppose $\delta$ is a limit ordinal and that $(A_i)_{i=1}^{\infty}$, $(a_i)_{i=1}^{\infty}$, $C_2$, $r_\beta^\gamma$, and $I_i$ have been defined for all $\lambda \leq \delta$ so that:

1. For each positive integer $i$ the continuum $A_i^\lambda$ is irreducible from $a_i^\lambda$ to $1_i$.
2. $C_2 = \bigcup_{i=1}^{\infty} A_i^\lambda$.
3. If $\beta < \lambda$ then $r_\beta^\gamma(A_\beta^\lambda) = A_\beta^\lambda$, $r_\beta^\gamma(a_\beta^\lambda) = a_\beta^\lambda$, and $(a_i)_{i=1}^{\infty}$ converges to $\alpha$.
4. If $\beta < \lambda$ then $r_\beta^\gamma(I_\lambda - I_\beta) = C_\beta$.
5. $C_2 = \text{Cmps}(I_\delta, 1_\delta) = \{P |$ there exists a $\beta < \delta$ such that $\pi_\delta(P) = 1$ for all $\gamma > \beta\}$.

Then construct $I_{\delta+n}$ for all positive integers $n$ by substituting $I_\delta$ for $I_0$, $A_i^\delta$ for $A_i^0$, $a_i^\delta$ for $a_i^0$, and $1_\delta$ for $1_0$ in the construction of $I_\delta$ above. Compare condition 4 with a similar condition in Bellamy [1].
Suppose that $\mu$ is a limit ordinal and $I_\gamma$ has been defined for all $\gamma < \mu$. Let $I_\mu = \lim \{I_\gamma, \gamma\}_{\gamma < \mu}$, $A_\mu^\gamma = \lim \{a_\gamma, r\}_{\gamma < \mu}$, and for each $\beta < \mu$ let $r_\beta^\mu$ be the projection of $I_\mu$ onto $I_\beta$. As above identify $I_\gamma$ with $\lim \{I_\gamma, r\}_{\gamma < \alpha < \mu}$ and $a$ with $\bar{a}$. The argument of Claim 1 can be used to prove that $I_\mu$ is indecomposable. Claim 2 also generalizes for $I_\mu$ as follows:

Claim 5. If $x \in I_\mu$ and there is an ordinal $j < \mu$ which is not a limit ordinal such that $\pi_j(x_j) = 0$ then $\pi_i(x_i) = 0$ for all ordinals $i, j < i < \mu$, which are not limit ordinals; and hence $x \in I_j$.

Proof. Suppose $x \in I_\mu$ and $j = \lambda + q$ for some limit ordinal $\lambda$ and positive integer $q$. If $\alpha = \lambda' + r$ for some limit ordinal $\lambda' \geq \lambda$ with $\lambda' + r > \lambda$ and $r > 0$ and it is true that $\pi_\alpha(x_\alpha) \neq 0$, then there exists an integer $n$ so that $x_\alpha \in A_n^\alpha$. But $r_\alpha^\mu(A_n^\alpha) = A_n^\mu$ and either $\pi_j(A_n^\mu) = [1/(2n - 1), 1]$ or $\pi_j(A_n^\mu) = 1$ (by P4). In either case $\pi_j(x_\alpha) \neq 0$. But $\pi_j(x_j) = \pi_j(x_\alpha)$, so that $\pi_j(x_j) \neq 0$, which is a contradiction.

Claims 6, 7, and 8 are concerned with the continuum $I_\mu$.

Claim 6. If $K$ is a subcontinuum of $I_\mu$ and $a \in \Pi_1(K)$ then $a \in K$.

Proof. If $a \in \Pi_1(K)$ then $(a, 0) \in \Pi_\mu(K)$ so $a \in \Pi_\mu(K)$. From Claim 5 it follows that $a \in \Pi_\gamma(K)$ for all $\gamma < \mu$ since $a$ is identified with $a \times \{0\}$. Thus $a$ must belong to $K$.

Claim 7.

\[ \text{Cmps}(I_\mu, 1_\mu) = \bigcup_{i=1}^{\infty} A_i^\mu. \]

Proof. Suppose that $K$ is a proper subcontinuum of $I_\mu$ containing $1_\mu$. If it is true that there is an integer $n$ so that if $\gamma < \mu$ then $a_n^\gamma \in \Pi_\gamma(K)$, then it would follow that $\Pi_\gamma(K) \subset A_n^\gamma$ for all $\gamma < \mu$, and so $K \subset A_n^\gamma$. So suppose that this is not true. Thus for each integer $n$ there exists an ordinal $\gamma_n < \mu$ such that $a_n^{\gamma_n} \in \Pi_\gamma(K)$. But then $a_n^{\gamma_n} \in \Pi_1(K)$ for all $n$, since $r_\gamma^\mu(a_n^{\gamma_n}) = a_n^{\gamma_n}$. So $a \in \Pi_1(K)$ and $a \in K$ by Claim 6. But then $K = I_\mu$ since $I_\mu$ is irreducible from $a$ to $1_\mu$. So the claim is true.

Claim 8. $I_\mu$ satisfies the following for each ordinal $\beta, \beta < \mu$, and each positive integer $i$:

1. $A_i^\mu$ is irreducible from $a_i^\mu$ to $1_\mu$.
2. $C = \text{Cmps}(I_\mu, 1_\mu) = \bigcup_{i=1}^{\infty} A_i^\mu$.
3. $r_\beta^\mu(A_i^\mu) = A_i^\mu$, $r_\beta^\mu(a_i^\gamma) = a_i^\beta$, and $\{a_i^{\gamma_n}\}_{n=1}^{\infty}$ converges to $a$.
\[
(4) \quad r^\omega_\beta(I_\alpha - I_\beta) = C_\beta.
\]

**Proof.** Part (1) follows from the irreducibility of \( \prod_I(A^\omega) \) for each \( \gamma < \mu \), and part (2) follows from Claim 7. Since for each ordinal \( \gamma < \mu \) the sequence \( (\prod_I(A^\omega))^\omega_{\gamma+1} \) converges to \( a \), it follows that \( \{a^\omega_i\}_{i=1}^\omega \) converges to \( a \) which is identified with \( a \). The rest of (3) follows from the definitions of \( r^\omega_\beta, A^\omega_\beta, \) and \( A^\omega_\beta \). To prove (4) suppose that \( x \in I_\alpha - I_\beta \). Then by Claim 5, \( \pi_{\beta+1}(x_{\beta+1}) \neq 0 \) so \( x_{\beta+1} \in A^\beta_{\beta+1} \) for some integer \( n \), but \( r^\beta_{\beta+1}(A^\beta_{\beta+1}) \subseteq C_\beta \), thus \( r^\beta_{\beta+1}(x_{\beta+1}) \in C_\beta \) so \( r^\omega_\beta(x) \in C_\beta \); equality follows from parts (2) and (3).

**Claim 9.** The continuum \( I_{\omega_1} = \lim_{\leftarrow} \{I_\alpha, r_\beta\}_\omega \) has exactly two composants.

**Proof.** From the construction, \( \{I_\alpha\}_\omega \) is a monotonic collection of continua. (a) If \( \beta > \gamma \) then \( I_\beta \) does not intersect \( C_\gamma \) because \( I_\gamma \) does not intersect \( C_\gamma+1 \) and if \( \beta > \gamma \), \( C_\gamma+1 = r^{\gamma+1}_\beta(C_\beta) \). (b) From (4) of Claim 8 it follows that \( r^\omega_\beta(I_\alpha - I_\beta) = C_\beta \) for \( \alpha > \beta \). Let \( W = \{x\} \) there is a \( \gamma \) so that if \( \alpha > \gamma \) then \( \pi_{\alpha}(x) = 0 \). If \( x \in W \) and \( \gamma \) is the ordinal specified in the definition of \( W \) then \( x \in I_\gamma \). So \( x \) lies in the same composant as \( a \).

Now \( I_{\omega_1} \) is irreducible from \( a \) to \( 1_{\omega_1} \), it will now be shown that if \( y \) is a point of \( I_{\omega_1} \) not in \( W \) then \( y \) lies in \( \text{Cmps}(I_{\omega_1}, 1_{\omega_1}) \). Suppose \( y \in W \). The following two conditions need to be established: (i) if \( \alpha > \beta \) then \( y_\alpha \notin I_\beta \), and (ii) \( y_\alpha \in C_\alpha \). If \( \alpha > \beta \) there exists an ordinal \( \delta > \alpha \) such that \( y_\alpha \neq y_\beta \) or else \( y \in W \) (in particular \( y \in I_\omega \)). Suppose that \( y_\alpha \in I_\beta \), then \( y_\alpha \in C_\alpha \) by (a) above. But \( r^\omega_\alpha(I_\alpha - I_\alpha) \subseteq C_\alpha \) so \( y_\alpha \notin I_\alpha - I_\alpha \), so \( y_\alpha \notin I_\alpha \). But \( r^\omega_\alpha I_\alpha \) is the identity which contradicts the fact that \( y_\alpha \neq y_\alpha \). Thus (i) has been shown, also it has been shown that if \( \alpha > \beta \) then there exists a \( \delta > \alpha \) such that \( y_\delta \notin I_\alpha \). So \( y_\delta \in I_\delta - I_\alpha \), \( r^\omega_\delta(I_\delta - I_\alpha) \subseteq C_\alpha \), and so (ii) has been shown.

Suppose that \( y \in W \). By (i) if \( \alpha > 1 \) then \( y_\alpha \notin I_1 \), and by (ii) \( y_\alpha \in C_\alpha \). Thus by (2) of Claim 8 there exists an integer \( n_\alpha \) so that \( y_\alpha \notin A^n_\omega \). There exists an uncountable subset \( J \) of \( \omega_1 \) and an integer \( n \) so that \( n_\alpha = n \) for all \( \alpha \in J \). But since \( r^\omega_\beta(A^n_\omega) = A^n_\beta \) it follows that \( y \in \lim_{\leftarrow} \{A^n_\omega, r_\beta\}_\omega \) which is a proper subcontinuum of \( I \) containing \( 1_{\omega_1} \). Thus it has been shown that if \( y \in W \) then \( y \in \text{Cmps}(I_{\omega_1}, 1_{\omega_1}) \). So \( I_{\omega_1} \) has exactly two composants \( W \) and \( C_{\omega_1} \).

One can see that \( X \) is a retract of each \( I_\alpha \) and hence of \( I_{\omega_1} \). In order to construct a continuum with only one composant which has \( X \) as a retract it is only necessary to construct \( I_\alpha \) and a retraction \( r \) from \( I_0 \) onto \( X \) that maps \( 1_0 \) onto \( a \), then by identifying \( a \) and the point \( 1_{\omega_1} \) the continuum \( I_{\omega_1} \) satisfies the desired condition.
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