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Let H denote a Hilbert space, and let $B(H)$ denote the algebra of all bounded linear operators on H. In this note, an intrinsic characterization of those Banach subalgebras of $B(H)$ which are similar to approximately finite C^*-subalgebras of $B(H)$ is obtained.

This can be viewed as a noncommutative analog of theorems of Mackey ([9], p. 131) and Wermer ([10], Theorem 1). These authors gave conditions on certain families of idempotents $\{E_\alpha\}_{\alpha \in A}$ in $B(H)$ which insured the existence of an invertible T in $B(H)$ such that $TE_\alpha T^{-1}$ is a projection for all α in A. The main idea of the present paper involves finding conditions on certain families of matrix units $\{e(i, j)\}$ in $B(H)$ which guarantee the existence of an invertible T in $B(H)$ for which $\{Te(i, j)T^{-1}\}$ spans a C^*-algebra. This technique also has interesting applications to the orthogonalization of continuous representations of C^*-algebras (cf. [11]).

2. Preliminary definitions and lemmas. We begin by recalling the definition of an approximately finite C^*-algebra. A C^*-algebra $\mathscr{C}$ is *approximately finite* if there is an increasing sequence of finite dimensional C^*-subalgebras of $\mathscr{C}$ whose union is norm dense in $\mathscr{C}$. These algebras were defined and studied by Ola Bratteli in 1972 ([1]) as a generalization of the UHF algebras of Glimm ([7]), and have become popular objects of study among C^*-algebra enthusiasts (cf. [2], [3], [4], [5], and [8]).

The definition of approximate finiteness can be extended slightly to the context of Banach algebras as follows:

DEFINITION 2.1. A Banach algebra $\mathscr{A}$ is *approximately finite* if there is an increasing sequence $\{\mathscr{A}_n\}_{n=1}^\infty$ of finite dimensional, semisimple subalgebras such that $\mathscr{A} = (\bigcup_n \mathscr{A}_n)^-$, where $^-$ denotes norm closure.

Note that the most natural definition of approximate finiteness for Banach algebras would not include the hypothesis of semisimplicity on the $\mathscr{A}_n$'s. It is included here primarily to simplify the statement of Theorem 3.1 below.

Consider then an approximately finite Banach algebra $\mathscr{A} = (\bigcup_n \mathscr{A}_n)^-$. Since each $\mathscr{A}_n$ is by definition finite dimensional and semisimple, it has a Wedderburn decomposition

$$\mathscr{A}_n = \dot{+}\{\mathscr{A}_k^{(n)}: k = 1, \cdots, r_n\} ,$$

where $\mathscr{A}_k^{(n)}$ is isomorphic to the full complex matrix algebra $M_{[n,k]}$ of order $[n, k]$. (This notation is the same as [1].) One may hence select matrix units $\{e_k^{(n)}(i, j): i, j=1, \cdots, [n, k]\{$ for each $\mathscr{A}_k^{(n)}$ for which

$$\mathscr{A}_k^{(n)} = \text{linear span of } \{e_k^{(n)}(i, j): i, j = 1, \cdots, [n, k]\} .$$

Now $\mathscr{A}_n \subseteqq \mathscr{A}_{n+1}$, and the selection of matrix units $\{e_k^{(n)}(i, j)\}$ can be made to reflect this inclusion. This is accomplished in the following proposition, whose proof, left to the reader, is a straightforward modification of the proof of Proposition 1.7 of [1].

PROPOSITION 2.2. *Let $\mathscr{A}_1$ and $\mathscr{A}_2$ be finite dimensional, semisimple algebras, with Wedderburn decompositions*

$$\mathscr{A}_i = \dot{+}\{\mathscr{A}_k^{(i)}, k = 1, \cdots, n_i\} , \quad i = 1, 2 ,$$
$$\mathscr{A}_k^{(i)} \cong M_{[i,k]} , \quad i = 1, 2 .$$

Let $\{e_l^{(1)}(i, j): i, j = 1, \cdots, [1, l]\}$, $l = 1, \cdots, n_1$, be matrix units for $\mathscr{A}_1$. If $\mathscr{A}_1 \subseteqq \mathscr{A}_2$, then there exists unique nonnegative integers n_{ki}, $k = 1, \cdots, n_2$, $i = 1, \cdots, n_1$, and matrix units $\{e_k^{(2)}(i, j): i, j = 1, \cdots, [2, k]\}$, $k = 1, \cdots, n_2$ for $\mathscr{A}_2$ such that

$$\sum_{p=1}^{n_1} n_{kp}[1, p] \leqq [2, k]$$

and

$$e_l^{(1)}(i, j) = \sum_{q=1}^{n_2} \sum_{m=1}^{n_{ql}} e_q^{(2)}\Big(\sum_{p=1}^{l-1} n_{qp}[1, p] + (m-1)n_{ql} + i ,$$
$$\sum_{p=1}^{l-1} n_{qp}[1, p] + (m-1)n_{ql} + j\Big) . \tag{2.1}$$

The matrix units for $\mathscr{A}_n$ are now chosen inductively by applying Proposition 2.2 at the nth inclusion, so that for each n the matrix units for $\mathscr{A}_{n+1}$ satisfy (2.1) relative to the matrix units for $\mathscr{A}_n$. Such a selection of matrix units will be called an *admissable selection of matrix units for $\mathscr{A}$.*

We turn now to the problem of orthogonalization of matrix units in $B(H)$ for a fixed Hilbert space H. Recall that a set of bounded operators $\{e(i, j): i, j = 1, \cdots, n\}$ on H is said to form a *system of matrix units on H* if

(i) $\sum_{i=1}^{n} e(i, i) =$ identity operator on H,

(ii) $e(i, j)e(k, l) = \delta_{jk} \cdot e(i, l)$, $i, j, k, l = 1, \cdots, n$,

where δ_{jk} denotes the Kronecker delta. $\{e(i, j): i, j = 1, \cdots, n\}$ is said to form a *C^*-system of matrix units* if in addition to (i) and (ii), one has

(iii) $e(i, j)^* = e(j, i),\ i, j = 1, \cdots, n.$

DEFINITION 2.3. Let $\{e(i, j): i, j = 1, \cdots, n\}$ be a system of matrix units on H. An invertible operator T on H is said to *orthogonalize* $\{e(i, j): i, j = 1, \cdots, n\}$ if

$$(Te(i, j)T^{-1})^* = Te(j, i)T^{-1},\ i, j = 1, \cdots, n\ ,$$

i.e., if $\{Te(i, j)T^{-1}: i, j = 1, \cdots, n\}$ is a C^*-system of matrix units on H.

LEMMA 2.4. *Let $\{e(i, j): i, j = 1, \cdots, n\}$ be a system of matrix units on H. Then there exists an invertible operator T on H which orthogonalizes $\{e(i, j): i, j = 1, \cdots, n\}$.*

Proof. Set

$$T = \Big(\sum_{1 \leqq i, j \leqq n} e(i, j)^* e(i, j)\Big)^{1/2} .$$

We claim that T is invertible. For x in H,

$$\begin{aligned} \|Tx\|^2 &= \sum_{1 \leqq i, j \leqq n} \|e(i, j)x\|^2 \\ &\geqq \sum_{i=1}^{n} \|e(i, i)x\|^2 \\ &\geqq n^{-2}\Big(\sum_{i=1}^{n} \|e(i, i)x\|\Big)^2 \\ &\geqq n^{-2}\|x\|^2 , \end{aligned}$$

since $x = \sum_i e(i, i)x$; T is thus bounded below. By a theorem of T. Crimmins ([6], Theorem 2.2),

$$\begin{aligned} \text{range of } T &= \text{range of } \Big(\sum_{1 \leqq i, j \leqq n} e(i, j)^* e(i, j)\Big)^{1/2} \\ &= \sum_{1 \leqq i, j \leqq n} \text{range of } e(i, j)^* . \end{aligned} \tag{2.2}$$

Since $\sum_i e(i, i)^* = I$, $H = \sum_i$ range of $e(i, i)^*$, and therefore by (2.2), T is surjective. T is hence invertible.

Let k and l be fixed positive integers between 1 and n. Then

$$\begin{aligned} T^2 e(k, l) &= \sum_{i, j} (e(i, j)^* e(i, j)) e(k, l) \\ &= \sum_{j=1}^{n} e(j, k)^* e(j, l) , \end{aligned}$$

so that if $f(k, l) = Te(k, l)T^{-1}$,

$$f(k, l) = T^{-1}\Big(\sum_{j=1}^{n} e(j, k)^* e(j, l)\Big)T^{-1} . \tag{2.3}$$

Since T is positive, (2.3) yields

$$\begin{aligned} f(k, l)^* &= (Te(k, l)T^{-1})^* \\ &= T^{-1}\Big(\sum_{j=1}^{n} e(j, l)^* e(j, k)\Big)T^{-1} \\ &= f(l, k) . \end{aligned}$$

The next lemma is the basic orthogonalization lemma of Mackey (see [9], p. 135).

LEMMA 2.5. *Let $\{E_1, \cdots, E_n\}$ be a pairwise independent set of idempotents in $B(H)$* (i.e., $E_i^2 = E_i$, $E_iE_j = 0$, $i \neq j$) *such that $\sum_{i=1}^{n} E_i = I$, and let $M > 0$ be such that for every set $\{\varepsilon_1, \cdots, \varepsilon_n\}$ on zero's and one's,*

$$\Big\| \sum_{i=1}^{n} \varepsilon_i E_i \Big\| \leqq M .$$

Then for all x in H,

$$\frac{\|x\|^2}{4M^2} \leqq \sum_{i=1}^{n} \|E_i x\|^2 \leqq 4M^2\|x\|^2 .$$

We now extend Lemma 2.5 to matrix units:

LEMMA 2.6. *Let $\{e(i, j): i, j = 1, \cdots, n\}$ be a system of matrix units on H such that each $e(i, i)$ is a projection. Let $M > 0$ be such that $\|e(i, j)\| \leqq M$, $i, j = 1, \cdots, n$. Then for all x in H,*

$$\frac{n}{M^2}\|x\|^2 \leqq \sum_{1 \leqq i, j \leqq n} \|e(i, j)x\|^2 \leqq nM^2\|x\|^2 .$$

Proof. $\{e(i, i): i = 1, \cdots, n\}$ is a set of pairwise orthogonal projections with sum I, so if $\mathscr{M}_i =$ range of $e(i, i)$, $i = 1, \cdots, n$, then $H = \mathscr{M}_1 \oplus \cdots \oplus \mathscr{M}_n$.

Since $e(i, j)e(k, k) = 0$, $k \neq j$, $e(i, j)e(j, i) = e(i, i)$, and $e(i, j)e(i, j) = e(i, j)$, we have

$$\text{(2.4)} \qquad \text{kernel of } e(i, j) \supseteqq \bigoplus_{\substack{1 \leqq k \leqq n \\ k \neq j}} \mathscr{M}_k$$

$$\text{(2.5)} \qquad \text{range of } e(i, j) = \mathscr{M}_i .$$

Suppose $e(i, j)x = 0$, with $x = \bigoplus_{i=1}^{n} m_i$, $m_i \in \mathscr{M}_i$.

Then

$$\text{(2.6)} \qquad m_j = e(j, j)x = e(j, i)e(i, j)x = 0,\ i \neq j .$$

(2.4) and (2.6) imply

$$\text{(7)} \qquad \text{kernel of } e(i, j) = \bigoplus_{\substack{1 \leqq k \leqq n \\ k \neq j}} \mathscr{M}_k .$$

From (2.5) and (2.7), it follows that $e(i, j)$ maps $\mathscr{M}_j$ bijectively onto $\mathscr{M}_i$. Therefore if $e(i, j)$ is represented as an operator matrix relative to the decomposition $H = \mathscr{M}_1 \oplus \cdots \oplus \mathscr{M}_n$, then there exists an invertible linear transformation $T_{ij}: \mathscr{M}_j \to \mathscr{M}_i$ such that

(2.8) $e(i, j)$ has a matrix with T_{ij} in the (i, j)th position and zeros elsewhere.

Set

$$T = \sum_{1 \leq i\ j \leq n} e(i, j)^* e(i, j) .$$

From (2.8), we find that the operator matrix of $e(i, j)^* e(i, j)$ has $T_{ij}^* T_{ij}$ in the (j, j)th position and zeros elsewhere, so that T is the diagonal matrix

$$T = \begin{pmatrix} I_{\mathscr{M}_1} + A_1 & & & \\ & I_{\mathscr{M}_2} + A_2 & & \\ & & \ddots & \\ & & & I_{\mathscr{M}_n} + A_n \end{pmatrix} \tag{2.9}$$

where

$$A_k = \sum_{\substack{1 \leq i \leq n \\ i \neq k}} T_{ik}^* T_{ik}, \quad k = 1, \cdots, n .$$

Let $x \in H$, $x = \bigoplus_{i=1}^n x_i$, $x_i \in \mathscr{M}_i$. By (2.9),

$$\begin{aligned} \sum_{1 \leq i, j \leq n} \| e(i, j)x \|^2 &= (Tx, x) \\ &= \sum_{i=1}^n \Big(\| x_i \|^2 + \sum_{\substack{1 \leq j \leq n \\ j \neq i}} \| T_{ji} x_i \|^2 \Big) . \end{aligned} \tag{2.10}$$

Since $\| e(i, j) \| \leq M$, $i, j = 1, \cdots, n$,

$$\| T_{ij} \| = \| e(i, j) \| \leq M, \ i, j = 1, \cdots, n . \tag{2.11}$$

Also $e(i, j)e(j, i) = e(i, i)$ implies $T_{ij} T_{ji} = I_{\mathscr{M}_i}$, so that by (2.11), for x in $\mathscr{M}_j$,

$$\begin{aligned} \| T_{ij} x \|^2 &= \| T_{ji}^{-1} x \|^2 \\ &\geq \frac{\| x \|^2}{\| T_{ji} \|^2} \\ &\geq \frac{\| x \|^2}{M^2} . \end{aligned} \tag{2.12}$$

Therefore by (2.10) and (2.12),

$$\sum_{1 \leqq i, j \leqq n} \|e(i, j)x\|^2 \geqq \sum_{i=1}^{n} \Big(\|x_i\|^2 + \frac{1}{M^2} \sum_{\substack{1 \leqq j \leqq n \\ i \neq i}} \|x_i\|^2 \Big)$$
$$\geqq nM^{-2} \sum_{i=1}^{n} \|x_i\|^2$$
$$= nM^{-2} \|x\|^2 .$$

By (2.10) and (2.11),

$$\sum_{1 \leqq i, j \leqq n} \|e(i, j)x\|^2 \leqq \sum_{i=1}^{n} \Big(\|x_i\|^2 + M^2 \sum_{\substack{1 \leqq j \leqq n \\ j \neq i}} \|x_i\|^2 \Big)$$
$$\leqq nM^2 \sum_{i=1}^{n} \|x_i\|^2$$
$$= nM^2 \|x\|^2 .$$

LEMMA 2.7. *Let $\{E_i\}_{i=1}^{n}$ be a pairwise independent set of idempotents in $B(H)$ and $M > 0$ a constant satisfying the hypotheses of Lemma 2.5. Then there exists an invertible T in $B(H)$ such that TE_iT^{-1} is self-adjoint, $i = 1, \cdots, n$, and $\|T\|^{\pm 1} \leqq 2M$.*

Proof. The proof is similar to the proof of Lemma 2.4. Set

$$T = \Big(\sum_{i=1}^{n} E_i^* E_i \Big)^{1/2} .$$

Then for x in H, Lemma 2.5 gives

$$\frac{\|x\|^2}{4M^2} \leqq \|Tx\|^2 = \sum_{i=1}^{n} \|E_i x\|^2 \leqq 4M^2 \|x\|^2 ,$$

whence $\|T^{\pm 1}\| < 2M$. One shows that $(TE_iT^{-1})^* = TE_iT^{-1}$, $i = 1, \cdots, n$, as before.

3. The theorem.

The following theorem can now be stated and proved.

THEOREM 3.1. *Let $\mathscr{A}$ be a Banach subalgebra of $B(H)$. Then $\mathscr{A}$ is similar to an approximately finite C^*-subalgebra of $B(H)$ if and only if $\mathscr{A}$ is approximately finite and the following condition holds: there is an admissable selection of matrix units $\{e_k^{(n)}(i, j): i, j = 1, \cdots, [k, n]\}$, $k = 1, \cdots, r_n$ for $\mathscr{A}$ and a constant $M > 0$ such that:*

(i) *for each fixed n and for all sets $\{\delta_i^{(k)}: i = 1, \cdots, [n, k]\}$ of zero's and one's,*

$$\Big\| \sum_{k=1}^{r_n} \sum_{i=1}^{[n,k]} \delta_i^{(k)} e_k^{(n)}(i, i) \Big\| \leqq M ,$$

(ii) *for each k and n,*

$$\|e_k^{(n)}(i, j)\| \leqq M,\ i, j = 1, \cdots, [n, k] .$$

Moreover, if these conditions are met, an invertible operator implementing the similarity can be chosen in the von Neumann algebra generated by $\mathscr{A}$.

Proof. ($\Rightarrow$). Suppose $\mathscr{C} = T\mathscr{A}T^{-1}$ is an approximately finite C^*-algebra for some invertible T in $B(H)$. Thus, $\mathscr{C} = (\bigcup_n \mathscr{C}_n)^-$, where $\{\mathscr{C}_n\}$ is an ascending sequence of finite dimensional C^*-subalgebras. By Proposition 1.7 of [1], there is an admissable selection of C^*-matrix units $\{f_k^{(n)}(i, j): i, j = 1, \cdots, [n, k]\}, k = 1, \cdots, r_n$ for $\mathscr{C}$ relative to $\{\mathscr{C}_n\}$. If $\mathscr{A}_n = T^{-1}\mathscr{C}_n T$, then $\{\mathscr{A}_n\}$ is an increasing sequence of finite dimensional, semisimple subalgebras of $\mathscr{A}$ such that $\mathscr{A} = (\bigcup_n \mathscr{A}_n)^-$, so that $\mathscr{A}$ is approximately finite, and if $e_k^{(n)}(i, j) = T^{-1}f_k^{(n)}(i, j)T$, then $\{e_k^{(n)}(i, j): i, j=1, \cdots, [n, k]\}$, $k=1, \cdots, r_n$ is an admissable selection of matrix units for $\mathscr{A}$.

For each positive integer n, let $\operatorname{diag}(\lambda_1, \cdots, \lambda_n)$ denote the $n \times n$ diagonal matrix with main diagonal $\{\lambda_1, \cdots, \lambda_n\}$. Let $\{\delta_i^{(k)}: i = 1, \cdots, [n, k]\}$, $k = 1, \cdots, r_n$ be sets of zero's and one's. Let

$$A = \sum_{k=1}^{r_n} \sum_{i=1}^{[n,k]} \delta_i^{(k)} e_k^{(n)}(i, i) .$$

Then

$$\begin{aligned} \|TAT^{-1}\| &= \Big\| \bigoplus_{k=1}^{r_n} \bigoplus_{i=1}^{[n,k]} \delta_i^{(k)} f_k^{(n)}(i, i) \Big\| \\ &= \max_{1 \leqq k \leqq r_n} \|\operatorname{diag}(\delta_1^{(k)}, \cdots, \delta_{[n,k]}^{(k)})\| \\ &\leqq 1 . \end{aligned}$$

It therefore follows that (i) obtains with $M = \|T\|\,\|T^{-1}\|$. (ii) follows on noticing that $\|f_k^{(n)}(i, j)\| = 1$ for all i, j, k, and n.

($\Leftarrow$). It will first be shown that there exists an invertible T in the von Neumann algebra generated by $\mathscr{A}$ such that $Te_k^{(n)}(i, i)T^{-1}$ is self-adjoint for all i, k, and n. Set

$$F_n = \sum_{k=1}^{r_n} \sum_{i=1}^{[n,k]} e_k^{(n)}(i, i) .$$

Then if $E_n = I - F_n$, $\mathscr{E}_n = \{E_n\} \cup \{e_k^{(n)}(i, i): i = 1, \cdots, [n, k], k = 1, \cdots, r_n\}$ is a pairwise independent set of idempotents in $B(H)$ with sum I. It follows by (i) that $\mathscr{E}_n$ satisfies the hypotheses of Lemma 2.7 with constant $2M + 1$. By that lemma, an invertible T_n in the C^*-algebra generated by $\mathscr{A}$ and I may hence be chosen such that $T_n e_k^{(n)}(i, i)T_n^{-1}$ is self-adjoint for $i = 1, \cdots, [n, k]$, $k = 1, \cdots, r_n$, and such that

$$(3.1)\qquad \|T_n^{\pm 1}\| \leqq 2(2M+1)\,, \qquad n = 1, 2, \cdots .$$

Since the selection of matrix units is admissable, for each fixed i, k, and n, $e_k^{(n)}(i, i)$ is a sum of a subfamily of idempotents $e_k^{(n+1)}(j, j)$. It follows that

$$(3.2)\qquad T_m e_k^{(n)}(i, i) T_m^{-1} \text{ is self-adjoint, for all } m \geqq n .$$

Since closed balls are compact in the weak operator topology on $B(H)$, (3.1) implies the existence of a subsequence $\{n_k\}$ and an invertible T in the von Neumann algebra generated by $\mathscr{A}$ for which

$$(3.3)\qquad T_{n_k}^2 \longrightarrow T^2(WOT)\,, \quad k \longrightarrow \infty .$$

Since each T_n is positive, we may assume T is positive.

Now fix i, k, and n. By (3.2), for x, y in H,

$$(3.4)\qquad (T_m e_k^{(n)}(i, i)x, T_m y) = (T_m x, T_m e_k^{(n)}(i, i)y)\,, \quad m \geqq n .$$

The self-adjointness of T and each T_n together with (3.3) and (3.4) hence yield

$$(Te_k^{(n)}(i, i)x, Ty) = (Tx, Te_k^{(n)}(i, i)y)\,,$$

i.e., $Te_k^{(n)}(i, i)T^{-1}$ is self-adjoint. There is therefore no loss of generality in assuming that $e_k^{(n)}(i, i)$ is a projection for each i, k, and n.

We have $\mathscr{A} = (\bigcup_n \mathscr{A}_n)^-$, where

$$\mathscr{A}_n = \bigoplus \{\mathscr{A}_k^{(n)}: k = 1, \cdots, r_n\}\,,$$
$$\mathscr{A}_k^{(n)} = \bigvee \{e_k^{(n)}(i, j): i, j = 1, \cdots, [n, k]\} .$$

Set

$$P_k^{(n)} = \bigoplus \{e_k^{(n)}(i, i): i = 1, \cdots, [n, k]\}\,,$$
$$P_n = \bigoplus \{P_k^{(n)}: k = 1, \cdots, r_n\}\,,$$
$$M_k^{(n)} = \text{range of } P_k^{(n)} .$$

For each k and n, $\{e_k^{(n)}(i, j): i, j = 1, \cdots, [n, k]\}$ can be considered as a system of matrix units in $B(M_k^{(n)})$. By (ii), $\{e_k^{(n)}(i, j)\}$ satisfies the hypotheses of Lemma 2.6, so if

$$T_k^{(n)} = \frac{1}{[k, n]^{1/2}}\Big(\sum_{1\leqq i, j\leqq [n,k]} e_k^{(n)}(i, j)^* e_k^{(n)}(i, j)\Big)^{1/2}\Big)\,,$$

then by that lemma,

$$(3.5)\qquad \frac{\|x\|^2}{M^2} \leqq \|T_k^{(n)}x\|^2 \leqq M^2\|x\|^2\,, \quad x \in M_k^{(n)} .$$

Now $H = (\text{range of } P_n)^\perp \oplus (\bigoplus \{M_k^{(n)}: k = 1, \cdots, r_n\})$, and therefore if

$$T_n = (I - P_n) \oplus (\oplus \{T_k^{(n)}: k = 1, \cdots, r_n\}) ,$$

then by (3.5)

$$\frac{\|x\|^2}{M^2} \leqq \| T_n x \|^2 \leqq M^2 \|x\|^2, x \in H, n = 1, 2, \cdots . \tag{3.6}$$

The proof of Lemma 2.4 shows that T_n orthogonalizes $\{e_k^{(n)}(i, j): i, j = 1, \cdots, [n, k], k = 1, \cdots, r_n\}$. Since the selection of matrix units is admissable, each $e_k^{(n)}(i, j)$ is a sum of the form (2.1) of a subfamily of matrix units of $\mathscr{A}_{n+1}$. (3.6) hence allows one to use the previous compactness argument to find a invertible operator T in the von Neumann algebra generated by $\mathscr{A}$ which orthogonalizes $e_k^{(n)}(i, j)$ for all i, j, k, and n. It follows that $T\mathscr{A}T^{-1}$ is an approximately finite C^*-subalgebra of $B(H)$.

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