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A construction is given which yields to any quasi-metrizable not non-archimedeanly quasi-metrizable space another quasi-metrizable space which is not σ-orthocompact. It is shown that (σ-)orthocompactness does not imply non-archimedean quasi-metrizability and is neither summable nor multiplicative nor (CH) hereditary in completely regular quasi-metric spaces.

It is proved that quasi-metric spaces are preserved under perfect mappings.

0. Let T be the completely regular quasi-metric space without any σ-interior preserving base presented in [8]. T has been invented to show that a sufficient condition for quasi-metrizability due to S. Nedev ([12]) and to P. Fletcher and W. F. Lindgren ([3]), namely the existence of a σ-interior preserving base, is not necessary. In §1 of the present paper some further analogs of well known metric theorems are proved to be false. A general construction on quasi-metric spaces is given, which when applied to the space T yields a (completely regular perfect subparacompact submetrizable) quasi-metric non-σ-orthocompact extention $T^{\vee}$, while T is shown to be hereditarily orthocompact. $T^{\vee}$ supples i.a. an answer to a question of P. Fletcher concerning the σ-orthocompactness of quasi-metric spaces [private communication].

It is shown further that (σ-)orthocompactness is neither multiplicative nor summable in completely regular quasi-metric spaces. In fact, the product of the space T with the Sorgenfrey line is shown to be non-σ-orthocompact, and $T^{\vee}$ is shown to be the union of an open set homeomorphic to T and a discrete set of cardinality of the continuum. Together with the continuum hypothesis the above construction provides an example of a regular Lindelöf quasi-metric space that is not hereditarily σ-orthocompact.

In §2[1] it is shown that a perfect image of quasi-metrizable space is quasi-metrizable, in analogy to the metric case. This result answers a question posed first by S. Nedev and M. M. Čoban ([13]). It is further proved that non-archimedean quasi-metric spaces are preserved under perfect mappings. In [13] S. Nedev and M. M. Čoban have proved the same result for γ-spaces. Hence each of the three increasing classes of spaces, namely non-archimedean quasi-metric spaces, quasimetric spaces and γ-spaces, is preserved under perfect mappings.

[1] The results of §2 had been included in [9].

All spaces below are T_1. D denotes the set $\{0\} \cup \{1/j\colon j = 1, 2, \cdots\}$.

1. A generalized metric d on a space X is a *quasi-metric* (*non-archimedean quasi-metric*) provided that always $d(x, z) \leqq d(x, y) + d(y, z)(d(x, z) \leqq \max\{d(x, y), d(y, z)\})$.

A collection α of subsets of a space X is *interior preserving* provided that $\operatorname{int} \cap \{A\colon A \in \alpha'\} = \bigcap \{\operatorname{int} A\colon A \in \alpha'\}$ for every $\alpha' \subset \alpha$, and it is *σ-interior preserving* provided that α is countable union of interior preserving collections. A space is non-archimedeanly quasi-metrizable iff it has a σ-interior preserving base ([8], [3]). A space is *(σ-)orthocompact* provided that every open cover has a (σ-) interior preserving open refinement ([14], [2]). A space X is *perfect*, provided that any open set of X is F_σ and *subparacompact* provided that every cover of X has a σ-discrete closed refinement. A space X is *submetrizable* provided that there exists a metrizable topology which is coarser than that of X.

The space T has the complex plane as its underlying set. A base of neighborhoods in a point $t \in T$ consists of the sets $C(t, r) = \{t\} \cup \{t'\colon |t' - (t + ri)| < r\}$, $r > 0$, i.e., $C(t, r)$ is an open circle with radius r together with its "southern pole" t. It is shown in [8] that the space T is submetrizable, quasi-metrizable via a quasi-metric which is continuous in the second variable, but not non-archimedeanly quasi-metrizable. Moreover, the same arguments as in [8] prove the following lemma.

LEMMA. *Let T_0 be a subset of the second category in the plane topology and let $\mathscr{C} = \{U(t)\colon t \in T_0\}$ be a collection of subsets of T such that for each $t \in T_0$, $U(t)$ is a T-neighborhood of t contained in $C(t, r_t)$. Then $\mathscr{C}$ is not σ-interior preserving in T and $\{U(t) \cap T_0\colon t \in T_0\}$ is not σ-interior preserving in the subspace T_0 of T.*

PROPOSITION 1. *T is a perfect, subparacompact and hereditarily orthocompact space*[2].

Proof. Let us show that any open cover ζ of an open set V has a closed σ-discrete refinement in T, so that T is perfect and subparacompact.

For each $G \in \zeta$ let G^0 denote the interior of G in the plane topology. Set $\zeta^0 = \{G^0\colon G \in \zeta\}$ and set $V' = \bigcup \zeta^0$. Note that ζ^0 has a closed σ-discrete refinement even in the plane topology. Set $F = V - V'$ and set $F_n = \{t \in F\colon C(t, 1/n) \subset G$ for some $G \in \zeta\}$. We have $F = \bigcup F_n$. Let us show that any F_n is σ-discrete.

For $t \in T$, $r > 0$ set $C^{-1}(t, r) = \{t'\colon t \in C(t', r)\}$. Note that $C^{-1}(t, r)$

[2] After this result was obtained, I learned from H. Junnila that he also has proved that the space T is orthocompact in a different way.

is an open circle with radius r together with its "northern pole" t. Set $S(t, r) = C(t, r) \cup C^{-1}(t, r)$. The space X with the plane as its underlying set with the basic neighborhoods $S(x, r)$ is semimetrizable ([6], [7]) and its topology is coarser than that of T yet finer than the plane topology. Now for any $t \in F_n$ we have $S(t, 1/n) \cap F_n = \{t\}$. Otherwise pick some $t' \in S(t, 1/n) \cap F_n$, $t' \neq t$. Then $t' \in C(t, 1/n)$ or $t' \in C^{-1}(t, 1/n)$ and $t \in C(t', 1/n)$. In the first case, for instance, $t' \in C(t, 1/n) - \{t\} \subset G^0 \in \zeta^0$, and $t' \in V'$ and this contradicts $t' \in F$. Hence for every $t \in F_n$ the trace of $S(t, 1/n)$ on F_n is $\{t\}$.

Since the open collection $\{S(t, 1/n): t \in F_n\}$ in the semi-metrizable space X has a closed σ-discrete refinement, F_n is a union of countably many sets that are closed and discrete in X, and hence in T.

We have proved that ζ has a σ-discrete closed refinement. Thus T is perfect and subparacompact. A perfect space is hereditarily orthocompact if it is σ-orthocompact ([2]). Let η be an open cover of T. For each $H \in \eta$ let H^0 denote the interior of H in the plane topology. Set $\eta^0 = \{H^0: H \in \eta\}$ and set $T_0 = \bigcup \eta^0$. Note η^0 has a σ-locally finite open refinement even in the plane topology. Let $E = T - T_0$, $E_n = \{t \in E: C(t, 1/n) \in H$ for some $H \in \eta\}$. We have $E = \bigcup_{n=1}^{\infty} E_n$. We shall construct for every E_n an open interior preserving collection which refines $\{C(t, 1/n): t \in E_n\}$ and covers E_n.

Let β be a base of the plane topology, $\beta = \bigcup_{k=1}^{\infty} \beta_k$, where β_k are point finite and for every $U \in \beta_k$ diam $U = \sup\{|t - t'|: t, t' \in U\} < 1/k$.

For $t \in E_n$, $t' \in C(t, 1/2n)$ let $k(t, t')$ be the smallest k such that there exists $U \in \beta_k$ with $t' \in U \subset C(t, 1/n)$, and let $U(t, t')$ be such a U. Let us note that if one has sequences $t_j \in E_n$, $t'_j \in C(t, 1/2n)$, $t_j \neq t'_j$, then (1) $k(t_j, t'_j) \to \infty \Rightarrow |t_j - t'_j| \to 0$.

We put $U(t) = \bigcup\{U(t, t'): t' \in C(t, 1/2n)\} \cup \{t\}$. Obviously, $U(t) \subset C(t, 1/n)$.

The collection $\{U(t): t \in E_n\}$ is interior preserving. Otherwise pick some $t_0 \in T$ such that $\bigcap\{U(t): t_0 \in U(t)\}$ is not a neighborhood of t_0. Since any $U(t)$ is an union of some $U(t, t') \in \beta_{k(t,t')}$ and any β_k is interior preserving, there exist sequences $t_j \in E_n$ and $t'_j \in C(t_j, 1/2n)$ such that (2) $t_0 \in U(t_j, t'_j)$ for any $j = 1, 2, \cdots$ and (3) $k(t_j, t'_j) \to \infty$. From (3) and (1) it follows that $|t_j - t'_j| \to 0$, from (2) and from the definitions of $U(t, t')$ and β_k it follows that $|t'_j - t_0| < \operatorname{diam} U(t_j, t'_j) < 1/k(t_j, t'_j) \to 0$, so $|t_0 - t_j| \to 0$ and since all $t_j \in E$ and E is closed in the plane topology we have $t_0 \in E$. However, for some t_j we have $t_0 \neq t_j$, and $t_0 \in U(t_j) - \{t_j\} \subset C(t_j, 1/n) - \{t_j\} \subset H^0 \in \eta^0$, hence $t_0 \in T_0$. Thus $t_0 \in T_0 \cap E$ — *a* contradiction.

We have proved that η has a σ-interior preserving refinement. Hence T is σ-orthocompact, and therefore, as mentioned above, it is hereditary orthocompact.

REMARK. By the same arguments it can be proved that the space X of H. W. Martin's Example 3 of [11] is orthocompact; this answers Question 1 of [11].

Let (X, d) be a quasi-metric space, $B(x, r)$ be a d-sphere, and set $X^{\vee} = X \times D$. We define a generalized metric $d^{\vee}$ on $X^{\vee}$ such that for $r \leqq 1$ the $d^{\vee}$-spheres $B^{\vee}(\langle x, 1/j\rangle, r) = B(x, r/j) \times \{1/j\}$ and $B^{\vee}(\langle x, 0\rangle, r) = \bigcup_{1/j<r} B^{\vee}(\langle x, 1/j\rangle, r) \cup \{\langle x, 0\rangle\}$. For $r > 1$ we put all $d^{\vee}$-spheres $B^{\vee}(\langle x, y\rangle, r) = X^{\vee}$. It follows that $d^{\vee}$ is a quasi-metric and that if X is Hausdorff that so is $X^{\vee}$.

THEOREM 1. (i) *$X^{\vee}$ is a union of countably many disjoint clopen subspaces homeomorphic to X and a discrete subspace of the same cardinality as that of X.*

(ii) *If d is continuous in the second variable then so is $d^{\vee}$.*

(iii) *If X is perfect (subparacompact, submetrizable) then so is $X^{\vee}$.*

(iv) *If X is not non-archimedeanly quasi-metrizable then $X^{\vee}$ is not σ-orthocompact.*[3]

Proof. (i) is obvious. (ii) follows from the following criterion due to R. Stoltenberg [15]: a quasi-metric d is continuous in the second variable iff for every $x \in X$, $0 < r < r'$ one has $\operatorname{cl} B(x, r) \subset B(x, r')$. (iii) The topology of the product of X with the metric space D is coarser than that of $X^{\vee}$, hence $X^{\vee}$ is submetrizable if X is. The rest follows from (i). (iv) Let ζ be a σ-interior preserving refinement of the open cover $\{B^{\vee}(\langle x, 0\rangle, 1): x \in X\}$, and let $\pi_j: X \to X^{\vee}$ be defined by $\pi_j(x) = \langle x, 1/j\rangle$. Then $\{\pi_j^{-1}G: G \in \zeta, j = 1, 2, \cdots\}$ is a σ-interior preserving open collection in X. It is also a base in X because if U is a neighborhood of $x \in X$, $B(x, 1/j) \subset U$ and $\langle x, 0\rangle \in G \in \zeta$, where $G \subset B^{\vee}(\langle x, 0\rangle, 1)$, then one has $x \in \pi_j^{-1}(G) \subset \pi_j^{-1}(B^{\vee}(\langle x, 0\rangle, 1)) = B(x, 1/j) \subset U$, $\pi_j^{-1}(G) \in \pi_j^{-1}(\zeta)$. This completes the proof.

The following proposition is a consequence of Proposition 1 and Theorem 1.

PROPOSITION 2. *The space $T^{\vee}$ is perfect, subparacompact, submetrizable, quasi-metrizable via a quasi-metric which is continuous in the second variable, but is not σ-orthocompact.*

The notion of neighbornet due to H. Junnila ([5]) helps to unify some definitions.

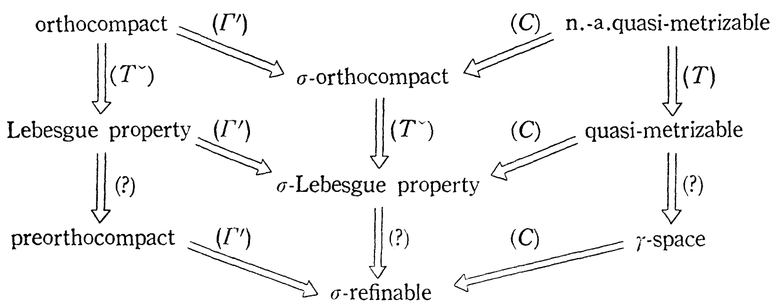
A reflexive binary relation U on a space X is a *neighbornet* provided that for any $x \in X$ the set $U(x)$ is a neighborhood of x. A sequence $\langle U_n\rangle$ of neighbornets is *basic* provided that for any

[3] Some constructions non-orthocompact *non-quasi-metrizable* spaces based on quite a different idea have been given in [4].

$x \in X$ the sequence $\langle U_n(x) \rangle$ is a base of neighborhoods of x ([5]). A neighbornet U (a sequence of neighbornets $\langle U_n \rangle$) *refines* a cover ζ of X provided that for any $x \in X$ there exists some $G \in \zeta$ such that $U(x) \subset G(U_n(x) \subset G$ for some n). A neighbornet U_0 is *double* (*normal*) provided that there exists another neighbornet U_1 (a sequence of neighbornets $U_1, U_2, \cdots$) such that $U_0 \supset U_1^2$ ($U_n \supset U_{n+1}^2$ any $n \geqq 0$). A space X is orthocompact (has the *Lebesque property*, is *pre-orthocompact*) iff for any open cover ζ of X there exists a transitive (normal, double) neighbornet which refines ζ, *countably orthocompact* (*countably preorthocompact*) iff for any countably open cover ζ of X there exists a transitive (double) neighbornet which refines X; σ-orthocompact (has the *σ-Lebesque property, is σ-refinable*) iff for any open cover ζ of X there exists a sequence of transitive (normal, double) neighbornets which refines ζ, and non-archimedeanly quasi-metrizable (quasi-metrizable, a *γ-space*) iff there exists a basic sequence transitive (normal, double) neighbornets ([5], [2], [3], [8]).

REMARKS. H. Junnila mentioned in a letter to the author that the $\check{}$-construction preserves the Lebesgue property, any since T is orthocompact and therefore has the Lebesgue property, $T\check{}$ is an example of a quasi-metric space with the Lebesgue property which is not *σ-orthocompact.*

We note that σ-orthocompactness does not imply the orthocompactness in quasi-metric spaces. A σ-orthocompact non-orthocompact quasi-metric space was found by P. Fletcher. E. K. van Douwen and H. H. Wicke [1] have constructed an example of a regular non-archimedianly quasi-metrizable space Γ' which is not countably orthocompact. Moreover, it can be shown that Γ' is not even countably preorthocompact.



Let us consider the following diagram:

All implications of the diagram are obvious. Those marked with T, $T\check{}$, Γ', and C are irreversible by the counterexamples indicated, where C is an arbitrary compact nonmetrizable space. The problem of the reversibility of the other implications is open.

PROPOSITION 3. *(σ-)orthocompactness is not summable in quasi-metric spaces, namely, $T^{\vee}$ is a sum of an open set homeomorphic to T and a discrete subspace.*

Proof. Since T is the countable union of disjoint open and closed mutually homeomorphic subsets, the desired result follows from Theorem 1.

PROPOSITION 4. *(σ-)orthocompactness is not multiplicative, namely the product of the space T with the Sorgenfrey line Z is not σ-orthocompact; neither is $T \times T$.*

Proof. Z is homeomorphic to closed subspace of T, hence it is enough to show that $T \times Z$ is not σ-orthocompact. Let $t \in T$, $z \in Z$, $a = \langle t, z \rangle \in T \times Z$. The sets $S(a, r) = C(t, r) \times [z, z + r)$, $r > 0$ form a base of neighborhoods of a. The set $\{t\} \times [z, z + r)$ will be called $I(a, r)$.

The underlying set of T is a plane. Any line parallel to x-axis is a discrete set in T. Hence the plane $P_1 = \{\langle x, y, z \rangle : y + z = 0, \langle x, y \rangle \in T, z \in Z\}$ is a discrete set in $T \times Z$ while any subspace of $T \times Z$ whose underlying set is a plane P parallel to $P_2 = \{\langle x, y, z \rangle : y - z = 0, \langle x, y \rangle \in T, z \in Z\}$ is homeomorphic to T, and the orthogonal projection $\pi: P \to T$ is a homeomorphism.

Let us show that the open cover $\zeta = \{S(a, 1) : a \in P_1\}$ of a clopen set $F = U\zeta$ has no σ-interior preserving refinement η. Otherwise let $S(a)$ be some member of η containing $a \in P_1$. One has $S(a) \subset S(a, 1)$. For some k the subset $P_1(k) = \{a \in P_1 : S(a, 1/k) \subset S(a)\}$ is of the second category in the plane topology of P_1. Since the sets $I(a, 1/k)$ with $a \in P_1(k)$ have the same "height", hence the intersection of $\cup \{I(a, 1/k) : a \in P_1(k)\}$ with some plane P parallel to P_2 denoted by P_0 is of the second category in the plane topology of P. Therefore the set $T_0 = \pi(P_0)$ with $T_0 \subset T$ is of the second category in the plane topology. Let $t \in T_0$, $t = \pi(b)$ and $\{b\} = I(a, 1/k) \cap P$. The set $\pi(S(a) \cap P)$ will be called $U(t)$. Since $t \in \pi(I(a, 1/k) \cap P) \subset \pi(S(a, 1/k) \cap P) \subset U(t)$ hence $U(t)$ is a neighborhood of t in T. Since the collection of all $S(a)$ is σ-interior preserving in $T \times Z$, hence the collection of all $S(a) \cap P$ is σ-interior preserving in the subspace P of $T \times Z$, and since π is a homeomorphism, we get that the collection of all $U(t)$, where $t \in T_0$, is σ-interior preserving. We also have $U(t) = \pi(S(a) \cap P) \subset \pi(S(a, 1) \cap P) \subset 0(t, 1)$, and since T_0 is of the second category in the plane topology, we get a contradiction to the lemma. Hence ζ has no σ-interior preserving refinement and $T \times Z$ is not σ-orthocompact.

PROPOSITION 5. *(CH) The (σ-)orthocompactness is not hereditary in quasi-metric spaces, namely there exists a regular Lindelöf non-archimedean quasi-metric space T^L which is not hereditary σ-orthocompact.*

Proof. If the continuum hypothesis is valid, then there exists an uncountable subspace T_0 of T such that the trace of T_0 on each nowhere dense subset in the plane topology is countable ([10]). Note that $T - T_0$ is dense. T_0 is of the second category in the plane topology, and the subspace T_0 of T is Lindelöf. Indeed, if ζ is an open cover of T_0 and ζ' is a subcollection of ζ which covers some dense set in T, then the complement of $U\zeta'$ in T_0 is countable by the definition of T_0. The same arguments imply that the spaces obtained from the plane by scattering the points of T_0 is also Lindelöf.

Let $B(t, r)(B_0(t, r))$ be spheres of some quasi-metric of the plane (of the subspace T_0 of T) and $B(t, r) \subset B_0(t, r)$ for $t \in T_0$. We define a space T^L with the underlying set $(T_0 \times D) \cup (S - T_0) \times \{0\})$, and with the generalized metric d^L on T^L such that for $r \leqq 1$ d^L-spheres $B^L(\langle t, 0\rangle, r) = B((t, r) \times [0, r)) \cap T^L$ for $t \in S - T_0$ and, $B^L(\langle t, x\rangle, r) = B_0^{\vee}(\langle t, x\rangle, r)$ for $t \in T_0$. For $r > 1$ all $B^L(\langle t, x\rangle, r) = T^{\perp}$. It follows that the subspace $T_0 \times D$ of T^L is isometric to $T_0^{\vee}$, d^L is a quasi-metric and T^L is regular. Since T_0 is of the second categoy in the plane topology, it follows from the lemma that T_0 is not non-archimedeanly quasi-metrizable and $T_0^{\vee}$ is not σ-orthocompact, neither is the subspace $T_0 \times D$ of T^L. Since $S \times \{0\}$ is Lindelöf and for each $x \in D - \{0\}$, $T_0 \times \{x\}$ is Lindelöf, the desired space T^L is Lindelöf.

2. A quasi-uniformity on X is *transitive* provided that it has a base consisting of transitive binary relations ([3]).

THEOREM 2. *Let f be a perfect map from X onto Y. If the space X is quasi-uniformizable via a (transitive) quasi-uniformity with a base of cardinality $\leqq m$, then so is Y.*

Proof. Let $\mathscr{U}$ be a quasi-uniformity on X with a base $\mathscr{B}$ of cardinality $\leqq m$. For any $U \in \mathscr{U}$ the binary relation $U^Y = \{(y, y') \in Y \times Y: U(f^{-1}(y)) \supset f^{-1}(y')\}$ is reflexive in Y. If $U_1 \subset U_2$ then $U_1^Y \subset U_2^Y$. If $U_1 \circ U_1 \subset U_2$, then $U_1^Y \circ U_1^Y \subset U_2^Y$. Thus $\{U^Y | U \in \mathscr{B}\}$ is a base of cardinality $\leqq m$ for a quasi-uniformity $\mathscr{U}^Y$ on Y which is transitive if $\mathscr{U}$ is transitive.

We shall show that $\mathscr{U}^Y$ is compatible with the topology of Y, i.e., for any $y \in Y$, $E \subset Y$ one has that $y \in \operatorname{cl} E$ if, and only if, for each $U \in \mathscr{U}$, $U^Y(y) \cap E \neq \emptyset$.

For $U \in \mathscr{U}$, $y \in Y$ define $U_Y(y) = f(U(f^{-1}(y)))$. As $U^Y(y) = Y -$

$f(X - U(f^{-1}(y)))$ and f is a surjection, we obtain $U^Y(y) \subset U_Y(y)$.

If now $U^Y(y) \cap E \neq \emptyset$ for every $U \in \mathscr{U}$, then also $U_Y(y) \cap E \neq \emptyset$ and $A(U) = \{z \in f^{-1}(y)\colon U(z) \cap f^{-1}(E) \neq \emptyset\} \neq \emptyset$.

Since $A(U_1) \subset A(U_2)$, whenever $U_1 \subset U_2$, it follows that $\{A(U) \mid U \in \mathscr{U}\}$ is a filter base on the compact set $f^{-1}(y)$ with a limit point $x \in f^{-1}(y)$.

For any $U \in \mathscr{U}$ let $U' \in \mathscr{U}$, $U' \circ U' \subset U$ and let $z \in U'(x) \cap A(U')$. One obtains $U'(z) \cap f^{-1}(E) \neq \emptyset$ and $U(x) \cap f^{-1}(E) \neq \emptyset$. Since $\mathscr{U}$ is compatible with the topology of X, $x \in \operatorname{cl} f^{-1}(E)$, and since f is continuous, $y = f(x) \in \operatorname{cl} ff^{-1}(E) = \operatorname{cl} E$.

Let now $y \in \operatorname{cl} E$ and let $U \in \mathscr{U}$. Then $U^Y(y) \cap E \neq \emptyset$. Indeed, $(f^{-1}(y))$ is a neighborhood of $f^{-1}(y)$, i.e., $f^{-1}(y) \subset \operatorname{int} U(f^{-1}(y))$. Thus $E \not\subset f(X - U(f^{-1}(y)))$. Otherwise $E \subset f(X - \operatorname{int} U(f^{-1}(y)))$ and since f is closed, $\operatorname{cl} E \subset f(X - \operatorname{int}(f^{-1}(y))) \subset f(X - f^{-1}(y)) = Y - \{y\}$.

Therefore, $\emptyset = E \cap (Y - f(X - U(f^{-1}(y)))$ and as it was mentioned above, $Y - f(X - U(f^{-1}(y))) = U^Y(y)$, so that $\emptyset \neq E \cap U^Y(y)$. The theorem is proved.

Since a space is (non-archimedeanly) quasi-metrizable iff it is uniformizable via a (transitive) quasi-uniformity with a countable base, the two following corollaries are valid.

COROLLARY 1. *A perfect image of a quasi-metrizable space is quasi-metrizable.*

COROLLARY 2. *A perfect image of a non-archimedeanly quasi-metrizable space is non-archimedeanly quasi-metrizable.*

REMARK. A direct proof of the last result is given by the author in [9][4].

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