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**ON BERRY-ESSEEN APPROXIMATION AND A FUNCTIONAL
LIL FOR A CLASS OF DEPENDENT RANDOM FIELDS**

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ON BERRY-ESSEEN APPROXIMATION AND A FUNCTIONAL LIL FOR A CLASS OF DEPENDENT RANDOM FIELDS

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In this paper we derive a Berry-Esseen type approximation for a class of dependent random fields and use it to obtain a functional law of the iterated logarithm.

1. **Introduction.** In recent years there has been considerable interest in multiparameter stochastic collections or the so-called random fields. In this note we deal with stationary, dependent discrete-parameter random field. In [3] a concept of ϕ -mixing was introduced for such random fields and a functional central limit theorem was proved for them. Here we obtain a Berry-Esseen type approximation for such random fields and use it to prove a functional law of the iterated logarithm.

The set-up and the basic notation is as in [3]. Z^q is the set of all q -tuples of integers ($q \geq 1$). We denote the points in Z^q by i, n etc. or sometimes explicitly by $(i_1, i_2, \dots, i_q), (n_1, n_2, \dots, n_q)$ etc. Let $\{\xi_n: n \in Z^q\}$ be a stationary, ϕ -mixing random field as defined in [3]. We denote the partial sums of this random field by S_n or S_{n_1, n_2, \dots, n_q} i.e.,

$$S_{n_1, n_2, \dots, n_q} = \sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \dots \sum_{i_q=1}^{n_q} \xi_{i_1, i_2, \dots, i_q}$$

where $n_i \geq 1$. If some n_i are zero and others ≥ 1 then it is convenient to set $S_{n_1, n_2, \dots, n_q} = 0$.

Let $T^q = [0, 1]^q$ be the q -fold Cartesian product of the unit interval, and let D_q be the Skorohod function space on T^q . We use the uniform metric d on D_q i.e., if $x, y \in D_q$ then $d(x, y) = \sup_t |x(t) - y(t)|$.

A block B in T^q is a product of half-closed intervals i.e., a set of the form $\prod_{i=1}^q (s_i, t_i]$. If x is a function on T^q then $x(B)$ denotes increment of x around B .

We will assume throughout that:

$$(1) \quad E(\xi_n) = 0 \quad \text{and} \quad E|\xi_n|^{2+\eta} < \infty \quad \text{for some } \eta > 0.$$

We will also assume the following condition in [3] on the rate of ϕ -mixing:

$$(2) \quad \sum_{q=1}^{\infty} r^{q-1} \phi^{1/2}(r) < \infty.$$

It is proved in [3] that under these conditions: $\lim_{n \rightarrow \infty} n^{-q} \text{Var}(S_{n,n,\dots,n}) = \sigma^2 (< \infty)$ where

$$\sigma^2 = \sum_{j \in \mathbb{Z}^q} E(\xi_0 \xi_j) . \quad (\text{Here } \xi_0 = \xi_{0,0,\dots,0}) .$$

To avoid trivial complications we will assume $\sigma^2 > 0$.

We denote by K_σ the Strassen's set of continuous functions on T^q :

$$K_\sigma = \left\{ x: x(t_1, t_2, \dots, t_q) = \int_0^{t_1} \int_0^{t_2} \dots \int_0^{t_q} y(u_1, u_2, \dots, u_q) du_1 du_2 \dots du_q \right. \\ \left. \text{where } \int_0^1 \dots \int_0^1 y^2(u_1, u_2, \dots, u_q) du_1 \dots du_q \leq \sigma^2 \right\} .$$

Theorem 1 below is a Berry-Esseen type theorem dealing with the speed of convergence of (normalized) $S_{n,n,\dots,n}$ to normality. Theorem 2 is a functional LIL for these partial sums.

Denote by $(H_n: n \geq 1)$ the sequence of random functions in D_q defined by

$$H_n(t) = (2n^q \log \log n)^{-1/2} S_{[nt_1], [nt_2], \dots, [nt_q]}$$

where $t = (t_1, t_2, \dots, t_q) \in T^q$ and $[\cdot]$ is the usual greatest-integer function.

2. Theorems and proofs.

THEOREM 1. *Let Φ be the standard normal distribution function. Then under (1) and (2) there exists $C > 0, \alpha > 0$ such that*

$$\sup_x |P\{\sigma^{-1} n^{-q/2} S_{n,n,\dots,n} < t\} - \Phi(t)| < Cn^{-\alpha} , \quad \text{for all } n .$$

Proof. For simplicity suppose $q = 2$.

For given integers $n, a = a(n)$ and $b = b(n)$, let μ be the largest integer such that $\mu(a + b) \leq n$. Then subdivide the square $(0, n] \times (0, n]$ into blocks by taking the product of 2 copies of the partition $0 < a < a + b < 2a + b < \dots < \mu(a + b) < n$. If $1 \leq m \leq \mu$, denote by I_{ma} the interval $((m - 1)(a + b), (m - 1)(a + b) + a]$, by I_{mb} the interval $((m - 1)(a + b) + a, m(a + b)]$ and $I_{(\mu+1)a} = (\mu(a + b), n]$.

Set

$$\alpha_m(n) = \sum_{j \in I_{m_1 a} \times I_{m_2 a}} \xi_j \quad (1, 1) \leq m \leq ((\mu + 1), \mu + 1)$$

$$\beta'_m(n) = \sum_{j \in I_{m_1} \times I_{m_2 b}} \xi_j \quad (1, 1) \leq m \leq (\mu + \mu)$$

$$\beta''_m(n) = \sum_{j \in I_{m_1 a} \times I_{m_2 b}} \xi_j \quad (1, 1) \leq m \leq (\mu + 1, \mu)$$

$$\beta_m'''(n) = \sum_{j \in I_{m_1 b} \times I_{m_2 a}} \xi_j \quad (1, 1) \leq m \leq (\mu, \mu + 1)$$

$$u_n = \sum_{m_i=1}^{\mu+1} \alpha_m(n) \quad v'_n = \sum_{m_i=1}^{\mu} \beta'_m(n) \quad v''_n = \sum_m \beta''_m(n) \quad v'''_n = \sum_m \beta'''_m(n).$$

Then $S_{n,n} = u_n + v'_n + v''_n + v'''_n$.

Because of condition (2), by Proposition 1.1.20 of [6], we have $E(\gamma_m^2(n)) = \#(\gamma_m)(\alpha^2 + \rho_{\#(\gamma_m)})$ where γ_m stands for one of the α_m or $\beta'_m, \beta''_m, \beta'''_m$ and $\#(\gamma_m)$ is the “size” of the block γ_m and $\rho_{\#(\gamma_m)} \rightarrow 0$ if $\#(\gamma_m) \rightarrow \infty$.

Furthermore, as in Theorem 1.1.22 of [6] we get:

- (i) $E v_n'^2 \leq [1 + 4\mu^2 \phi^{1/2}(a)][\mu^2 b^2(\alpha^2 + \rho_a)]$
- (ii) $E v_n''^2 \leq [1 + 4(\mu + 1)^2 \phi^{1/2}(b)][\mu^2 ab(\sigma^2 + \rho_{ab}) + \mu b(n - \mu(a + b))(\sigma^2 + \rho_{n-\mu(a+b)})]$.
- (iii) $E v_n'''^2 \leq [1 + 4(\mu + 1)^2 \phi^{1/2}(b)][\mu^2 ab(\alpha^2 + \rho_{ab}) + \mu b(n - \mu(a + b))(\sigma^2 + \rho_{n-\mu(a+b)})]$.

For $(1, 1) \leq m \leq (\mu, \mu)$ define $\alpha'_m(n)$ to be independent random variables having the same law as $\alpha_{(1,1)}(n)$; then as in Theorem 1.1.22 of [6] we get:

$$(iv) \quad \left| P\left(\frac{u_n}{\sigma\sqrt{n^2}} < t\right) - P\left(\frac{\sum_{m_i=1}^{\mu} \alpha'_m(n)}{\sigma\sqrt{n^2}} < t\right) \right| \leq (\mu + 1)^2 \phi(b).$$

For this computation, it is easy to show that the “end blocks” $\alpha_m(n)$ (with m_1 or m_2 equal to $\mu + 1$) become negligible for large n .

$$(v) \quad \left| P\left(\frac{\sum_{m=1}^{\mu} \alpha'_m(n)}{E^{1/2}(\sum \alpha'_m(n))^2} < \frac{t\sigma\sqrt{n^2}}{E^{1/2}(\sum \alpha'_m(n))^2}\right) - \Phi\left(\frac{t\sigma\sqrt{n^2}}{E^{1/2}(\sum \alpha'_m(n))^2}\right) \right| \leq \frac{c_\delta(\mu + 1)^2 E|\alpha'_{(1,1)}|^{2+\delta}}{[(\mu + 1)^2 E(\alpha_{(1,1)}^2)]^{1+\delta/2}} \leq A c_\delta (1 + \mu)^{-\delta}$$

because by [4, Lemma 7]. $E(|\alpha_{(1,1)}|^{2+\delta}) \leq A(E(\alpha_{(1,1)}^2))^{1+\delta/2}$

$$(vi) \quad \left| \Phi\left(\frac{\sqrt{n^2}}{\sqrt{(\mu + 1)^2 a(\sigma^2 + \rho_a)}} \cdot t\sigma\right) - \Phi(t) \right| \leq \frac{1}{2\pi e} \max\left(1, \sqrt{\frac{(\mu + 1)^2 a^2}{n^2}} \left(1 + \frac{\rho_a}{\sigma^2}\right) \left| \sqrt{\frac{n^2 \sigma^2}{(\mu + 1)^2 a^2 (\sigma^2 + \rho_a)}} - 1 \right| \right) = \psi(n).$$

From (i)-(vi) and using a similar argument as in Theorem 1.1.22 of [6], for $\tau > 0$

$$\left| P\left(\frac{S_{n,n}}{\sigma\sqrt{n^2}} < t\right) - \Phi(t) \right| \leq (\mu + 1)^2 \phi(b) + A c_\delta (\mu + 1)^{-\delta} + \psi(n) + \frac{\tau}{\sigma\sqrt{n^2}} + \frac{E v_n'^2}{\tau^2/9} + \frac{E v_n''^2}{\tau^2/9} + \frac{E v_n'''^2}{\tau^2/9}.$$

If we choose $a = [n^{\cdot 6}]$ $b = [n^{\cdot 4}]$ $\tau = [n^{1-\varepsilon}]$ $0 < \varepsilon < 1$ then $\mu = O(n^{\cdot 4})$ and $n - \mu(a + b) = O(n^{\cdot 6})$. Since condition (2) implies that $r^q \phi^{1/2}(r) \rightarrow 0$; then

$$(\mu + 1)^2 \phi(b) = ((\mu + 1)^4 \phi(b))(\mu + 1)^{-2} = O(n^{\cdot -8}).$$

$$Ac_\delta(\mu + 1)^{-\delta} = O(n^{-\delta(\cdot 4)}) = O(n^{\cdot -4\delta}).$$

$$\begin{aligned} \psi(n) &= O\left(\left|\sqrt{\frac{n^2 \sigma^2}{(\mu + 1)^2 a^2 (\sigma^2 + \rho_a)}} - 1\right|\right) = O\left(\left|\sqrt{\frac{n^2}{(\mu + 1)^2 a^2}} - 1\right|\right) \\ &= O\left(\frac{b}{a}\right) = O(n^{\cdot -2}) \end{aligned}$$

$$\frac{\tau}{\sigma \sqrt{n^2}} = O(n^{-\varepsilon}).$$

$$\begin{aligned} \frac{E v_n'^2}{\tau^2/9} &\leq (\text{constant}) \frac{\mu^2 b^2 (\sigma^2 + \rho_b)}{\tau^2/9} \text{ since } \mu^2 \phi^{1/2}(a) \rightarrow 0. \\ &= O\left(\frac{n^{2\cdot -4}}{n^{2\cdot -2\varepsilon}}\right) = O(n^{2\varepsilon - \cdot 4}). \end{aligned}$$

$$\begin{aligned} \frac{E v_n''^2}{\tau^2/9} &\leq \frac{(\text{constant})}{\tau^2/9} [\mu^2 ab (\sigma^2 + \rho_{ab}) + \mu b (n - \mu(a + b)) (\sigma^2 + \rho_{n - \mu(a + b)})] \\ &= O(n^{2\varepsilon - \cdot 2}). \end{aligned}$$

Similarly

$$\frac{E v_n'''^2}{\tau^2/9} = O(n^{2\varepsilon - \cdot 2}).$$

Then

$$\left| P\left(\frac{S_{n,n}}{\sigma \sqrt{n^2}} < t\right) - \Phi(t) \right| \leq C n^{-\alpha}$$

$$\text{if we set: } \varepsilon = \alpha = \frac{1}{15} \text{ whenever } \delta \geq \frac{1}{6}.$$

$$\varepsilon = \alpha = .48 \text{ whenever } 0 < \delta < \frac{1}{6}.$$

An analogous proof is valid for the $q > 2$, in that case take

$$\varepsilon = \alpha = \frac{1}{15} \text{ if } \delta \geq \frac{1}{3q}$$

$$\varepsilon = \alpha = (.2q\delta) \text{ if not.}$$

REMARK. From the proof, it can be seen that a more general theorem can be obtained if we replace $S_{n,n,\dots,n}$ by S_n where $n' = (n\theta_1, n\theta_2, \dots, n\theta_q)$ $0 < \theta_i \leq 1$. Then we have

$$\sup_n |P\{\sigma^{-1}n^{-q/2}(\theta_1 \cdots \theta_q)^{-1/2}S_n, < t\} - \Phi(t)| < Cn^{-\alpha} \quad \forall n .$$

In fact it is in this stronger form that we will use it in the proof of Theorem 2.

THEOREM 2. *Let (1) and (2) be satisfied. Then*

$$(a) \quad P\{\limsup_{n \rightarrow \infty} d(H_n, K_\sigma) = 0\} = 1 ,$$

and

$$(b) \quad P\{\bigcap_{x \in K_\sigma} [\liminf_{n \rightarrow \infty} d(H_n, x) = 0]\} = 1 .$$

Proof. We will give only a very brief sketch of the proof since the arguments used are fairly standard and can be found e.g., in Chover (1967) and Wichura (1973). Take $q = 2$ for simplicity and $\sigma = 1$ without loss of generality.

We begin by showing a kind of asymptotic equi-continuity in the following form: Let $B = \prod_i (s_i, t_i]$ be a block in T^q ; write $m(B) = \min_{1 \leq i \leq q} (t_i - s_i)$. Then

LEMMA 1. *Given $\varepsilon > 0$, $\exists \delta > 0$ such that if B is any block with $m(B) < \delta$ then the event $\{|H_n(A)| > \varepsilon\}$ occurs only finitely often wp.1.*

Proof. Standard arguments (using the triangle inequality) such as those appearing on pp.56-59 of Billingsley (1968) show that it suffices to prove the following: Given $\varepsilon > 0$, $\exists \delta > 0$ such that

$$\sum_n [P\{\max_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n}} |S_{i,j}| > \varepsilon \sqrt{2n^2 \log \log n}\} + P\{\max_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n\delta}} |S_{i,j}| > \varepsilon \sqrt{2n^2 \log \log n}\}] < \infty .$$

But this can be proved in a straightforward manner using the maximal inequality developed on pp.713-714 of [3], Theorem 1 above and the arguments in §3 of Chover (1967). We omit the details.

Let now m be a positive integer. Consider a partition of the unit square (T^2) into $m \times m$ squares with corners $(i/m, j/m)$, $0 \leq i, j \leq m$. We enumerate these squares (blocks) arbitrarily as B_{im} , $1 \leq i \leq m^2$. Let $\gamma > 0$ be a small positive number and denote by $B_{im}^* = B_{im}^*(\gamma)$ the square which is concentric with B_{im} (and is contained in B_{im}) with each side being equal to $(1 - 2\gamma)/m$.

If x is a function on T^2 we denote by $\pi_m x$ the function on T^2 defined by

$$(\pi_m x)(t_1, t_2) = \int_0^{t_1} \int_0^{t_2} \sum_{i=1}^{m^2} m^2 x(B_{im}) I_{B_{im}}(u_1, u_2) du_1 du_2$$

where I_B stands for the indicator of the block B .

Lemmas 2 and 3 below follow easily from the arguments used in proving Corollaries 1 and 2 in Chover (1967). Lemma 4 is immediate from Lemma 1.

LEMMA 2. *Given $\varepsilon > 0$, $\exists m$ such that*

$$P\{d(\pi_m H_n, H_n) > \varepsilon \text{ only finitely often (in } n)\} = 1 .$$

LEMMA 3. *Given $\varepsilon > 0$, $\exists c > 1$ such that, wp. 1, $\max_{c^n \leq m \leq c^{n+1}} d(H_m, H_{\lceil c^n \rceil}) > \varepsilon$ for only finitely many n .*

LEMMA 4. *Given $\varepsilon > 0$, $\exists \gamma > 0$ such that for each m and i ($1 \leq i \leq m^2$),*

$$P\{|H_n(B_{im}) - H_n(B_{im}^*)| > \varepsilon \text{ for only finitely many } n\} = 1 .$$

We now proceed to prove (a) of the theorem. Let $\{\theta_i: 1 \leq i \leq m^2\}$ be real numbers such that $\sum_{i=1}^{m^2} \theta_i^2 = 1$. To prove (a) it suffices to show that for each m ,

$$P\left\{\sum_{i=1}^{m^2} \theta_i (\pi_m H_n)(B_{im}) < (1 + \varepsilon) \text{ for all large } n\right\} = 1 .$$

In view of the preceding lemmas it thus suffices to prove (with $c > 1$ sufficiently close to 1 and $\gamma > 0$ sufficiently small)

$$\sum_{n=1}^{\infty} P\left\{m \sum_{i=1}^{m^2} \theta_i H_{\lceil c^n \rceil}(B_{im}^*) > (1 + \varepsilon)\right\} < \infty .$$

But the proof of this is essentially the same as given in §4 of Chover (1967). The only complication here is that the m^2 random variables $\{H_{\lceil c^n \rceil}(B_{im}^*): 1 \leq i \leq m^2\}$ are not independent. But there is enough separation among these and it suffices to apply Lemma 1.1.5 in Iosifescu and Theodorescu (1969).

To prove (b) take $x \in K$ with $\int_0^1 \int_0^1 (\partial^2 x / \partial t_1 \partial t_2)^2 dt_1 dt_2 < 1$. We need to show that $\forall \varepsilon > 0$, $P(\liminf d(H_n, x) < \varepsilon) = 1$. Again in view of the preceding lemmas and the arguments in Sec.5 of Chover (1967) it is enough to prove for sufficiently small $\delta > 0$, $\gamma > 0$

$$P(\lim_{n \rightarrow \infty} \sup F_n) = 1 \text{ where}$$

$$F_n = \{|H_{\lceil c^n \rceil}(B_{im}^*) - x(B_{im})| < \delta, \text{ all } i, 1 \leq i \leq m^2\} .$$

[It might be noted here that (35) in [2] is insufficient; it should be strengthened to $P(\lim_{r \rightarrow \infty} \sup \bigcap_r C_r^{(\nu)}) = 1$.] Now if the probability of F_n is computed on the assumption that the m^2 random variables $\{H_{\lceil c^n \rceil}(B_{im}^*): 1 \leq i \leq m^2\}$ are independent then the error committed is

at most $m^2 \phi_{[v, e^n]}$ which forms a term of a convergent series in n . Hence using part (a) of the lemma on page 142 of [5] it is enough to show $\sum_n P(F_n) = \infty$. But given Theorem 1 this follows from computations which are standard in the proof of Strassen's theorem. This completes the proof of Theorem 2.

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