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HOLOMORPHY ON SPACES OF DISTRIBUTION

PHILIP J. BOLAND AND SEAN DINEEN

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If E is a locally convex space and $U \subset E$ is open, then $H(U)$ is the space of holomorphic functions on U (i.e., $H(U) = \{f: U \rightarrow \mathbb{C}, f \text{ } G\text{-analytic and continuous}\}$). τ_0 is the topology of uniform convergence on compact subsets of U . τ_ω is the Nachbin ported topology defined by all semi-norms on $H(U)$ ported by compact subsets of U . (A semi-norm p on $H(U)$ is ported by K if whenever V is open and $K \subset V \subset U$, there exists C_V such that $p(f) \leq C_V |f|_V$ for all $f \in H(U)$.) τ_δ is the topology defined by all semi-norms p on $H(U)$ with the following property: if (U_n) is a countable increasing open cover of U , there exist $C > 0$ and U_N such that $p(f) \leq C |f|_{U_N}$ for all $f \in H(U)$. $H_{HY}(U)$ is the space of hypoanalytic functions on U — that is $H_{HY}(U) = \{f: f \text{ is } G\text{-analytic and the restriction of } f \text{ to any compact set } K \subset U \text{ is continuous}\}$.

If Ω is open in R^n , then $\mathcal{D}(\Omega)$ and $\mathcal{D}'(\Omega)$ are respectively the Schwartz space of test functions and the Schwartz space of distributions on Ω . We prove that $H(\mathcal{D}(\Omega)) \neq H_{HY}(\mathcal{D}(\Omega))$ and that $\tau_0 = \tau_\omega = \tau_\delta$ on $H(\mathcal{D}(\Omega))$ while $H_{HY}(\mathcal{D}'(\Omega)) = H(\mathcal{D}'(\Omega))$ but $\tau_0 \neq \tau_\omega \neq \tau_\delta$ on $H(\mathcal{D}'(\Omega))$.

1. Holomorphic and hypoanalytic functions on countable direct sums. In this section, we will prove two lemmas which are useful in the construction of holomorphic and hypoanalytic functions on countable direct sums. If $E_n \neq 0$ is a locally convex space for each n , then $E = \sum_{n=0}^\infty E_n$ is the countable direct sum of the E_n with the finest locally convex topology such that $E_n \rightarrow E$ is continuous for all n . $\prod_{n=0}^\infty E_n$ is the product of the E_n with the product topology.

LEMMA 1. Let $E = \sum_{n=0}^\infty E_n = E_0 \oplus E_1 \oplus E_2 \oplus \dots$ where each $E_n \neq 0$. For each $n > 0$, let $\psi_n \in E'_n$, $\psi_n \neq 0$. Let $(\phi_n)_{n=1}^\infty \subseteq E'_0$. Then $p = \sum_{n=1}^\infty \phi_n \psi_n \in P_{HY}({}^2E)$ (i.e., is a hypocontinuous polynomial of degree 2 on E), and $p \in P({}^2E)$ if and only if there exists an absolutely convex neighborhood V_0 of zero in E_0 such that $|\phi_n|_{V_0} < +\infty$ for each n , (i.e., $(\phi_n)_{n=1}^\infty \subseteq E'_0(V_0)$).

Proof. Since each compact subset of E is contained in $E_0 \oplus \dots \oplus E_m$ for some m , it follows that p is a hypoanalytic polynomial of degree 2. Now if p is continuous ($p \in P({}^2E)$) there exists an absolutely convex neighborhood of zero $V = V_0 \oplus V_1 \oplus V_2 \oplus \dots$ such that $|p|_V \leq 1$. Now if $n \geq 1$ is given and $y_n \in V_n$ is such that $\psi_n(y_n) \neq 0$, then $|\phi_n|_{V_0} \leq 1/|\psi_n(y_n)|$.

Conversely suppose V_0 is an absolutely convex neighborhood of zero in E_0 such that for each n , $|\phi_n|_{V_0} \leq C_n$. Now choose V_n an absolutely convex neighborhood of zero in E_n such that $|\psi_n|_{V_n} \leq 1/C_n 2^n$. Then p is bounded on the neighborhood $V = V_0 \oplus V_1 \oplus V_2 \oplus \dots$ of zero in E and hence p is continuous at zero. Similarly, one may show that p is bounded on a neighborhood of an arbitrary point, and hence p is continuous on E .

COROLLARY 2. *Let $E = E_0 \oplus E_1 \oplus E_2 \oplus \dots = \sum_{n=0}^{\infty} E_n$ where E_0 is a metrizable nonnormed space. Then $P(^2E) \neq P_{HY}(^2E)$, and hence $H(E) \neq H_{HY}(E)$.*

Proof. Since E_0 is metrizable and nonnormed, there exists a neighborhood basis (W_n) of zero in E_0 and a sequence (ϕ_n) of continuous linear forms on E_0 such that $|\phi_n|_{W_n} < +\infty$, but $|\phi_n|_{W_{n-1}} = +\infty$ for each n . Hence, constructing (ϕ_n) as in Lemma 1, we see $p = \sum_{n=1}^{\infty} \phi_n \psi_n \in P_{HY}(^2E) \setminus P(^2E)$.

LEMMA 3. *Let $E = E_1 \oplus E_2 \oplus E_3 \oplus \dots = \sum_{n=1}^{\infty} E_n$. Suppose $f_n \in H(E_n)$, $f_n(0) = 0$ for each n , and let g_n be a continuous polynomial on $E_1 \oplus \dots \oplus E_n$ (i.e., $g_n \in P(E_1 \oplus \dots \oplus E_n)$) for each n . Then if there exists a neighborhood U of zero in E such that $|g_n|_U < +\infty$ for all n , then $F = \sum_{n=1}^{\infty} g_n f_n \in H(E)$.*

Proof. By the nature of compact sets in E , it is clear that F is always hypoanalytic (on each compact subset K of E , F reduces to a finite sum of analytic functions).

Now suppose U is a neighborhood of zero in E such that $|g_n|_U < +\infty$ for each n . Without loss of generality we may assume $U = U_1 \oplus U_2 \oplus U_3 \oplus \dots$ where U_n is an absolutely convex neighborhood of zero in E_n for each n . Now let V_n be an absolutely convex neighborhood of zero in E_n such that $V_n \subset U_n$ and

$$|f_n|_{V_n} \leq \frac{1}{2^n |g_n|_U}$$

for each $n \geq 1$. Then $V = V_1 \oplus V_2 \oplus \dots$ is an absolutely convex neighborhood of zero in E and

$$|F|_V \leq \sum_{n=1}^{\infty} |g_n|_V |f_n|_V \leq \sum_{n=1}^{\infty} |g_n|_U |f_n|_{V_n} \leq \sum_{n=1}^{\infty} \frac{1}{2^n} = 1.$$

Hence F is locally bounded and therefore continuous at zero. We now show that F is also continuous at any $x \in E$.

Given $x \in E$, there exists an m such that $x = x_1 + \dots + x_m \in E_1 \oplus \dots \oplus E_m$. Since g_n is a polynomial and is bounded on U , it

follows that g_n is bounded on $x + U$.

For each n let W_n be an absolutely convex neighborhood of zero in E_n such that $W_n \subset U_n$ and

$$|f_n|_{x+W_n} < \frac{1}{2^n |g_n|_{x+U}} .$$

Let $W = W_1 \oplus W_2 \oplus \dots \oplus W_m \oplus W_{m+1} \oplus \dots$.

Then $x + W \subset x + U$, and

$$\begin{aligned} |F|_{x+W} &= \left| \sum_{n=1}^{\infty} g_n f_n \right|_{x+W} \\ &\leq \sum_{n=1}^{\infty} |g_n f_n|_{x+W} \\ &\leq \sum_{n=1}^{\infty} |g_n|_{x+U} |f_n|_{x+W_n} < \sum_{n=1}^{\infty} \frac{1}{2^n} = 1 . \end{aligned}$$

EXAMPLE 4. Let $g_n \in P(E_1 \oplus \dots \oplus E_n)$ for each n , and $f_n \in H(E_n)$ such that $f_n(0) = 0$ for each n . Then if E_n is normed for each n , $F = \sum_{n=1}^{\infty} g_n f_n \in H(E)$.

EXAMPLE 5. Let $\phi_n \in E'_n$, and $f_n \in H(E_n)$ for each n where $f_n(0) = 0$. Then $F = \sum_{n=1}^{\infty} \phi_1 \cdots \phi_n f_n \in H(E)$.

2. Holomorphic and hypoanalytic functions on \mathcal{D} and \mathcal{D}' .

If Ω is an open subset of R^n , $\mathcal{D}(\Omega)$ and $\mathcal{D}'(\Omega)$ are respectively the Schwartz space of test functions and the Schwartz space of distributions on Ω . The topologies on these spaces are normally described in terms of inductive and projective limits. Valdivia [7] has shown that for any n and any open $\Omega \subset R^n$, $\mathcal{D}(\Omega) \cong \sum_{(N)} s$ and $\mathcal{D}'(\Omega) \cong \prod_{(N)} s'$ (where s is the Fréchet nuclear space of rapidly decreasing sequences and s' is its dual). As we will see, this simplifies considerably the study of holomorphic functions on these spaces. Because of this characterization of Valdivia, we will use the notation \mathcal{D} and \mathcal{D}' in place of respectively $\mathcal{D}(\Omega)$ and $\mathcal{D}'(\Omega)$. Hence we will write $\mathcal{D} \cong \sum_{(N)} s$ and $\mathcal{D}' \cong \prod_{(N)} s$.

In this section, we will show that $H(U) \neq H_{HY}(U)$ for U open in \mathcal{D} but that $H(U) = H_{HY}(U)$ for U open in \mathcal{D}' .

Since $\mathcal{D} \cong \sum_{(N)} s$, where s is the (nonnormed) Fréchet nuclear space of rapidly decreasing sequences, we see that by Corollary 2, $P^2(\mathcal{D}) \neq P_{HY}(\mathcal{D})$ and hence $H(U) \neq H_{HY}(U)$ for U open in \mathcal{D} .

PROPOSITION 6. Let U be open in \mathcal{D}' . Then there exists a compact subset K of U with the property that if $f \in H_{HY}(U)$ and $f|_K = 0$, then $f \equiv 0$.

Proof. It suffices to consider the case when U is an absolutely convex neighborhood of zero in \mathcal{D}' . For each n , let $s_n \cong s$. Then $\mathcal{D}' \cong \prod_{n=1}^{\infty} s'_n$, and we may assume U is of the form $U_1 \times \cdots \times U_m \times \prod_{n=m+1}^{\infty} s'_n$ where U_n is an absolutely convex neighborhood of zero in s'_n . Since s_n has a continuous norm, there exists a compact set $K_n = V_n^0 \subset U_n$ which has dense linear span in s'_n for each n . Now $K = K_1 \times K_2 \times K_3 \times \cdots$ is compact in \mathcal{D}' and we claim that if $f \in H_{HY}(U)$ and $f|_K = 0$, then $f \equiv 0$.

If $f \in H_{HY}(U)$ where $f|_K = 0$ and $f = \sum_{r=0}^{\infty} p_r$ is the Taylor series expansion of f at 0, it follows from the Cauchy inequalities that $|p_r|_K = 0$ for each r . It suffices to show that $p_r \equiv 0$ for all r . Now $s'_1 \times \cdots \times s'_n$ is a \mathcal{DFN} space (in particular a k -space) and hence $(p_r)_n = p_r|_{s'_1 \times \cdots \times s'_n}$ is continuous for each r and n . But $K_1 \times \cdots \times K_n$ has dense linear span in $s'_1 \times \cdots \times s'_n$ and thus $(p_r)_n \equiv 0$ since $(p_r)_n|_{K_1 \times \cdots \times K_n} = 0$. Now if $x \in \mathcal{D}'$ and $(x)_n = (x_1, \cdots, x_n, 0, \cdots) \in \prod_{n=1}^{\infty} s'_n$, then $(x)_n \rightarrow x$ and $\{x, (x)_1, (x)_2, \cdots\}$ is compact. Therefore, since p_r is continuous on compact sets, $p_r(x) = \lim_n p_r((x)_n) = \lim_n (p_r)_n((x)_n) = 0$. Hence $p_r \equiv 0$ for all r , and $f \equiv 0$.

PROPOSITION 7. *If U is open in \mathcal{D}' , then $H_{HY}(U) = H(U)$.*

Proof. Without loss of generality, we may assume U is of the form $U_1 \times \cdots \times U_m \times \prod_{n=m+1}^{\infty} s'_n$ as in Proposition 6. Let $\Pi_n: \prod_{n=1}^{\infty} s'_n \rightarrow s'_1 \times \cdots \times s'_n$ for each n . We claim that if $f \in H_{HY}(U)$, then f factors through some Π_n , i.e., there exists $n \geq m$ and \tilde{f} a mapping from $\Pi_n(U)$ to C making the following diagram commutative:

$$\begin{array}{ccc} U & \xrightarrow{\Pi_n} & \Pi_n(U) \\ & \searrow f & \swarrow \tilde{f} \\ & & C \end{array}$$

If such a factorization does exist, then clearly \tilde{f} is G -analytic. Since $\Pi_n(U)$ is a k -space ($s'_1 \times \cdots \times s'_n$ is \mathcal{DFN}), and compact subsets of $\Pi_n(U)$ can be pulled back to compact subsets of U , it would follow that \tilde{f} and hence f is continuous.

Suppose f does not factor for any n . Then there exist two sequences $(x_n), (y_n) \subset U$ where $\Pi_n(x_n) = x_n$, $\Pi_n(y_n) = 0$ and $f(x_n + y_n) - f(x_n) \neq 0$ for each n .

Let K be a hypoanalytically determining set for $H_{HY}(U)$ as in Proposition 6. Now for $n > m$,

$$g_n: z \longrightarrow f(z + y_n) - f(z) \in H_{HY}(U)$$

and therefore there exists $z_n \in K$ such that

$$f(z_n + y_n) - f(z_n) \neq 0 .$$

Furthermore, for fixed $n > m$, $\lambda \rightarrow f(z_n + \lambda y_n) - f(z_n)$ is analytic on C and nonconstant. By Liouville's theorem there exists $\lambda_n \in C$ such that $|f(z_n + \lambda_n y_n)| > n$. But then f is unbounded on the relatively compact subset $(z_n + \lambda_n y_n)_{n > m}$ in U , contradicting the fact that $f \in H_{HY}(U)$.

COROLLARY 8. *Let U be open in \mathcal{D}' . Then τ_0 bounded subsets of $H(U)$ are locally uniformly bounded (i.e., uniformly bounded in a neighborhood of each point in U).*

Proof. Without loss of generality $U = U_1 \times \dots \times U_m \times \prod_{n=m+1}^{\infty} s'_n$ as in Proposition 7. We proceed as in the proof of Proposition 7. If $(f_\alpha)_{\alpha \in A}$ is τ_0 bounded in $H(U)$, we claim it is uniformly factorizable through some $\Pi_n (n > m)$. If not, we may as before find a relatively compact subset $(z_r + \lambda_r y_r)_r$ in U on which $(f_\alpha)_{\alpha \in A}$ is not uniformly bounded. Hence $(f_\alpha)_{\alpha \in A}$ factors uniformly through some $\Pi_n, (n > m)$.

But if $U \subset E$ where E is a \mathcal{DFN} space or a Fréchet space, then τ_0 bounded subsets of $H(U)$ are locally uniformly bounded [1]. Hence $(\tilde{f}_\alpha \circ \Pi_n)_{\alpha \in A}$ is locally uniformly bounded and therefore so is $(f_\alpha)_{\alpha \in A}$.

3. Topologies on $H(\mathcal{D})$ and $H(\mathcal{D}')$. In this section, we will prove the promised results concerning the three topologies τ_0, τ_w and τ_s on the spaces \mathcal{D} and \mathcal{D}' . We will use duality theory and some results already known about topologies on $H(E)$ when E is a Fréchet nuclear space or a \mathcal{DFN} space, ([3], [4], [6]).

Initially, we review some basic results in duality theory for holomorphic functions.

A locally convex space E is fully nuclear if E and E' are complete reflexive nuclear spaces. $\mathcal{D}, \mathcal{D}'$ and Fréchet nuclear or \mathcal{DFN} spaces are fully nuclear. E is fully nuclear with a basis if E is fully nuclear and has a Schauder (and hence absolute) basis. \mathcal{D} and \mathcal{D}' are fully nuclear with a basis. If $(e_n)_{n=1}^{\infty}$ is a basis for the fully nuclear space E and $m \in N^{(N)}$, then

$$z = \sum_{n=1}^{\infty} z_n e_n \longrightarrow z^m = z_1^{m_1} \dots z_n^{m_n}$$

is a monomial in z . In [3] it is shown that if E is fully nuclear with a basis, then the monomials form an absolute basis for $H(E)$ for both of the topologies τ_0 and τ_w . Furthermore, $\tau_0 = \tau_w$ on $H(E)$ if and only if τ_0 and τ_w are compatible if and only if $H_{HY}(0_{E'}) = H(0_{E'})$ (the space of hypoanalytic and analytic germs at 0 in E'). In

particular, it follows that $\tau_0 = \tau_\omega$ on $H(E)$ whenever E is a Fréchet nuclear or dual of Fréchet nuclear space with a basis.

There exist Fréchet nuclear spaces with a basis E where $\tau_0 = \tau_\omega \neq \tau_s$ on $H(E)$. However, Dineen has shown in [6] that if $E = s$, then $\tau_0 = \tau_\omega = \tau_s$ on $H(s)$. We will use this fact together with the above duality theory to show that $\tau_0 = \tau_\omega = \tau_s$ on $H(\mathcal{D})$.

PROPOSITION 9. $\tau_0 = \tau_\omega = \tau_s$ on $H(\mathcal{D})$.

Proof. We have seen that $H(U) = H_{HY}(U)$ for all U open in \mathcal{D}' . Hence $H(0_{\mathcal{D}'}) = H_{HY}(0_{\mathcal{D}'})$. Since \mathcal{D} is a fully nuclear space with a basis, it follows by the above that $\tau_0 = \tau_\omega$ on $H(\mathcal{D})$. We will now show that $\tau_0 = \tau_s$ on $H(\mathcal{D})$. Initially however, we will introduce some further notation concerning $\mathcal{D} \cong \sum_{n=1}^{\infty} s_n$.

For each n , let $(e_{in})_i$ be an absolute basis for s_n . Then if $z = (z_n) \in \mathcal{D}$, each z_n is of the form $z_n = (z_{in})_i$. If $m_i \in \mathbb{N}^{(N)}$ then $z_i^{m_i} = z_{i1}^{m_{i1}} z_{i2}^{m_{i2}} \dots$. Now let $m = (m_1, m_2, \dots) \in \sum_{(N)N}(N)$. Then $z \in \mathcal{D} \rightarrow z^m = z_1^{m_1} z_2^{m_2} \dots$ is a monomial, and the set of monomials form a basis for $H(\mathcal{D})$, $\tau_0 = H(\mathcal{D})$, τ_ω . Note that a monomial on \mathcal{D} is simply a finite product of monomials on s .

Now let p be a τ_s continuous semi-norm on $H(\mathcal{D})$. We wish to show that p is a τ_0 continuous semi-norm. It suffices to show that p is the extension of a τ_s continuous semi-norm on $H(s_1 \oplus \dots \oplus s_n)$ for some n . For in this case, since $s_1 \oplus \dots \oplus s_n \cong s$, and $\tau_0 = \tau_s$ on $H(s)$ ([6]), it would follow that p is τ_0 continuous.

As p is τ_s continuous on $H(\mathcal{D})$, p is of the form $p(f) = \sum_{j=0}^{\infty} p_j(\hat{d}^j f(0)/j!)$ where p_j is a continuous semi-norm on $P^j(\mathcal{D})$. But $\tau_s = \tau_\omega = \tau_0$ on $P^j(\mathcal{D})$ for each j , and since every compact subset of \mathcal{D} is contained in $s_1 \oplus \dots \oplus s_n$ for some n , it follows that there exists a minimal n_j such that $f|_{s_1 + \dots + s_{n_j}} = 0 \Rightarrow p_j(f) = 0$ for all $f \in H(\mathcal{D})$. We claim that the sequence $\{n_j\}_j$ is bounded.

If not there exist two increasing sequences of integers $(r_j)_j$ and $(s_j)_j$ and a sequence of monomials $h_j = z_1^{m_{s_j,1}} \dots z_{s_j}^{m_{s_j,s_j}} \in P^{(r_j)}(\mathcal{D})$ such that

$$m_{s_j} \neq 0 \quad \text{and} \quad p_{r_j}(h_j) \neq 0.$$

Now let $k_j = h_j/p_{r_j}(h_j)$. Then $k_j = g_{s_j} f_{s_j}$ where g_{s_j} is a monomial and $f_{s_j} \in H(E_{s_j})$ such that $f_{s_j}(0) = 0$. Now there exists a neighborhood of zero U in $\sum_{n=1}^{\infty} s_n$ on which each monomial is bounded (each s_n has a continuous norm). Hence if $F \equiv \sum_{j=1}^{\infty} k_j$, then $F \in H(\mathcal{D})$ by Lemma 3. But $p(F) = \sum_{j=1}^{\infty} p_{r_j}(k_j) = +\infty$, a contradiction. Hence $(n_j)_j$ is bounded, and there exists an n such that if $f \in H(\mathcal{D})$, $f|_{s_1 \oplus \dots \oplus s_n} = 0$,

then $p(f) = 0$. It is clear then that $p|_{H(s_1 \oplus \dots \oplus s_n)}$ is a τ_δ continuous semi-norm on $H(s_1 \oplus \dots \oplus s_n)$ from the definition of the τ_δ topology, and this completes the proof.

PROPOSITION 10. $\tau_0 \neq \tau_\omega \neq \tau_\delta = \tau_{0,b} = \tau_{\omega,b}$ on $H(\mathcal{D}')$.

Proof. We have seen that $H(\mathcal{D}) \neq H_{HY}(\mathcal{D})$ and therefore it follows that $H(0_{\mathcal{D}}) \neq H_{HY}(0_{\mathcal{D}})$. Hence in particular τ_0 and τ_ω are not compatible on $H(\mathcal{D}')$, and therefore $\tau_0 \neq \tau_\omega$.

Now $\mathcal{D}' \cong \prod_{n=1}^{\infty} s'_n$, and in particular contains a sequence $(y_n)_n (y_n \neq 0)$ which converges to zero very strongly (i.e., $\lambda_n y_n \rightarrow 0$ for each sequence $(\lambda_n)_n$ of complex numbers). Hence $\tau_\omega \neq \tau_{\omega,b}$ (bornological topology associated with τ_ω): see for example [5]. In particular $\tau_\omega \neq \tau_\delta$ since τ_δ is bornological [5].

Finally we show that $\tau_{0,b} = \tau_{\omega,b} = \tau_\delta$. Since it is easy to see from the definition of τ_δ that sets of holomorphic functions which are locally uniformly bounded are τ_δ bounded on $H(\mathcal{D}')$, it suffices to show that τ_0 bounded subsets of $H(\mathcal{D}')$ are locally uniformly bounded. But this is precisely Corollary 8.

REMARK 11. It is interesting to note that τ_0 , τ_ω and τ_δ are three distinct nuclear topologies on $H(\mathcal{D}')$ ([2], [3], [6], [8]) while $\tau_0 = \tau_\omega = \tau_\delta$ on $H(\mathcal{D})$ is nuclear but not sequentially complete.

REMARK 12. Since \mathcal{D} and \mathcal{D}' are fully nuclear with a basis, we may define open polydiscs in these spaces. If U is an open polydisc in the fully nuclear space E , then U^M (the multiplicative polar of U) is a compact subset of E' . [4].

The results of Propositions 9 and 10 may be extended to show that $\tau_0 = \tau_\omega = \tau_\delta$ on $H(U)$ whenever U is an open polydisc in \mathcal{D} and that $\tau_0 \neq \tau_\omega \neq \tau_\delta = \tau_{0,b} = \tau_{\omega,b}$ on $H(U)$ whenever U is an open subset in \mathcal{D}' .

In [4] it is shown that τ_0 is bornological on $H(U)$ if and only if the space of germs $H(U^M) = \lim_{V \supset U^M} H^\infty(V)$ is a regular inductive limit.

Hence if U is an open polydisc in \mathcal{D} (respectively \mathcal{D}') then $H(U^M)$ is regular ($H(U^M)$ is not regular).

REFERENBES

1. J. A. Barroso, M. C. Matos and L. Nachbin, *On bounded sets of holomorphic mappings*, Proceedings on Infinite Dimensional Holomorphy, University of Kentucky 1973, Springer Verlag Lecture Notes in Mathematics, 364, 1974.
2. P. J. Boland, *An example of a nuclear space in infinite dimensional holomorphy*, Arkiv för matematik, **15** (1), (1977), 87-91.

3. P. J. Boland and S. Dineen, *Holomorphic Functions on Fully Nuclear Spaces*, Bull. Soc. Math. France, **106** (1978), 311-336.
4. P. J. Boland and S. Dineen, *Duality Theory for Spaces of Germs and Holomorphic Functions on Nuclear Spaces*, Advances in Holomorphy, Ed. J. A. Barroso, North Holland Mathematics Studies, **34**, 179-207.
5. S. Dineen, *Holomorphic functions on locally convex topological vector spaces I, Locally convex topologies on $H(U)$* , Ann. Inst. Fourier, **23** (1973), 1, 19-54.
6. S. Dineen, *Analytic functionals on fully nuclear spaces*, to appear in Studia Math.
7. M. Valdivia, *Representaciones de Los Espacios $\mathcal{D}(\Omega)$ y $\mathcal{D}'(\Omega)$* , Revista Real Academia de Ciencias Exactas, Físicas y Naturales—Madrid, 1978, **72**, 385-414.
8. L. Waelbroeck, *The nuclearity of $\mathcal{O}(U)$* , Infinite Dimensional Holomorphy and Applications, Ed. M.C. Matos, North Holland Mathematical Studies, **12** (1977), 425-634.

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| | |
|---|-----|
| Michael E. Adams and J. Sichler, Lattices with unique complementation | 1 |
| Walter Allegretto, Positive solutions and spectral properties of second order elliptic operators | 15 |
| Philip J. Boland and Sean Dineen, Holomorphy on spaces of distribution . . . | 27 |
| Duncan Alan Buell, Philip A. Leonard and Kenneth S. Williams, Note on the quadratic character of a quadratic unit | 35 |
| Herbert Busemann and Bhalchandra B. Phadke, Two theorems on general symmetric spaces | 39 |
| Emeric Deutsch, Bounds for the Perron root of a nonnegative irreducible partitioned matrix | 49 |
| Charles F. Dunkl, A difference equation and Hahn polynomials in two variables | 57 |
| Gustave Adam Efroymson, The Riemann mapping theorem for planar Nash rings | 73 |
| John Albert Fridy and Robert Ellis Powell, Tauberian theorems for matrices generated by analytic functions | 79 |
| Denton Elwood Hewgill, John Hamilton Reeder and Marvin Shinbrot, Some exact solutions of the nonlinear problem of water waves | 87 |
| Bessie Hershberger Kirkwood and Bernard Robert McDonald, The symplectic group over a ring with one in its stable range | 111 |
| Esther Portnoy, Transitive groups of isometries on H^n | 127 |
| Jerry Ridenhour, On the sign of Green's functions for multipoint boundary value problems | 141 |
| Nina M. Roy, An M -ideal characterization of G -spaces | 151 |
| Edward Barry Saff and Richard Steven Varga, On incomplete polynomials. II | 161 |
| Takeyoshi Satō, The equations $\Delta u = Pu$ ($P \geq 0$) on Riemann surfaces and isomorphisms between relative Hardy spaces | 173 |
| James Henry Schmerl, Correction to: "Peano models with many generic classes" | 195 |
| Charles Madison Stanton, On the closed ideals in $A(W)$ | 199 |
| Viakalathur Shankar Sunder, Unitary equivalence to integral operators . . . | 211 |
| Pavel G. Todorov, New explicit formulas for the n th derivative of composite functions | 217 |
| James Li-Ming Wang, Approximation by rational modules on boundary sets | 237 |
| Kenneth S. Williams, The class number of $Q(\sqrt{p})$ modulo 4, for $p \equiv 5 \pmod{8}$ a prime | 241 |