DIFFERENTIABLY $k$-NORMAL ANALYTIC SPACES AND EXTENSIONS OF HOLOMORPHIC DIFFERENTIAL FORMS

Lawrence James Brenton
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In this paper the concept of normality for a complex analytic space $X$ is strengthened to the requirement that every local holomorphic $p$-form, for all $0 \leq p \leq$ some integer $k$, defined on the regular points of $X$ extend across the singular variety. A condition for when this occurs is given in terms of a notion of independence, in the exterior algebra $\Omega^N_\mathbf{R}$, of the differentials $dF_1, \ldots, dF_r$ of local generating functions $F_i$ of the ideal of $X$ in some ambient polydisc $\Delta^N \subset \mathbb{C}^N$. One result is that for a complete intersection, "$k$-independent implies $(k-2)$-normal" (precise definitions are given below), which extends some ideas of Oka, Abhyankar, Thimm, and Markoe on criteria for normality.

Recall that a complex space $(X, \mathcal{O}_X)$ is normal at a point $x \in X$ if every bounded holomorphic function defined on the regular points in a punctured neighborhood of $x$ extends analytically to the full neighborhood. This is equivalent to the condition that the ring $\mathcal{O}_{X,x}$ be integrally closed in its field of quotients, and except for regular points $x$ in dimension 1 the boundedness requirement is irrelevant: if $\dim X > 1$, $x \in X$ is normal $\iff$ for all sufficiently small neighborhoods $U$ of $x$ the restriction of sections $\Gamma(U, \mathcal{O}_X) \to \Gamma(U - \Sigma, \mathcal{O}_X)$ is an isomorphism, for $\Sigma$ the set of singular points of $X$. In 1974 A. Markoe [6] observed that the basic modern ideas in the topic of cohomology with supports gives a very simple criterion of normality in terms of the homological codimension of the structure sheaf:

**Theorem (Markoe).** Let $(X, \mathcal{O}_X)$ be a reduced complex space with singular set $\Sigma$. Then $\forall x \in X$, if $\text{codh}_x \mathcal{O}_X > \dim_x \Sigma + 1$, then $X$ is normal at $x$.

Here $\text{codh}_x \mathcal{O}_X = \max\{k | \exists \text{ germs } f_1, \ldots, f_k \text{ in the maximal ideal of } \mathcal{O}_{X,x} \text{ such that } \forall i \leq k, \text{ the coset } f_i + \sum_{j<i} f_j \mathcal{O}_{X,x} \text{ is not a zero divisor in the ring } \mathcal{O}_{X,x}[\sum_{j<i} f_j \mathcal{O}_{X,x}]\}$. For the standard concepts of sheaf cohomology with supports and their relation to the algebraic properties of the stalks the reader may consult [5], [8], [9] or [11]. This generalizes earlier results of Oka [7], Abhyankar [1], and Thimm [10] for hypersurfaces and complete intersections.

At about the same time the present writer became interested in the question of extending holomorphic differential forms across sub-
varieties of analytic spaces in an effort to understand the local contribution of singular points to the groups \( H^q(X, \Omega_X^p) \), especially for compact spaces where the dimensions of these groups are important numerical invariants (see [2] and [3] for some results of this sort for hypersurfaces). Since in particular a 0-form is just an analytic function it seems natural to consider spaces with a higher degree of "normality" and to extend and relate Markoe's result to statements about higher order differential forms. For instance we will see below (Proposition 6) that if \( X \) is a complete intersection at each point, then \( X \) is normal if and only if there are no local holomorphic 1-forms supported on the singular set.

**Definition 1.** Let \((X, \mathcal{O}_X)\) be a reduced complex subspace of a domain \( D \subset \mathbb{C}^N \), with ideal sheaf \( \mathcal{I}_X \subset \mathcal{O}_D \). By the sheaf of germs of local holomorphic \( p \)-forms on \( X \) we mean the sheaf on \( X \)

\[
\Omega_X^p = \Omega_D^p / (\mathcal{I}_X \Omega_D^p + d\mathcal{I}_X \wedge \Omega_D^{p-1})
\]

where \( d\mathcal{I}_X \wedge \Omega_D^{p-1} \) is the subsheaf of \( \Omega_D^p \) consisting of those germs of the form \( df \wedge \phi^{p-1} \), \( f \in \mathcal{I}_X \). For \( U \subset X \) an open set, by a holomorphic \( p \)-form on \( U \) we shall mean a section on \( U \) of this sheaf.

**Definition 2.** For \( X \subset D \) as above and \( k \) a non-negative integer, a point \( x \in X \) is said to be differentiably \( k \)-normal if for any integer \( p \leq k \) and any sufficiently small neighborhood \( U \) of \( x \), every holomorphic \( p \)-form \( \omega^p \) defined on the regular points of \( U \) extends to a holomorphic \( p \)-form \( \omega^p \) on all of \( U \). \( X \) itself is differentiably \( k \)-normal if each of its points is differentiably \( k \)-normal. That is, \( X \) is differentiably \( k \)-normal if \( \forall p \leq k \) the restriction of sections \( \Gamma(U, \Omega_X^p) \rightarrow \Gamma(U - \Sigma, \Omega_X^p) \) is surjective for all open sets \( U \), where \( \Sigma \) is the singular set of \( X \).

**Remarks.** It is clear that \( \Omega_X^p \) is coherent and (hence) that \( \forall k \) the set \( \Sigma_k \) of points of \( X \) that are not differentiably \( k \)-normal is a subvariety of \( X \). If \( \dim X > 1 \), then differentiably 0-normal is the same as normal, and \( \Sigma_0 \supseteq \Sigma_1 \supseteq \Sigma_{k-1} \supseteq \cdots \supseteq \Sigma_0 \). The adverb "differently" is used here to distinguish the concept under view from that of the "\( k \)-normality" of Andreotti and Siu [2]. There a space is \( k \)-normal if the \( k \)th gap sheaf \( \mathcal{O}_X^{(k)} \) is equal to the structure sheaf \( \mathcal{O}_X \)—that is, if holomorphic functions always extend across subvarieties of dimension \( \leq k \). I thank the referee for drawing my attention to this terminology.

The main result of [3] has the consequence that if \( X \) is locally a hypersurface, then \( X \) is differentiably \( k \)-normal but not differentiably
To give a concrete example, put
\[ F(z_1, \ldots, z_{n+1}) = (z_1)^m + \cdots + (z_{n+1})^m \]
for some integer \( m \geq 2 \), and let \( X \subset C^{n+1} \) be the Fermat cone defined by \( F = 0 \). \( X \) has one singular point, the origin in \( C^{n+1} \), and it is easy to establish (by Corollary 5 below, for instance) that \( X \) is differentiably \((n - 2)\)-normal.

To show that \( X \) is not, however, differentiably \((n - 1)\)-normal, denote by \( U_i, i = 1, \ldots, n + 1 \), the affine set \( \{ z_i \neq 0 \} \subset C^{n+1} \), and define a holomorphic \((n - 1)\)-form \( \omega_i^{n-1} \) on \( U_i \) by
\[
\omega_i^{n-1} = (z_i)^{1-m} \sum_{l \neq i} (-1)^{\sigma_{il}} z_l dz_1 \wedge \cdots \wedge \hat{dz}_i \cdots \wedge dz_{n+1}.
\]
Here \( \hat{\cdot} \) means "omit this factor", and \( \sigma_{il} = 0 \) if \((i, l, 1, 2, \ldots \) \( i \ldots \), \( n + 1) \) is an even permutation of \( (1, 2, \ldots, n + 1) \), 1 if an odd permutation. Direct computation verifies that on \( U_i \cap U_j (i < j) \),
\[
\omega_i^{n-1} - \omega_j^{n-1} = F \psi_i^{n-1} + dF \wedge \varphi_i^{n-2}
\]
for
\[
\psi_i^{n-1} = (-1)^{\tau_{ij}} (z_i z_j)^{1-m} dz_1 \wedge \cdots \wedge \hat{dz}_i \cdots \wedge dz_{n+1}
\]
and for
\[
\varphi_i^{n-2} = (m - 1)^{-1} (z_i z_j)^{1-m} \sum_{l \neq i, j} (-1)^{\tau_{ij}} z_l dz_1 \wedge \cdots \wedge \hat{dz}_i \cdots \wedge dz_{n+1},
\]
where \( \tau_{ij} = 0 \) if \( 1 < l < j \), 1 otherwise. Thus the \( \omega_i^{n-1} \) together comprise a well-defined section \( \Omega_i^{n-1} \) of \( \Omega_X^{n-1} \) on \( X - \{0\} \).

But \( \omega_i^{n-1} \) does not extend across 0. For if it did, then (since \( C^{n+1} \) is Stein) there would exist a globally defined \((n - 1)\)-form \( \tilde{\omega}_i^{n-1} \) on \( C^{n+1} \) satisfying, for all \( i \),
\[
(\ast) \quad \tilde{\omega}_i^{n-1} = \omega_i^{n-1} + F \psi_i^{n-1} + dF \wedge \varphi_i^{n-2}
\]
for some \((n - 1)\)-, \((n - 2)\)-forms \( \psi_i^{n-1}, \varphi_i^{n-2} \) on \( U_i \). Put
\[
\tilde{\omega}_i^{n-1} = \sum_{1 \leq k, l \leq n+1} f_{kl} dz_1 \wedge \cdots \wedge \hat{dz}_k \cdots \wedge dz_{n+1},
\]
\[
\psi_i^{n-1} = \sum_{1 \leq k, l \leq n+1} g_{kl} dz_1 \wedge \cdots \wedge \hat{dz}_k \cdots \wedge dz_{n+1},
\]
\[
\varphi_i^{n-2} = \sum_{1 \leq p < q \leq n+1} h_{pq} dz_1 \wedge \cdots \wedge \hat{dz}_p \cdots \wedge dz_{n+1},
\]
where the \( f \)'s are entire holomorphic functions on \( C^{n+1} \) and the \( g \)'s and \( h \)'s are defined and homomorphic on \( U_i \). Then for \( i < j \), equating
coefficients of $dz_1 \wedge \cdots \wedge dz_i \cdots \wedge dz_{n+1}$ in (*) gives, on $U_i$,

$$f_{ij} = (-1)^{s_{ij}} z_j(z_i)^{1-m} + F g_{ij} + (m - 1) \sum_{l \neq i, j} z_l \tilde{h}_{iljl}$$

for

$$\tilde{h}_{iljl} = \begin{cases} (-1)^{l+1} h_{iljl} & \text{for } l < i \\ (-1)^i h_{iljl} & \text{for } i < l < j \\ (-1)^{l+1} h_{iljl} & \text{for } l > j \end{cases}$$

Now let $\alpha$ be an $m$th root of $-1$, and evaluate (**) along the punctured line $L_\alpha - \{0\}$, for $L_\alpha$ defined by $z_j = \alpha z_i$, $z_l = 0$ for $l \neq i, j$, to conclude

$$f_{ij} = (-1)^{s_{ij}} \alpha z^{2-m}$$
on $L_\alpha - \{0\}$. Thus if $m > 2$, $f_{ij}$ cannot be defined at the origin, a contradiction. If $m = 2$, then $f_{ij} = (-1)^{s_{ij}} \alpha$ on $L_\alpha - \{0\}$, while if $\beta \neq \alpha$ is the other square root of $-1$, then similarly $f_{ij} = (-1)^{s_{ij}} \beta$ on $L_\beta - \{0\}$. Thus in this case also $f_{ij}$ cannot be defined at 0. This contradiction shows that $\omega^{n-1}$ cannot be extended from $X - \{0\}$ to all of $X$, and hence that $X$ is not differentiably $(n-1)$-normal.

Actually, much more can be said about $(n-1)$-forms on this space $X$. For $m_1, \ldots, m_{n+1}$ integers, with $0 \leq m_l \leq m - 2$ for all $l$, define a holomorphic $(n-1)$-form on $X - \{0\}$ by

$$\omega^{n-1}_{m_1, \ldots, m_{n+1}} = \left( \prod_{l=1}^{m} (z_l)^{m_l} \right) \omega^{n-1}.$$ 

By the same argument as above for $\omega^{n-1}$, $\omega^{n-1}_{m_1, \ldots, m_{n+1}}$ does not extend across 0. In fact, the set $\{\omega^{n-1}_{m_1, \ldots, m_{n+1}}\}$ for all such indices $m_1, \ldots, m_{n+1}$ forms a basis over $C$ for the quotient of stalks $\Omega_{X - \{0\}, 0}^{n-1} / \Omega_{X, 0}^{n-1}$. That is, if $U$ is any neighborhood of 0 in $X$, then every holomorphic $(n-1)$-form $\zeta^{n-1}$ on $U - \{0\}$ can be written uniquely

$$\zeta^{n-1} = p \omega^{n-1} + \eta^{n-1}$$

where $p$ is a polynomial in $z_i, \cdots, z_{n+1}$ with constant coefficients and of degree at most $m - 2$ in each variable $z_i$, and where $\eta^{n-1}$ is a holomorphic $(n-1)$-form on all of $U$. This example quite easily generalizes to the Brieskorn varieties $(z_1)^{p_1} + \cdots + (z_{n+1})^{p_{n+1}} = 0$.

We want next to introduce a notion of independence of differential forms, which is our main tool in studying differentiable $k$-normality.

**DEFINITION 3.** Let $R$ be a commutative ring, let $M$ be the free $R$-module on generators $dx^1, \cdots, dx^N$, and denote by $\Lambda^* M$ the total exterior algebra of $M$. A sequence $\Phi_1, \cdots, \Phi_r \in M$ is called $k$-inde-
pendent over $R$ if $\forall p \leq k$ and $\forall i \leq r$, if

$$\omega_i^p \wedge \phi_i = \sum_{j \leq i} \omega_j^p \wedge \phi_j$$

for some $\omega_j^p \in \Lambda^p M$, $j = 1, \ldots, i$, then $\exists \varphi_j^{p-1} \in \Lambda^{p-1} M$, $j = 1, \ldots, i$, such that

$$\omega_i^p = \sum_{j \leq i} \varphi_j^{p-1} \wedge \phi_j.$$ 

**Theorem 4.** Let $(X, \mathcal{O}_X)$ be a reduced subspace of a domain $D$ in $\mathbb{C}^n$. Denote by $\mathcal{I}_X \subset \mathcal{O}_D$ the ideal sheaf defining $X$ and by $\sum$ the set of singular points of $X$. Let $x \in X$ and suppose that for some integer $k \geq 0$,

(i) $\text{codh}_z \mathcal{O}_X > \dim_x \sum + k + 1$, and

(ii) there exist generators $F_1, \ldots, F_r$ of $\mathcal{I}_{x,z}$ such that $dF_1, \ldots, dF_r$ are $k$-independent over $\mathcal{O}_{x,z}$.

Then for all integers $p, q$ with $p + q \leq k + 1$,

$$(\mathcal{X}_z^g \Omega_X^p)_{x} = 0.$$  

**Proof.** Without loss of generality assume that the functions $F_i$ are defined throughout $D$ and generate $\mathcal{I}_X$ at each point of $D$. Put $X_0 = D$, $X_1 = V(F_1)$, $X_2 = V(F_1, F_2)$, $\ldots$, $X_r = X$, where $V(F_1, \ldots, F_i)$ means the variety of $F_1, \ldots, F_i$ with ideal sheaf $\sum_{j \leq i} F_j \mathcal{O}_D$. Fix $k' \leq k + 1$. We will prove by induction on $i$ that $\forall i$

$$(*) \quad \mathcal{X}_z^g (\Omega_{X_i}^p \otimes \mathcal{O}_X)_{x} = 0 \forall q + p = k'.$$

The case $i = r$, then, is the desired result.

If $i = 0$, $\Omega_{X_i}^p \otimes \mathcal{O}_X = \Omega_{D}^p \otimes \mathcal{O}_X$ is free, so $(*)$ holds by the condition $q \leq k' < \text{codh}_z \mathcal{O}_X - \dim_x \sum$ ([9], Theorem 1.14). Now let $i > 0$ and assume inductively that $\mathcal{X}_z^g (\Omega_{X_i}^p \otimes \mathcal{O}_X)_{x} = 0 \forall q + p = k'$. We have the complex

$$(**) \quad 0 \rightarrow \mathcal{O}_X^{p-1} \otimes \mathcal{O}_X \xrightarrow{\rho_{dF}^{p-1}} \Omega_{X_{i-1}}^{p-1} \otimes \mathcal{O}_X \xrightarrow{\rho_{dF}^{(1)}_{i-1}} \Omega_{X_{i-1}}^{p} \otimes \mathcal{O}_X \rightarrow \cdots $$

$$\rightarrow \Omega_{X_{i-2}}^{p} \otimes \mathcal{O}_X \xrightarrow{\rho_{dF}^{(p-2)}_{i-1}} \Omega_{X_{i-1}}^{p} \otimes \mathcal{O}_X \xrightarrow{\pi_i^{(p)}} \Omega_X^p \otimes \mathcal{O}_X \rightarrow 0,$$

where $\rho_{dF}^{(j)} : \Omega_{X_{i-1}}^j \otimes \mathcal{O}_X \rightarrow \Omega_{X_{i-1}}^{j+1} \otimes \mathcal{O}_X$ is induced by right wedge multiplication by $dF_i$ and where $\pi_i^{(p)}$ is the natural projection. Since $dF_1, \ldots, dF_r$ are at least $(p - 1)$-independent over $\mathcal{O}_{x,z}$ at $x$, $(**)$ is exact at $x$. Hence for $j = 1, \ldots, p - 1$, the sequences

$$0 \rightarrow \text{im} \rho_{dF}^{(j-1)} \rightarrow \Omega_{X_{i-1}}^{j} \otimes \mathcal{O}_X \rightarrow \text{im} \rho_{dF}^{(j)} \rightarrow 0$$

are exact at $x$, and at the last stage, so also is
0 \longrightarrow \text{im } \rho^{(p-1)}_{dF_i} \longrightarrow \Omega_{X_{i-1}}^p \otimes \mathcal{O}_X \longrightarrow \Omega_{X_i}^p \otimes \mathcal{O}_X \longrightarrow 0 .

Taking \( \mathcal{H}^0 \) at \( x \), this yields

\[ \cdots \longrightarrow \mathcal{H}_{X_{i-1}}^{p+q-i}(\Omega_{X_{i-1}}^i \otimes \mathcal{O}_X)_x \longrightarrow \mathcal{H}_{X_i}^{p+q-i}(\text{im } \rho^{(j)}_{dF_i})_x \longrightarrow \mathcal{H}_{X_i}^{p+q-i+1}(\text{im } \rho^{(j-1)}_{dF_i})_x \longrightarrow \cdots \]

and

\[ \cdots \longrightarrow \mathcal{H}_{X_i}^p(\Omega_{X_{i-1}}^i \otimes \mathcal{O}_X)_x \longrightarrow \mathcal{H}_{X_i}^p(\Omega_{X_i}^p \otimes \mathcal{O}_X)_x \longrightarrow \mathcal{H}_{X_i}^{p+1}(\text{im } \rho^{(p-1)}_{dF_i})_x \longrightarrow \cdots . \]

By the inductive hypothesis, the first group in each of these triples vanishes, so the induced maps

\[ \mathcal{H}_{X_i}^p(\Omega_{X_{i-1}}^i \otimes \mathcal{O}_X)_x \longrightarrow \mathcal{H}_{X_i}^{p+1}(\text{im } \rho^{(p-1)}_{dF_i})_x \longrightarrow \mathcal{H}_{X_i}^{p+2}(\text{im } \rho^{(p+1)}_{dF_i})_x \longrightarrow \cdots \]

\[ \longrightarrow \mathcal{H}_{X_i}^{p+q-i}(\text{im } \rho^{(j)}_{dF_i})_x \longrightarrow \mathcal{H}_{X_i}^{p+q}(\text{im } \rho^{(0)}_{dF_i})_x \cong (\mathcal{H}_{X_i}^{p+q} \mathcal{O}_X)_x = 0 , \]

are all injective. Hence in particular \( \mathcal{H}_{X_i}^p(\Omega_{X_{i-1}}^i \otimes \mathcal{O}_X)_x = 0 \).

**COROLLARY 5.** Let \( X \subset D \subset \mathbb{C}^N \) be a complete intersection of dimension \( n \). Suppose that \( \mathcal{I}_X \subset \mathcal{O}_D \) has generators \( F_1, \cdots, F_{N-n} \) whose varieties meet transversally and are such that \( dF_1, \cdots, dF_{N-n} \) are \( k \)-independent in any order over \( \mathcal{O}_X \) at each point of \( X \). Then \( X \) is differentiably \((k-2)\)-normal.

**Proof.** It is shown in [3] (Lemmas 1 and 2 and Remark 6) that the single function \( F_i \) is \( k \)-independent over \( \mathcal{O}_X \) at \( x \) for some choice of local co-ordinates \( z^1, \cdots, z^N \) in \( D \) the derivatives \( \partial F_i/\partial z^1, \cdots, \partial F_i/\partial z^k \) form a regular \( \mathcal{O}_{X,x} \)-sequence \( \Longleftrightarrow \text{codim}_x(\sum_i \cap X) \geq k \) at \( x \), for \( x \) the singular set of the variety \( X \) of \( F_i \). Since the \( V(F_i) \) meet transversally, \( \sum = \bigcup_{i=1}^n (\sum_i \cap X) \). Thus \( \dim \sum = \max(\dim_z(\sum_i \cap X)) \leq n - k = \text{codim}_z \mathcal{O}_X - k \), at each point \( x \in X \). By the theorem, then, \( \mathcal{H}_{X}^q \Omega_X^p = 0 \ \forall \, p + q \geq k - 1 \). Taking \( q = 1 \) we conclude that \( \mathcal{H}_{X}^1 \Omega_X^p = 0 \ \forall \, p \leq k - 2 \). That is, for every open set \( U, \mathcal{H}_X^1(U, \Omega_X^p) = 0 \). The conclusion now follows from the sequence

\[ H^0(U, \Omega_X^p) \longrightarrow H^0(U - \sum, \Omega_X^p) \longrightarrow H^1(U, \Omega_X^p) . \]

**REMARK.** Taking \( q = 0 \) in the conclusion to Theorem 4 (respectively, in its application in Corollary 5) shows that such spaces have no local holomorphic \( p \)-forms supported on \( \sum \) for \( p = 0, 1, \cdots, k + 1 \) (respectively, for \( p = 0, 1, \cdots, k - 1 \)). This observation suggests the characterization of normality mentioned in the introduction:

**PROPOSITION 6.** Let \( x \) be a point of a reduced analytic space
(X, \mathcal{O}_X) such that X is a complete intersection at x. Then X is normal at x if \( H^2_x(U, \Omega^1_X) = 0 \) for all sufficiently small neighborhoods U of x, for \( \sum \) the set of singular points of X.

**Proof.** Complete intersections have 0-independent generators. Namely, if locally X is an n-dimensional subvariety of N-dimensional polydisc \( \Delta^N \), and if \( F_1, \ldots, F_{N-n} \) generate the ideal of X in \( \Delta^N \), then at the regular points of X near x the differentials \( dF_i, \ldots, dF_{N-n} \) are independent over C. That is, if \( g_i dF_i = \sum_{j<i} g_j dF_j \), then the \( g_i \)'s are identically 0 most places in a neighborhood of x, hence everywhere.

Now Theorem 4 applies. X is normal at x if \( \text{codim}_x \sum > 1 \Rightarrow (\mathbb{H}^0_x \Omega^1_X)_x = 0 \Rightarrow H^2_x(U, \Omega^1_X) = 0 \forall \) sufficiently small neighborhoods U of x.

Conversely, \( (\mathbb{H}^0_x \Omega^1_X)_x = 0 \Rightarrow dh_x \Omega^1_x < \text{codim}_x \sum \) ([9], Theorem 1.14). If \( \text{codim}_x \sum \) were equal to 1, this would mean that \( dh_x \Omega^1_x = 0 \) and \( \Omega^1_x, x \) is free. But then x is a regular point of X, contradicting \( \text{codim}_x \sum = 1 \). (If \( \text{dim} X = 1 \), we should look at \( \mathbb{H}^0_x \Omega^1_X \) throughout, and at this point achieve not a contradiction but the assertion that x is regular, hence normal.) The alternative is \( \text{codim}_x \sum > 1 \), which implies normality by the Oka-Abhyankar-Thimm-Markoe criterion, or by Theorem 4.

**Added in proof.** It has recently come to the author's attention that similar results have been obtained by G.-M. Greuel, *Der Gauss-Manin-Zusammenhang isolierter Singularitäten von vollständigen Durchschnitten* Math. Ann., 214 (1975), 235–266. For isolated singularities of hypersurfaces the topic was first considered from the present point of view by Brieskorn, *Die Monodromie der isolierten Singularitäten von Hyperflächen*, Manuscripta Math., 2 (1970), 103–161.

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