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The number of Weierstrass points on a compact Riemann surface of finite genus g is at most $(g - 1)g(g + 1)$ and at least $2(g + 1)$. After the Riemann-Roch's theorem for the class of canonical semi-exact differentials, Watanabe considered the number of Weierstrass points on an open Riemann surface of class O_{KD}. In this paper it will be shown that Watanabe's estimate can be proved without any conception of principal operators.

Using the notation and terminology of [1], the following theorem [6, Theorem 2] will be proved without use of the results of Mori [3], Rodin [4] and Royden [5]. Note that a meromorphic function on an open Riemann surface is said to be rational if Re df is distinguished.

THEOREM. *Suppose that R is a Riemann surface of finite genus g on which $\Gamma_{he} \cap \Gamma_{hse}^{*} \subset \Gamma_{he}^{*}$ holds. Then the number of Weierstrass points on R is at most $(g - 1)g(g + 1)$.*

Let S be a compact continuation of R such that the genus of S is g. Suppose P is a Weierstrass points on R and f is a rational function on R which has the only singularity of order at most g at P. Let D be a closed disk with $P \in \overset{\circ}{D} \subset D \subset R$. Then the Dirichlet integral of f over $R - D$ is finite. Since $\int_{\partial D} d \operatorname{Re} f^{*} = 0$, there exist harmonic functions u on $S - D$ and v on R such that $u - v = \operatorname{Re} f$ on $R - D$. Thus we have $dv \in \Gamma_{he} \cap \Gamma_{h}^{*}$.

We wish to show that dv^{*} is semi-exact on R. If c is a dividing cycle on $R - D$, then c is homologous to zero on $S - D$. This gives that

$$\int_{c} dv^{*} = \int_{c} du^{*} - \int_{c} d \operatorname{Im} f = 0 \ .$$

Since $dv \in \Gamma_{he} \cap \Gamma_{hse}^{*}$, it follows from the assumption that $dv^{*} \in \Gamma_{he}$.
We define

$$\lambda = \begin{cases} du & \text{on } S - D \\ dv + d \operatorname{Re} f & \text{on } R \ . \end{cases}$$

Then λ and λ^{*} have no periods along any cycle b on S, where $b \not\ni P$. Therefore $\int \lambda + i\lambda^{*}$ is a meromorphic function on S. It is easy to

see that $\int \lambda + i\lambda^*$ has as its only singularity a pole of order at most g at P. This shows that P is a Weierstrass point on S. Due to the classical result on the Weierstrass points on compact Riemann surfaces our assertion is obtained.

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Pacific Journal of Mathematics