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ON EMBEDDING SEMIFLOWS INTO A RADIAL FLOW ON l_2

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Let π be a semiflow on a separable metric space X such that the negative escape time function is lower semicontinuous and $x \to x\pi t$ is a one-to-one mapping for each $t \in R^+$. If π has a globally uniformly asymptotically stable critical point, then π can be embedded into a radial flow on l_2. This generalizes known results on embedding flows or semiflows into radial flows on l_2.

1. Introduction. In [3] L. Janos showed that a semiflow π on a compact metric space X satisfying

(i) $\cdot \pi t$ is one-to-one for every $t \in R^+$

(ii) there is a $p \in X$ such that $\cap \{X\pi t: t \geqq 0\} = \{p\}$ can be embedded into a radial flow on l_2. In [2] M. Edelstein generalized this result to

THEOREM I. *Let π be a semiflow on a separable metric space X satisfying*

(a) *for each $t \in R^+$, $x \to x\pi t: X \to X$ is a homeomorphism, of X onto a closed subset of X,*

(b) *there is a $p \in X$ such that for each neighborhood U of p there is a $T \in R^+$ such that $X\pi t \subset U$ for all $t \geqq T$.*
Then π can be embedded into a radial flow on l_2.

Evidently properties (a) and (b) generalize properties (i) and (ii) respectively. Note that property (b) imposes a type of compactness on the semiflow. For example, a radial flow on l_2 can be embedded into itself trivially, but such a flow does not have property (b).

In this paper we further generalize properties (a) and (b) to

(c) $x \to x\pi t$ is one-to-one for each $t \in R^+$,

(d) the negative escape time function is lower semicontinuous,

(e) π has a globally uniformly asymptotically stable critical point p.

We will show (Corollary 8) that property (a) implies properties (c) and (d). Evidently property (b) implies property (e). Property (e) imposes a type of local compactness on the semiflow. Notice that a radial flow on l_2 does satisfy property (e).

The principal result of this paper, Theorem 7, generalizes every other result known to the author concerning embedding flows or semiflows into radial flows on l_2.

2. Notation and definitions. Throughout this paper R and R^+ will denote the reals and nonnegative reals respectively. A flow on a topological space X is a continuous mapping $\pi: X \times R \to X$ such that (where $x\pi t = \pi(x, t)$) $x\pi 0 = x$ for all $x \in X$ and $(x\pi t)\pi s = x\pi(t + s)$ for all $x \in X$ and $t, s \in R$. If R is replaced by R^+ in the previous sentence, then π is called a semiflow. A point p of X is called a critical point of π if $p\pi t = p$ for all $t \in R$ (or $t \in R^+$ if π is a semiflow). A compact subset M of X is said to be stable with respect to π if for any neighborhood U of M there is a neighborhood V of M such that $V\pi R^+ \subset U$. A compact subset M of X is said to be a global attractor if for any neighborhood U of M and any $x \in X$ there is a $d \in R^+$ such that $x\pi[d, \infty) \subset U$. The compact set M is called a global uniform attractor if it is a global attractor and if there is a neighborhood U of M such that for any neighborhood $V \subset U$ of M there is a $c \in R^+$ such that $U\pi[c, \infty) \subset V$. A stable global (uniform) attractor is said to be globally (uniformly) asymptotically stable.

A continuous function $L: X \to R^+$ is called a Liapunov function for a compact subset M of X if $L(x\pi t) < L(x)$ for every $x \in X - M$ and $0 < t$, $L(x\pi t) \to 0$ as $t \to \infty$ for every $x \in X$, and $L(x) = 0$ if $x \in M$. Let M be a compact asymptotically stable subset of X. A straightforward argument shows that if $x \in X - M$ and if U is any neighborhood of M, then there is a neighborhood V of x and a $T > 0$ such that $V\pi[T, \infty) \subset U$. With this observation the proof of the following theorem is essentially identical with that of Theorem 10 in [1].

THEOREM II. *A compact subset M of a metric space X is globally asymptotically stable with respect to a semiflow π if and only if there is Liapunov function for M.*

Let X and Y be topological spaces on which are defined flows (semiflows) π and ρ respectively. We say that π can be embedded into ρ if there is a homeomorphism h of X onto a subset of Y such that $h(x\pi t) = h(x)\rho t$ for every $x \in X$ and $t \in R (t \in R^+)$.

The set of all sequences $x = \{x_1, x_2, \cdots, x_n, \cdots\}$ of real numbers such that $\sum_{n=1}^{\infty} x_n^2$ converges is denoted by l_2. If addition and scalar multiplication are defined coordinatewise and if a norm is defined by $\|x\| = (\sum_{n=1}^{\infty} x_n^2)^{1/2}$, then l_2 is a real Banach space. A flow ρ on l_2 is called a radial flow if there is a $c \in (0, 1)$ such that $x\rho t = c^t x$ for every $(x, t) \in l_2 \times R$.

Let π be a semiflow on X. The function $\alpha: X \to [-\infty, 0]$ defined by $\alpha(x) = \inf\{-t\colon$ there exists $y \in X$ with $y\pi t = x\}$ is called the negative escape time function. Throughout this paper we shall assume that α is lower semicontinuous, i.e., $\alpha(x) \leqq \lim_{y\to x} \inf \alpha(y)$. It is an elementary exercise to show that $\alpha(x\pi t) = \alpha(x) - t$ for all $t \geqq 0$ and $x \in X$.

3. The embedding. Henceforth, π shall denote a semiflow on a separable metric space X satisfying

(1) $x \to x\pi t$ is one-to-one for each $t \in R^+$,

(2) the negative escape time function is lower semicontinuous,

(3) π has a globally unformly asymptotically stable critical point p.

Also, U shall denote a neighborhood of p such that for any neighborhood $V \subset U$ of p, there is a $T > 0$ such that $U\pi[T, \infty) \subset V$.

Let $t < 0$ and $x \in X$. Since $\cdot\pi(-t)$ is one-to-one there is at most one $y \in X$ with $y\pi(-t) = x$. If such a y exists then we shall denote this y by $x\pi t$. It is a straightforward exercise to show that if $s, t \in R$ and $x \in X$, then $(x\pi t)\pi s = x\pi(t + s)$ whenever each side of the equality is defined. Suppose that $\{x_i\}$ and $\{t_i\}$ are sequences in X and R converging to $x \in X$ and $t \in R$ respectively. Using property 2 it is easy to show that if $x_i\pi t_i$ is defined for each i, then $x\pi t$ is defined and $x_i\pi t_i \to x\pi t$ as $i \to \infty$.

LEMMA 1. *If* $x\pi(\alpha(x), 0] \subset U$, *then* $-\infty < \alpha(x)$.

Proof. Let $V \subset U$ be a neighborhood of p such that $V\pi R^+ = V$ and $x \notin V$. Then $x\pi(\alpha(x), 0] \cap V = \phi$. Let $T > 0$ be such that $U\pi T \subset V$. Then $x\pi(\alpha(x) + T, \infty) \subset V$. In order that this be consistent with $x\pi(\alpha(x), 0] \cap V = \phi$, we must have $\alpha(x) \neq -\infty$.

LEMMA 2. *Let* σ *be a semiflow on a metric space* Z. *If*

(i) *the negative escape time function* γ *is lower semicontinuous,*

(ii) *each trajectory contains a start point,* i.e., *for each* $x \in Z$ *there is a* $y \in Z$ *such that* $y\sigma(-\gamma(x)) = x$,

then $Z\pi t$ *is a closed subset of* Z *for each* $t \geqq 0$.

Proof. Let $t \geqq 0$ and let $\{x_i\}$ be a sequence in Z such that $x_i\sigma t \to y$ for some $y \in Z$. Then $\gamma(x_i\sigma t) \leqq -t$ for every i so that $\gamma(y) \leqq -t$. By (ii) there is a $z \in Z$ such that $z\sigma(-\gamma(y)) = y$. Then $y = (z\pi(-\gamma(y) - t))\sigma t \in Z\sigma t$. It follows that $Z\sigma t$ is a closed subset of Z.

Let L be a Liapunov function for p (Theorem II) and let λ be any number in the range of L such that $L^{-1}([0, \lambda]) \subset U$. Set

$$Y = \{x \in L^{-1}([0, \lambda]) \colon \alpha(x) \leqq -1 \text{ and } x\pi(-1, \infty) \subset L^{-1}([0, \lambda])\}$$

and let σ denote the semiflow obtained by restricting π to $Y \times R^+$. Let β denote the negative escape time function with respect to σ.

We will show that σ satisfies the hypotheses of Theorem I. Hence, σ can be embedded into a radial flow on l_2. We will then extend this embedding to an embedding of π into a radial flow.

LEMMA 3. *For every $x \in Y$ there is a $y \in Y$ such that $y\sigma(-\beta(x)) = x$.*

Proof. There are two cases to consider: $x\pi(\alpha(x), \infty) \subset L^{-1}([0, \lambda))$ and $x\pi(\alpha(x), \infty) \cap L^{-1}(\lambda) \neq \phi$. In the latter case there is a unique $z \in x\pi(\alpha(x), \infty) \cap L^{-1}(\lambda)$ and a unique $t \in R$ such that $z\pi t = x$. Since $x \in Y$ we must have $1 \leqq t$. Then $\beta(x) = -t + 1$. Set $y = z\pi 1$. Then $y \in Y$ and $y\sigma(-\beta(x)) = y\pi(-\beta(x)) = (z\pi 1)\pi(t - 1) = z\pi t = x$. Now suppose $x\pi(\alpha(x), \infty) \subset L^{-1}([0, \lambda))$. Then $x\pi(\alpha(x), \infty) \subset U$ so that, by Lemma 1, $-\infty < \alpha(x)$. Since $x \in Y$ we must have $\alpha(x) \leqq -1$. Let $y \in x\pi(\alpha(x), \infty)$ be such that $y\pi(-\alpha(x) - 1) = x$. Since $\alpha(x) = \alpha(y\pi(-\alpha(x) + 1)) = \alpha(y) + \alpha(x) + 1$ we have $\alpha(y) = -1$. If $y = z\pi t$ for some $t > 0$ then $-1 = \alpha(y) = \alpha(z\pi t) = \alpha(z) - t$ so that $\alpha(z) = t - 1 > -1$. It follows that $\beta(x) = \alpha(x) + 1$ and that $y\sigma(-\beta(x)) = x$. This completes the proof.

LEMMA 4. *Let $\{x_i\}$ be a sequence such that $x_i \to x$ for some $x \in X$. If there exists a $t \in R$ such that $x\pi t \in L^{-1}(\lambda)$, then either $t \leqq \liminf \alpha(x_i)$ or there are a subsequence $\{x_j\}$ of $\{x_i\}$ and a sequence $\{t_j\}$ in R such that $x_j\pi t_j \in L^{-1}(\lambda)$. In the latter case $t_j \to t$.*

Proof. Suppose $\liminf \alpha(x_i) < t$. Let $\{x_j\}$ be a subsequence of $\{x_i\}$ such that $\alpha(x_j) \to \liminf \alpha(x_i)$. For any $\delta \in (0, t - \liminf \alpha(x_i))$ eventually $\alpha(x_j) < t - \delta$. Also $\alpha(x) \leqq t - \delta$ because $\alpha(x) \leqq \liminf \alpha(x_i)$. Since $L(x\pi(t - \delta)) > L(x\pi t) = \lambda > L(x\pi(t + \delta))$ we have $L(x_j\pi(t - \delta)) > \lambda > L(x_j\pi(t + \delta))$ eventually. Hence, there are $t_j \in (t - \delta, t + \delta)$, eventually, such that $L(x_j\pi t_j) = \lambda$. Since δ can be chosen arbitrarily small we must have $t_j \to t$.

LEMMA 5. *β is lower semicontinuous.*

Proof. Let $x \in Y$ and let $\{x_i\}$ be a sequence in Y such that $x_j \to x$. Let $\{x_j\}$ be any subsequence of $\{x_i\}$ such that $\beta(x_j) \to \beta$ for some $\beta \in [-\infty, 0]$. There are two cases to consider: $x\pi t \in L^{-1}(\lambda)$ for some $t \in R$ and $x\pi(\alpha(x), \infty) \subset L^{-1}([0, \lambda))$. If $x\pi t \in L^{-1}(\lambda)$ for some t, then by Lemma 4 either $\alpha(x) \leqq t \leqq \liminf \alpha(x_j)$ or there are a subsequence $\{x_k\}$ of $\{x_j\}$ and a sequence $\{t_k\}$ in R such that $t_k \to t$ and $x_k\pi t_k \in$

$L^{-1}(\lambda)$. If $t \leqq \liminf \alpha(x_j)$, then $\beta(x) = t + 1$ and $\beta(x_j) = \alpha(x_j) + 1$ so that $\beta(x) \leqq \liminf \beta(x_j) = \beta$. If there are a subsequence $\{x_k\}$ of $\{x_j\}$ and a sequence $\{t_k\}$ in R such that $t_k \to t$ and $x_k\pi t_k \in L^{-1}(\lambda)$, then $\beta(x) = t+1$ and $\beta(x_k) = t_k + 1$. Then $\beta(x) = \lim \beta(x_k) = \beta$. Thus if $x\pi t \in L^{-1}(\lambda)$, then $\beta(x) \leqq \beta$. It follows that $\beta(x) \leqq \liminf \beta(x_i)$ whenever $x\pi t \in L^{-1}(\lambda)$ for some $t \in R$. Now suppose that $x\pi(\alpha(x), \infty) \subset L^{-1}([0, \lambda))$. Then $\beta(x) = \alpha(x) + 1$. Again there are two cases to consider: $x_i\pi(\alpha(x_i), \infty) \subset L^{-1}([0, \lambda))$ for every i and there exist a subsequence $\{x_n\}$ of $\{x_i\}$ and a sequence $\{s_n\}$ in R such that $x_n\pi s_n \in L^{-1}(\lambda)$ for every n. In the former case we have $\beta(x_i) = \alpha(x_i) + 1$ and $\beta(x) \leqq \liminf \beta(x_i)$ since α is lower semicontinuous. In the latter case, let $V \subset U$ be a neighborhood of p such that $x \notin \overline{V\pi R^+}$ and let $T > 0$ be such that $U\pi[T, \infty) \subset V$. Then $L^{-1}(\lambda)\pi[T, \infty) \subset V$ and we must have $s_n \in [0, T]$ for all n sufficiently large. Let s be any accumulation point of $\{s_n\}$ and let $\{s_j\}$ be a subsequence of $\{s_n\}$ such that $s_j \to s$. Then $x_j\pi s_j \in L^{-1}(\lambda)$ and $x_j\pi s_j \to x\pi s$. Hence, $x\pi s \in L^{-1}(\lambda)$ which contradicts our assumption that $x\pi(\alpha(x), \infty) \subset L^{-1}([0, \lambda))$. It follows that $\beta(x) \leqq \liminf \beta(x_i)$ whenever $x\pi(\alpha(x), \infty) \subset L^{-1}([0, \lambda))$. Combining this with the result $\beta(x) \leqq \liminf \beta(x_i)$ whenever $x\pi t \in L^{-1}(\lambda)$ for some $t \in R$ obtained earlier in the proof, we conclude that β is lower semicontinuous.

Collecting together the above results we have that

(i) σ is a semiflow on the separable metric space Y,

(ii) if V is a neighborhood in Y of p, then there is a $T > 0$ such that $Y\sigma[T, \infty) \subset V$, (This follows directly from the facts that $Y \subset U$ and σ is a restriction of π.)

(iii) $Y\sigma t$ is a closed subset of Y for every $t \geqq 0$ (Lemmas 3, 5, and 2).

In light of Theorem I the semiflow σ on Y can be embedded into a radial flow ρ on l_2. Let $c \in (0, 1)$ be such that $x\rho t = c^t x$ and let $h\colon Y \to l_2$ be a homeomorphism of Y onto $h(Y)$ such that $h(x\sigma t) = h(x)\rho t$ for every $(x, t) \in Y \times R^+$. Since σ is a restriction of π we have $h(x\pi t) = h(x)\rho t$ for every $(x, t) \in Y \times R^+$. Now define a mapping $H\colon X \to l_2$ by

$$H(x) = h(x\pi t)\rho(-t)$$

where $t \in R^+$ is such that $x\pi t \in Y$. (H will be shown to be well defined in the following lemma.)

LEMMA 6. *H is a homeomorphism of X onto $H(X)$.*

Proof. We will first show that H is well defined. Clearly for every $x \in X$, there is a $t \geqq 0$ such that $x\pi t \in Y$. Moreover, if $x\pi t \in Y$, then $x\pi(t+s) \in Y$ for every $s \geqq 0$. In order to show that H is well defined it suffices to show that $h(x\pi t)\rho(-t) = h(x\pi(t+s))\rho(-t-s)$ whenever $x\pi t \in Y$ and $s \geqq 0$. Since $x\pi t \in Y$ and $s \geqq 0$ we have $h(x\pi(t+s)) = h((x\pi t)\pi s) = h(x\pi t)\rho s$. Hence $h(x\pi(t+s))\rho(-t-s) = (h(x\pi t)\rho s)\rho(-t-s) = h(x\pi t)\rho(-t)$. The mapping H is well defined. We will now show that H is one-to-one. Suppose that $H(x) = h(x\pi t)\rho(-t)$, $H(y) = h(y\pi s)\rho(-s)$, and $H(x) = H(y)$. Without loss of generality we may assume that $t \geqq s$. Then $H(y) = h(y\pi t)\rho(-t)$ since $y\pi t \in Y$ whenever $y\pi s \in Y$ and $s \leqq t$. Since $H(x) = H(y)$ we must have $h(x\pi t) = h(y\pi t)$. Recalling that h is a homeomorphism we have $x\pi t = y\pi t$ so that $x = y$ since $\cdot\pi t$ is one-to-one. The mapping H is one-to-one. Next we will show that H is continuous. Let $x \in X$ and let $\{x_i\}$ be a sequence in X such that $x_i \to x$. Let $t \in R^+$ be such that $L(x\pi t) < \lambda$. Then $x\pi(t+1) \in Y$. Also for all i sufficiently large $L(x_i\pi t) < \lambda$ and $x_i\pi(t+1) \in Y$. Then $H(x_i) = h(x_i\pi(t+1))\rho(-t-1) \to h(x\pi(t+1))\rho(-t-1) = H(x)$. Hence, H is continuous. Finally we will prove that H^{-1} is continuous. Let $y \in X$ and let $\{y_i\}$ be a sequence in X such that $H(y_i) \to H(y)$. Then there exist $t, t_i \in R^+$ such that $H(y_i) = h(y_i\pi t_i)\rho(-t_i)$ and $H(y) = h(y\pi t)$. Let $s_i = \inf\{s \in R^+ : y_i\pi s \in Y\}$. We will show that $\{s_i\}$ is bounded. Suppose not. Then there is a subsequence $\{s_j\}$ of $\{s_i\}$ such that $s_j \to \infty$. If $y_i \in L^{-1}([0, \lambda])$, then $s_i \leqq 1$. Hence, we may assume $1 \leqq s_j$ and $y_j \notin L^{-1}([0, \lambda])$ for every j. Then $y_j\pi(s_j - 1) \in L^{-1}(\lambda)$. Note that $H(y) \leftarrow H(y_j) = h(y_j\pi s_j)\rho(-s_j) = c^{-s_j}h(y_j\pi s_j)$. Since $s_j \to \infty$ and $c \in (0, 1)$ we have $c^{-s_j} \to \infty$. In order that $c^{-s_j}h(y_j\pi s_j) \to H(y)$ we must also have $h(y_j\pi s_j) \to \bar{0}$ where $\bar{0}$ is the origin in l_2. Since h is a homeomorphism $y_j\pi s_j \to p$ so that $y_j\pi(s_j - 1) \to p$. This is impossible because $y_j\pi(s_j - 1) \in L^{-1}(\lambda)$ and $L(p) = 0$. Hence $\{s_i\}$ must be bounded. Without loss of generality we may suppose that $0 \leqq s_i \leqq t$ for every i. Then $H(y_i) = h(y_i\pi t)\rho(-t) \to h(y\pi t)\rho(-t) = H(y)$ so that $h(y_i\pi t) \to h(y\pi t)$. Since h is a homeomorphism, $y_i\pi t \to y\pi t$ and we have $y_i \to y$. Hence, H^{-1} is continuous and H is a homeomorphism of X onto $H(X) \subset l_2$.

THEOREM 7. *Let π be a semiflow on a separable metric space X such that the negative escape time function is lower semicontinuous and $\cdot\pi t$ is one-to-one for each $t \in R^+$. If π has a globally uniformly asymptotically stable critical point, then π can be embedded into a radial flow on l_2.*

Proof. In light of Lemma 6, we need only show that $H(x\pi s) = H(x)\rho s$ for every $(x, s) \in X \times R^+$. Let $x \in X$ and $t \geqq 0$ be such that $x\pi t \in Y$. Then $(x\pi s)\pi t = x\pi(t+s) \in Y$ and we have $H(x\pi s) =$

$h((x\pi s)\pi t)\rho(-t) = h((x\pi t)\pi s)\rho(-t) = (h(x\pi t)\rho s)\rho(-t) = (h(x\pi t)\rho(-t))\rho s = H(x)\rho s.$

COROLLARY 8. ([2, *Theorem* I].) *Let* π *be a semiflow on a separable metric space having the properties*

(i) $x \to x\pi t$ *is a homeomorphism of* X *onto a closed subset of* X *for each* $t \in R^+$,

(ii) *there is a* $p \in X$ *such that for any neighborhood* U *of* p *there is a* $T \in R^+$ *with* $X\pi t \subset U$ *for all* $t \geqq T$.

Then π *can be embedded into a radial flow on* l_2.

Proof. Clearly (i) and (ii) imply that $\cdot\pi t$ is one-to-one for all $t \in R^+$ and p is globally uniformly asymptotically stable respectively. It remains to show that (i) implies that the negative escape time function α is lower semicontinuous. Suppose that α is not lower semicontinuous. Then there exist $x \in X$, $\delta > 0$, and a sequence $\{x_i\}$ in X such that $x_i \to x$ and $\alpha(x_i) < \alpha(x) - \delta$ for every i. Thus $x_i\pi(\alpha(x)-\delta)$ is defined for every i. Then $(x_i\pi(\alpha(x)-\delta))\pi(-\alpha(x)+\delta) = x_i \to x$ so that $x \in \overline{X\pi(-\alpha(x)+\delta)} = X\pi(-\alpha(x)+\delta)$ since $X\pi t$ is closed for every $t \geqq 0$. Then there exists $z \in X$ such that $z\pi(\alpha(x)-\delta) = x$. This is impossible because $\alpha(x) - \delta < \alpha(x)$ and $\alpha(x) = \inf\{-t\colon$ there exists $y \in X$ with $y\pi t = x\}$. Therefore, we must have that α is lower semicontinuous. The desired result now follows from Theorem 7.

In the proof of Corollary 8 we showed that if $X\pi t$ is a closed subset of X for all $t \in R^+$ then the negative escape time function α is lower continuous. The converse of this is not valid. Let $X = [0, 1)$ and define $\pi\colon X \times R \to X$ by $x\pi t = e^{-t}x$. Evidently π is a semiflow on X. The negative escape time function is defined by

$$\alpha(x) = \begin{cases} \ln x & \text{if } x \neq 0 \\ -\infty & \text{if } x = 0 . \end{cases}$$

Thus α is lower semicontinuous. However $X\pi 1 = [0, e^{-1})$ is not a closed subset of $[0, 1)$. Thus the lower semicontinuity of α does not imply that $X\pi t$ is a closed subset of X for every $t \in R^+$.

COROLLARY 9. (*Theorem* 5 *of* [4].) *Let* π *be a flow on a separable metric space which has a globally asymptotically stable critical point* p. *Then* π *can be embedded into a radial flow on* l_2 *if and only if* p *is globally uniformly asymptotically stable.*

Proof. Since π is a flow $x \to x\pi t$ is one-to-one for every $t \in R^+$ and $\alpha(x) = -\infty$ for every $x \in X$. If p is globally uniformly asymptotically stable, then, by Theorem 7, π can be embedded into a radial flow on ℓ_2. The converse follows easily since the origin in ℓ_2 is globally uniformly asymptotically stable with respect to a radial flow.

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