ON POLYNOMIAL INVARIANTS OF FIBERED 2-KNOTS

KOUHEI ASANO AND KATSUYUKI YOSHIKAWA
ON POLYNOMIAL INVARIANTS OF FIBERED 2-KNOTS

KOUHEI ASANO AND KATSUYUKI YOSHIKAWA

Given any polynomial \( \lambda(t) = \sum_{j=0}^{m} c_j t^j \) satisfying the conditions that \( c_j \) is an integer, \( \lambda(1) = \pm 1, c_0 = 1 \) and \( c_m = \pm 1 \), we will construct a fibered 2-knot in the 4-sphere with the invariants \( \{ \lambda_i(t) \} \) such that \( \lambda_i(t) = \lambda(t) \) and \( \lambda_i(t) = 1 \) for \( i > 1 \)
and \( q = 1, 2 \).

1. Introduction. An \( n \)-knot \( K \) is a smooth submanifold of the \((n + 2)\)-sphere \( S^{n+2} \) which is homeomorphic to \( S^n \). By the exterior of \( K \), we mean the complement of an open tubular neighborhood of \( K \) in \( S^{n+2} \). If the exterior of \( K \) fibers over a 1-sphere, \( K \) is called a fibered \( n \)-knot.

Let \( E \) be the exterior of an \( n \)-knot and \( \tilde{E} \) the infinite cyclic covering of \( E \) with \( \langle t \rangle \) as the covering transformation group. Let \( A \) denote the integral group ring of \( \langle t \rangle \) and \( \Gamma = A \otimes_z Q \) the rational group ring of \( \langle t \rangle \). Since \( \Gamma \) is a principal ideal domain, \( H_q(\tilde{E}, Q) \cong \Gamma/\lambda_i(t) \oplus \cdots \oplus \Gamma/\lambda_n(t) \). In this decomposition, we can take \( \lambda_i(t) \) so that

(i) \( \lambda_i(t) \) is a primitive element and \( \lambda_{i+1}(t)|\lambda_i(t) \) in \( A \). Then \( \{ \lambda_i(t) : 1 \leq i \leq r \} \) are called the polynomial invariants of \( K \) in dimension \( q \), for \( 1 \leq q \leq n \) [3, 6, 7].

In [7], it is shown that polynomial invariants \( \{ \lambda_i(t) : 1 \leq i \leq r \} \), \( 1 \leq q \leq n \) of a fibered \( n \)-knot have the following properties:

(ii) If \( \lambda_i(t) = \sum_{j=0}^{m} c_j t^j \), then \( c_0 = \pm 1 \) and \( c_m = \pm 1 \).

(iii) \( \lambda_i(1) = \pm 1 \).

(iv) \( \lambda_i(t) = \varepsilon t^\alpha \lambda_{-\alpha+1}(t^{-1}) \), \( \varepsilon = \pm 1 \) and \( \alpha \) is an integer.

(v) If \( n = 2q - 1, q \) is even, \( \Delta(t) = \lambda_i(t) \cdots \lambda_q(t) \) is in normal form, i.e., \( \Delta(t) = \Delta(t^{-1}) \) and \( \Delta(1) > 0 \), then \( \Delta(-1) \) is an odd square.

Furthermore, the family \( \{ \lambda_i(t) \} \) satisfying (i)-(v) can be realized as the invariants of a fibered \( n \)-knot, if \( \lambda_i(t) = \lambda_i(t) = 1 \). In this paper, we will prove that

THEOREM. Given any polynomial \( \lambda(t) = \sum_{j=0}^{m} c_j t^j \) satisfying \( \lambda(t) \in A, \lambda(1) = \pm 1, c_0 = 1 \) and \( c_m = \pm 1 \), there exists a fibered 2-knot in the 4-sphere with the invariants \( \{ \lambda_i(t) \} \) such that \( \lambda_i(t) = \lambda(t) \) and \( \lambda_i(t) = 1 \), for \( i > 1 \) and \( q = 1, 2 \).

Using our theorem and the argument in [7], we can show that, for given family \( \{ \lambda_i(t) \} \) which satisfy (i)-(v), there is a fibered \( n \)-knot with \( \{ \lambda_i(t) \} \) as its invariants.
The authors wish to express their gratitude to Prof. A. Kawauchi for helpful suggestions.

2. Proof of Theorem. Let $V_m = B^4 \cup \bigcup \{h_i^{(1)}: 1 \leq i \leq m\}$, where $B^4$ is a 4-ball and $h_i^{(1)}$ is a 1-handle. Then $\pi_1(V_m)$ is a free group freely generated by the elements $x_1, \ldots, x_m$ corresponding to $h_1^{(1)}, \ldots, h_m^{(1)}$, respectively. By $\varphi$, we denote an automorphism of $\pi_1(V_m)$ defined by

$$\varphi(x_i) = \begin{cases} x_{i+1}, & 1 \leq i \leq m - 1, \\ (x_1^{e_1}x_2^{e_2} \cdots x_m^{e_m-1})^{e_m}, & i = m. \end{cases}$$

Clearly, there exists an automorphism $\tilde{\varphi}$ of $V_m$ which induces the automorphism $\varphi$ of $\pi_1(V_m)$. Without loss of generality, we may assume that $\varphi$ has a fixed point $p$ in $\partial V_m$. Let $X$ be the 5-manifold obtained from $V_m \times [0, 1]$ by identifying $V_m \times \{0\}$ and $V_m \times \{1\}$ via a homeomorphism $\tilde{\varphi}$. More precisely, $X$ is the quotient of $V_m \times [0, 1]$ by the equivalence relation $(x, 0) \sim (\tilde{\varphi}(x), 1)$.

We can show that $\pi_1(X)$ has a presentation

$$\langle t, x_1, \ldots, x_m: tx_1t^{-1}x_1^{-1}, \ldots, tx_{m-1}t^{-1}x_{m-1}^{-1}, tx_m t^{-1}(x_1^{e_1}x_2^{e_2} \cdots x_m^{e_m-1})^{e_m} \rangle.$$ 

As in [5], $H_1(X, \mathbb{Q})$ is isomorphic to $\Gamma/\lambda(t)$, as a $\Gamma$-module, where $\lambda$ denotes the infinite cyclic covering of $X$ with $\langle t \rangle$ as the covering transformation group. Adding a 2-handle $H_2^{(2)}$ to $X$ along a simple closed curve $\alpha = p \times [0, 1]/\sim$ representing $t$ in $\pi_1(X)$, we obtain a simply connected 5-manifold $Y$. We will show that $Y$ is homeomorphic to a 5-ball.

Let $H_1^{(2)} = h_1^{(1)} \times [0, 1/2]/\sim$, $H_1^{(1)} = h_1^{(1)} \times [1/2, 1]/\sim$, for $1 \leq i \leq m$, $\tilde{H}_0^{(1)} = B^4 \times [0, 1/2]/\sim$ and $\tilde{H}_0^{(2)} = B^4 \times [1/2, 1]/\sim$. Then

$$Y = H_0^{(0)} \cup \bigcup \{H_i^{(1)}: 0 \leq i \leq m\} \cup \bigcup \{H_i^{(2)}: 0 \leq i \leq m\},$$

is a handle decomposition of $Y$ such that $H_i^{(j)}$ is a $j$-handle.

Let $W_{m+1} = H_0^{(0)} \cup \bigcup \{H_i^{(1)}: 0 \leq i \leq m\}$. If we denote the elements of $\pi_1(W_{m+1})$ corresponding to $H_1^{(1)}, H_1^{(2)}, \ldots, H_m^{(1)}$ by $t, x_1, \ldots, x_m$, respectively, $\pi_1(W_{m+1})$ is a free group generated by $t, x_1, \ldots, x_m$. For $1 \leq i \leq m$, the attaching sphere of $H_i^{(j)}$ represents $tx_it^{-1}\varphi(x_i)^{-1}$.

The following transformations of a presentation $\langle y_1, \ldots, y_s: r_1, \ldots, r_t \rangle$ are called Andrews-Curtis moves [1], [2], [4]:

(i) Replace $r_i$ by $r_i^{-1}$.
(ii) Replace $r_i$ by $w r_i w^{-1}$, where $w$ is a word in $y_1, \ldots, y_s$.
(iii) Replace $r_i$ by $r_i r_j$, for $i \neq j$.
(iv) Add a generator $y$ and a relator $yw^{-1}$, where $w$ is a word in $y_1, \ldots, y_s$.
(v) Inverse transformation of (iv).
It is not difficult to show that a presentation
\[ \langle t, x_1, \cdots, x_m : t, tx_1t^{-1}x_2t^{-1}, \cdots, tx_{m-1}t^{-1}x_m^{-1}, tx_m(\alpha_0^{e_0} \cdots \alpha_m^{e_m})^{-1} \rangle \]
can be transformed to the trivial presentation by Andrews-Curtis moves. Hence one can slide 2-handles \( \{H^{(2)}_i\} \) to cancel 1-handles \( \{H^{(1)}_i\} \) [1]. Thus \( Y \) is homeomorphic to a 5-ball.

Let \( B^3 \) be a co-core of \( H^{(2)}_o \). Then \( H^{(3)}_o \) can be considered as a tubular neighborhood of \( \partial B^3 \) in \( Y \). Hence the exterior of a 2-knot \( \partial B^3 \) in \( \partial Y \) is \( (\partial X - (\partial X \cap H^{(2)}_o)) \). Since \( (\partial X - (\partial X \cap H^{(2)}_o)) \) fibers over a 1-sphere and
\[ \pi_1(X) \cong \pi_1(\partial X) \cong \pi_1(\partial X - (\partial X \cap H^{(2)}_o)) , \]
the proof is completed.

REFERENCES


Received November 3, 1980.

KWANSEI GAKUIN UNIVERSITY,
NISHINOMIYA, HYOGO 662,
JAPAN
PACIFIC JOURNAL OF MATHEMATICS

EDITORS

DONALD BABBITT (Managing Editor)
University of California
Los Angeles, CA 90024
HUGO ROSSI
University of Utah
Salt Lake City, UT 84112
C. C. MOORE and ANDREW OGG
University of California
Berkeley, CA 94720
J. DUGUNDJI
Department of Mathematics
University of Southern California
Los Angeles, CA 90007
R. FINN and J. MILGRAM
Stanford University
Stanford, CA 94305

ASSOCIATE EDITORS

R. ARENS    E. F. BECKENBACH    B. H. NEUMANN    F. WOLF    K. YOSHIDA

SUPPORTING INSTITUTIONS

UNIVERSITY OF ARIZONA
UNIVERSITY OF BRITISH COLUMBIA
CALIFORNIA INSTITUTE OF TECHNOLOGY
UNIVERSITY OF CALIFORNIA
MONTANA STATE UNIVERSITY
UNIVERSITY OF NEVADA, RENO
NEW MEXICO STATE UNIVERSITY
OREGON STATE UNIVERSITY
UNIVERSITY OF ARIZONA
UNIVERSITY OF SOUTHERN CALIFORNIA
STANFORD UNIVERSITY
UNIVERSITY OF HAWAII
UNIVERSITY OF TOKYO
UNIVERSITY OF UTAH
WASHINGTON STATE UNIVERSITY
UNIVERSITY OF WASHINGTON

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

Mathematical papers intended for publication in the Pacific Journal of Mathematics should be in typed form or offset-reproduced, (not dittoed), double spaced with large margins. Please do not use built up fractions in the text of the manuscript. However, you may use them in the displayed equations. Underline Greek letters in red, German in green, and script in blue. The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. Please propose a heading for the odd numbered pages of less than 35 characters. Manuscripts, in triplicate, may be sent to any one of the editors. Please classify according to the scheme of Math. Reviews, Index to Vol. 39. Supply name and address of author to whom proofs should be sent. All other communications should be addressed to the managing editor, or Elaine Barth, University of California, Los Angeles, California, 90024.

50 reprints to each author are provided free for each article, only if page charges have been substantially paid. Additional copies may be obtained at cost in multiples of 50.

The Pacific Journal of Mathematics is issued monthly as of January 1966. Regular subscription rate: $102.00 a year (6 Vols., 12 issues). Special rate: $51.00 a year to individual members of supporting institutions.

Subscriptions, orders for numbers issued in the last three calendar years, and changes of address should be sent to Pacific Journal of Mathematics, P.O. Box 969, Carmel Valley, CA 93924, U.S.A. Old back numbers obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.).
8-8, 8-chome, Takadanobaba, Shinjuku-ku, Tokyo 160, Japan.

Copyright © 1981 by Pacific Journal of Mathematics
Manufactured and first issued in Japan
<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patrick Robert Ahern and N. V. Rao, A note on real orthogonal measures</td>
<td>249</td>
</tr>
<tr>
<td>Kouhei Asano and Katsuyuki Yoshikawa, On polynomial invariants of fibered 2-knots</td>
<td>267</td>
</tr>
<tr>
<td>Charles A. Asmuth and Joe Repka, Tensor products for $SL_2(F)$. I. Complementary series and the special representation</td>
<td>271</td>
</tr>
<tr>
<td>Gary Francis Birkenmeier, Baer rings and quasicontinuous rings have a MDSN</td>
<td>283</td>
</tr>
<tr>
<td>Hans-Heinrich Brungs and Günter Törner, Right chain rings and the generalized semigroup of divisibility</td>
<td>293</td>
</tr>
<tr>
<td>Jia-Arng Chao and Svante Janson, A note on $H^1_q$-martingales</td>
<td>307</td>
</tr>
<tr>
<td>Joseph Eugene Collison, An analogue of Kolmogorov’s inequality for a class of additive arithmetic functions</td>
<td>319</td>
</tr>
<tr>
<td>Frank Rimi DeMeyer, An action of the automorphism group of a commutative ring on its Brauer group</td>
<td>327</td>
</tr>
<tr>
<td>H. P. Dikshit and Anil Kumar, Determination of bounds similar to the Lebesgue constants</td>
<td>339</td>
</tr>
<tr>
<td>Eric Karel van Douwen, The number of subcontinua of the remainder of the plane</td>
<td>349</td>
</tr>
<tr>
<td>D. W. Dubois, Second note on Artin’s solution of Hilbert’s 17th problem. Order spaces</td>
<td>357</td>
</tr>
<tr>
<td>Daniel Evans Flath, A comparison of the automorphic representations of GL(3) and its twisted forms</td>
<td>373</td>
</tr>
<tr>
<td>Frederick Michael Goodman, Translation invariant closed $\ast$ derivations</td>
<td>403</td>
</tr>
<tr>
<td>Richard Grassl, Polynomials in denumerable indeterminates</td>
<td>415</td>
</tr>
<tr>
<td>K. F. Lai, Orders of finite algebraic groups</td>
<td>425</td>
</tr>
<tr>
<td>George Kempf, Torsion divisors on algebraic curves</td>
<td>437</td>
</tr>
<tr>
<td>Arun Kumar and D. P. Sahu, Absolute convergence fields of some triangular matrix methods</td>
<td>443</td>
</tr>
<tr>
<td>Elias Saab, On measurable projections in Banach spaces</td>
<td>453</td>
</tr>
<tr>
<td>Chao-Liang Shen, Automorphisms of dimension groups and the construction of AF algebras</td>
<td>461</td>
</tr>
<tr>
<td>Barry Simon, Pointwise domination of matrices and comparison of $\mathcal{L}_p$ norms</td>
<td>471</td>
</tr>
<tr>
<td>Chi-Lin Yen, A minimax inequality and its applications to variational inequalities</td>
<td>477</td>
</tr>
<tr>
<td>Stephen D. Cohen, Corrections to: “The Galois group of a polynomial with two indeterminate coefficients”</td>
<td>483</td>
</tr>
<tr>
<td>Phillip Schultz, Correction to: “The typeset and cotypeset of a rank 2 abelian group”</td>
<td>486</td>
</tr>
<tr>
<td>Pavel G. Todorov, Correction to: “New explicit formulas for the $n$th derivative of composite functions”</td>
<td>486</td>
</tr>
<tr>
<td>Douglas S. Bridges, Correction to: “On the isolation of zeroes of an analytic function”</td>
<td>487</td>
</tr>
<tr>
<td>Stanley Stephen Page, Correction to: “Regular FPF rings”</td>
<td>488</td>
</tr>
</tbody>
</table>