CORONA PROBLEM FOR RIEMANN SURFACES OF PARREAU-WIDOM TYPE

MITSURU NAKAI
It is shown that there exists a hyperbolic regular Riemann surface $R$ of Parreau-Widom type that is not dense in the maximal ideal space $\mathcal{M}(R)$ of the Banach algebra $H^\infty(R)$ of bounded analytic functions on $R$.

It has been close to twenty years since Carleson [1] positively solved the corona problem for the unit disk. Since then various subsequent developments appeared. Among them we are particularly interested in investigations on the corona problem for Riemann surfaces of positive genus. As for the positive direction, Gamelin [2], e.g., proved by using his localization principle that the corona problem can be positively answered for finite Riemann surfaces with analytic borders. As for the negative direction, B. Cole constructed a Riemann surface for which the corona problem is negatively answered (see Gamelin [3]). In connection with these results, it is an interesting and also important problem to single out the class of Riemann surfaces of general genus for which the corona problem is positively settled. One might suspect that the class of Riemann surfaces of Parreau-Widom type falls into this category since the class $H^\infty(R)$ of bounded analytic functions on a Riemann surface $R$ of this class is known to share various nice properties with the class $H^\infty(D)$ on the unit disk $D$ (cf. Parreau [8], Widom [10, 11], Stanton [9], Hasumi [4, 5], Hayashi [6, 7], etc).

The purpose of this paper is to show that the above expectation is unfortunately incorrect.

Consider a hyperbolic Riemann surface $R$ so that there exists the Green's function $g(z, a)$ on $R$ with its pole at any point $a$ in $R$. By the maximum principle for harmonic functions the set $R(\alpha, a) = \{z \in R; g(z, a) > \alpha\}$ is a subregion of $R$ for any $\alpha > 0$ and $a$ in $R$. The surface $R$ is said to be regular if $R(\alpha, a)$ is relatively compact for any $\alpha > 0$ and $a$ in $R$. The first Betti number $B(\alpha, a)$ of $R(\alpha, a)$ is the minimum number of generators of the first singular homology group $H_1(R(\alpha, a))$ of $R(\alpha, a)$. A hyperbolic Riemann surface $R$ is said to be of Parreau-Widom type if $\int_0^\infty B(\alpha, a) d\alpha < +\infty$ for one and hence for every $a$ in $R$. We denote by $\mathcal{M}(R)$ the maximal ideal space of $H^\infty(R)$ equipped with the Gelfand topology. We may view $\mathcal{M}(R)$ as the space $\{q\}$ of multiplicative linear functionals $q$ on $H^\infty(R)$ with $q(1) = 1$ equipped with the weak star topo-
logy since \( q \mapsto q^{-1}(0) \) is the bijective homeomorphism between \( \{ q \} \) and \( \mathcal{M}(R) \). A point \( z \) in \( R \) corresponds to a functional \( q_z \) in \( \mathcal{M}(R) \) defined by \( q_z(f) = f(z) \) (point evaluation). If \( R \) is a hyperbolic Riemann surface of Parreau-Widom type, then this natural mapping \( z \mapsto q_z \) gives the injective homeomorphism \( R \to \mathcal{M}(R) \) and the image of \( R \) under this mapping is open in \( \mathcal{M}(R) \) (see Stanton [9]) and therefore we may view \( R \) as an open subset of \( \mathcal{M}(R) \). The corona problem asks whether \( R \) is dense in \( \mathcal{M}(R) \) or not. The main result of this paper is the following

**Theorem.** There exists a hyperbolic regular Riemann surface \( R \) of Parreau-Widom type that is not dense in the maximal ideal space \( \mathcal{M}(R) \) of \( H^\infty(R) \).

The surface \( R \) in the above theorem which we will construct is of infinite genus and infinite connectivity. It is obtained from the B. Cole example by making a minor modification. This modification is formulated as proposition in no. 1, and it is proved in nos. 2-4. The construction of \( R \) in the above theorem is carried over in nos. 5-9.

1. Consider a fixed sequence \((S_n)_{\infty}^\circ\) of interiors \( S_n \) of finite bordered Riemann surfaces \( \overline{S}_n \) with analytic borders \( \partial S_n \), two fixed sequences \((b_n)_{\infty}^\circ\) and \((c_n)_{\infty}^\circ\) of real numbers \( b_n \) and \( c_n \) with \( 0 < c_n < b_n \), and a variable sequence \((\eta_n)_{\infty}^\circ\) of real numbers \( \eta_n \) with \( 0 < \eta_n \leq \min(b_n - c_n) \). By using these sequences we will construct a Riemann surface as follows.

Let \( X_n \) be a rectangular strip \( \{ 0 \leq \text{Re} \ z \leq 2, 0 < \text{Im} \ z < b_n \} \) and \( X'_n \) a rectangular strip \( X_n \) less two vertical slits \( \sigma_n' = \{ \text{Re} \ z = 1, 0 \leq \text{Im} \ z \leq c_n - \eta_n \} \) and \( \sigma_n'' = \{ \text{Re} \ z = 1, c_n + \eta_n \leq \text{Im} \ z \leq b_n \} \), i.e., \( X_n' = X_n - \sigma_n' \cup \sigma_n'' \), for each \( n \). Observe that \( \tau_n = \{ \text{Re} \ z = 1, |\text{Im} \ z - c_n| < \eta_n \} \) is a cross-cut of \( X_n' \) with the length \( 2\eta_n \) for each \( n \). The left and right vertical sides of \( X_n' \) (and hence of \( X_n \)) will be denoted by \( \alpha_n \) and \( \beta_n \) respectively.

Weld \( X_n' \) to \( S_n \) and \( S_{n+1} \) by identifying the side \( \alpha_n \) of \( X_n' \) with an open arc in \( \partial S_n \) and the side \( \beta_n \) of \( X_n' \) with an open arc in \( \partial S_{n+1} \) for each \( n \). The resulting surface \( \bigcup_{n=1}^{\infty} (S_n \cup X_n') \) will be denoted by \( R = R((\eta_n)) = R((S_n), (X_n), (\eta_n)) \).

Here it is assumed that \( \overline{S}_n \cap \overline{S}_m = \phi \) \((n \neq m)\), \( \overline{X}_n' \cap \overline{X}_m' = \phi \) \((n \neq m)\), and \( \overline{X}_n' \cap \overline{S}_k = \phi \) \((k \neq n, n + 1)\) in \( R \). By using \( X_n \) instead of \( X_n' \) we construct the Riemann surface \( \bigcup_{n=1}^{\infty} (S_n \cup X_n) \) in the same fashion. 
as $R((S_n), (X_n), (\eta_n)) = \bigcup_{n=1}^{\infty} (S_n \cup X'_n)$. The resulting surface will be denoted by

$$R = R((S_n), (X_n)).$$

Hence $R((S_n), (X_n), (\eta_n)) = R((S_n), (X_n)) - \bigcup_{n=1}^{\infty} (\sigma'_n \cup \sigma''_n)$. Clearly the surfaces $R$ in (1) or (2) can be embedded in a larger Riemann surface $W$ such that $W - R \neq \phi$. Therefore the surfaces $R$ given by (1) or by (2) are hyperbolic.

We will prove the following

**Proposition.** If the sequence $(\eta_n)_{\infty}$ converges to zero sufficiently rapidly, then $R = R((S_n), (X_n), (\eta_n))$ is a hyperbolic regular Riemann surface of Parreau-Widom type.

It can happen that $R = R((S_n), (X_n))$ is neither regular nor of Parreau-Widom type. In such a case $R = R((S_n), (X_n), (\eta_n))$ is of the same sort if $(\eta_n)_{\infty}$ converges to zero not so rapidly. The proof of the proposition will be given in nos. 2–4.

2. We denote by $R_n = \bigcup_{i=1}^{n} (S_i \cup X'_i) - \beta_n$ the initial part of $R = R((S_n), (X_n), (\eta_n))$ and by $R'_n = R - R_n \cup \beta_n$ the terminal part of $R$. Recall that the first Betti number $B$ of a finite Riemann surface $W$ with border $\partial W$ is given by $B = 1 - \chi = 2g + m - 1$ where $\chi$ is the Euler characteristic of $W$, $g$ the genus of $W$, and $m$ the number of components of $\partial W$. Let $B_n$ be the first Betti number of $R_n$. Observe that $B_n$ is finite since $R_n$ is the interior of the finite bordered surface $\tilde{R}_n$ and that $B_n$ does not depend on the choice of $(\eta_n)$ since $R((S_n), (X_n), (\eta_n))$ are homeomorphic to each other for all choices of $(\eta_n)$. Then fix a sequence $(\varepsilon_n)_{\infty}$ of positive numbers $\varepsilon_n$ such that $\varepsilon_n > \varepsilon_{n+1}$ $(n=1, 2, \cdots)$, $\lim_n \varepsilon_n = 0$, and $\sum_{n=2}^{\infty} B_n(\varepsilon_{n-1} - \varepsilon_n) < +\infty$.

Fix a point $a$ in $S_1$ and let $g(z, a)$ be the Green's function of $R = R((S_n), (X_n), (\eta_n))$ with its pole at $a$. We also denote by $\hat{g}(z, a)$ the Green's function of $\tilde{R} = R((S'_n), (X'_n), (\eta'_n))$ with $\eta'_n = \min(\varepsilon_n, b_n - c_n)$. Then every $R$ is a subsurface of $\tilde{R}$ and therefore $g(z, a) \leq \hat{g}(z, a)$ on $R$. Choose and fix $M > 0$ so large that $\hat{U} = \{z; \hat{g}(z, a) > M\}$ is contained in $S$, and simply connected. Then $U = \{z; g(z, a) > M\}$ is a subset of $\hat{U}$ and also simply connected. Hence the Betti number $B_0$ of $U$ is zero.

Let $Y_n$ be the part $\{1 < \text{Re} z \leq 2, 0 < \text{Im} z < b_n\}$ of $X_n$, $\tilde{R}'_n$ the terminal part of $\tilde{R}$, and $w_n$ the harmonic measure of $\tau_n$ with respect to the region $Y_n \cup \tilde{R}'_n$. Then

$$g(z, a) \leq M w_n(z) \quad (z \in Y_n \cup \tilde{R}'_n)$$
and, in particular, \( \sup_{\sigma_n} g(\cdot, a) \leq M \sup_{\sigma_n} w_n \). It is clear that 
\[ \lim_{\varepsilon_n \to 0} w_n = 0 \] uniformly on \( Y_n \cup R_n \) less any neighborhood of the
left vertical side of \( Y_n \). We can thus choose a sequence \( (\eta_n)_i \) converging
to zero enough rapidly so that
\[
(3) \quad \sup_{\sigma_n} g(\cdot, a) \leq \varepsilon_n \quad (n = 1, 2, \ldots).
\]

3. We pause here to observe the following. Let \( W_0 \) be a sub-
surface of a Riemann surface \( W \). Consider the first singular homology
groups \( H_1(W_0) \) and \( H_1(W) \) of \( W_0 \) and \( W \), respectively. A cycle \( \gamma \) on \( W_0 \) is
automatically a cycle on \( W \) and this gives a natural group homomorphism
\( \gamma \mapsto \gamma \) of \( H_1(W_0) \) to \( H_1(W) \). Suppose that \( W_0 \) satisfies
the condition \( (N) \): Any connected component of \( W - W_0 \) is
not compact. Under this condition \( (N) \), the above natural homo-
morphism \( H_1(W_0) \to H_1(W) \) is \textit{injective}. To see this, let \( \gamma \) be a cycle
on \( W_0 \) which is homologous to zero on \( W \), i.e., \( \gamma \sim 0 \) on \( W \). We
have to show that \( \gamma \) is homologous to zero already on \( W_0 \), i.e., \( \gamma \sim 0 \)
on \( W_0 \). Since \( \gamma \sim 0 \) on \( W \) and \( \gamma \in H_1(W_0) \), we can express \( \gamma \) as
\( \gamma \sim \sum_{j=1}^{\infty} \partial \sigma_j^i \) where each \( \sigma_j^i \) is a \textit{simplicial} 2-simplex on \( W \) with
\( \partial \sigma_j^i \subset W_0 \). If \( \sigma_j^i \not\subset W_0 \) for some \( j \), then \( \sigma_j^i \) must contain
a component of \( W - W_0 \) since \( \partial \sigma_j^i \subset W_0 \). This contradicts the condition \( (N) \). We
have then \( \sigma_j^i \subset W_0 \) and therefore \( \gamma \sim 0 \) on \( W_0 \).

4. We have already remarked that \( \mathcal{R}(\alpha, a) = \{ z; g(z, a) > \alpha \} \)
is a region for any \( \alpha > 0 \) as a consequence of the maximum principle
for harmonic functions. In view of \( (3) \), \( \mathcal{R}(\alpha, a) \subset \mathcal{R}_n \) for \( \alpha > \varepsilon_n \).
Since \( g(\cdot, a) \leq \varepsilon_n \) on \( \partial \mathcal{R}_n \), we see that \( \mathcal{R}(\alpha, a) \) is relatively
compact for every \( \alpha > 0 \). This proves that \( \mathcal{R} \) is \textit{regular}.

Again by the maximum principle, it is readily seen that \( \mathcal{R} - \mathcal{R}(\alpha, a) \)
has no compact component. In particular, if \( \alpha > \alpha' \), then
\( \mathcal{R}(\alpha, a) \) is a subsurface of \( \mathcal{R}(\alpha', a) \) satisfying \( (N) \) with respect to
\( \mathcal{R}(\alpha', a) \). Therefore the natural homomorphism \( H_1(\mathcal{R}(\alpha, a)) \to H_1(\mathcal{R}(\alpha', a)) \) is injective and a fortiori
the minimum number of generators of \( H_1(\mathcal{R}(\alpha, a)) \), which is the first Betti number
\( B(\alpha, a) \) of \( \mathcal{R}(\alpha, a) \), is less than or equal to that of \( H_1(\mathcal{R}(\alpha', a)) \), which is \( B(\alpha', a) \). Therefore
we have
\[
(4) \quad B(\alpha, a) \leq B(\alpha', a) \quad (\alpha \geq \alpha') .
\]
Similarly \( \mathcal{R}(\alpha, a) \) is a subsurface of \( \mathcal{R}_n \) for \( \alpha > \varepsilon_n \) with the property
\( (N) \) and the natural homomorphism \( H_1(\mathcal{R}(\alpha, a)) \to H_1(\mathcal{R}_n) \) is injective.
Recall that the first Betti number of \( \mathcal{R}_n \) is \( B_n \) and hence we have
\[
(5) \quad B(\alpha, a) \leq B_n \quad (\alpha \geq \varepsilon_n) .
\]

For \( \alpha \geq M \), the region \( \mathcal{R}(\alpha, a) \) is contained in \( U \) and simply
connected. Therefore \( B(\alpha, a) = 0 \) for \( \alpha \geq M \) and hence \( \int_{-1}^{0} B(\alpha, a) d\alpha = 0. \) By (5) we see that \( \int_{-1}^{0} B(\alpha, a) d\alpha \leq B_1(M - \varepsilon) \). Again by (5), \( \int_{-1}^{0} B(\alpha, a) d\alpha \leq B_n(\varepsilon) \) for every \( n \geq 2 \). Hence we have

\[
\int_{0}^{\infty} B(\alpha, a) d\alpha \leq B_1(M - \varepsilon) + \sum_{n=2}^{\infty} B_n(\varepsilon) < + \infty,
\]

which proves that \( R = R((S_n), (X_n), (\eta_n)) \) is of Parreau-Widom type.

5. We proceed to the construction of \( R \) in our theorem. For the purpose we will briefly describe here how the \( B. \) Cole example \( W \) is constructed that is not dense in the maximal ideal space \( \mathcal{M}(W) \) of \( H^\infty(W) \). The construction is done based upon two crucial steps. The first is the following Existence of a malformed finite surface: Let \( \delta \) be an arbitrary real number with \( 0 \leq \delta < 1 \) and \( m \) an arbitrary positive integer. Then there exists a finite bordered Riemann surface \( \overline{W}_m \) with its interior \( W_m \) and with an analytic border \( \partial W_m \) and a pair \( f_m, g_m \) of functions in \( H^\infty(W_m) \) with \( |f_m|, |g_m| \leq 1 \) and \( |f_m| + |g_m| \geq \delta \) on \( W_m \) such that whenever \( f_m \phi + g_m \psi = 1 \) is satisfied on \( W_m \) for a pair \( \phi \) and \( \psi \) of functions in \( H^\infty(W_m) \), we have \( \sup_{W_m} |\phi| + \sup_{W_m} |\psi| \geq m \). For its proof see Gamelin [3, pp. 47-49].

6. Another important device for the construction is the following Approximation theorem which is easily deduced by a standard successive approximation procedure from the Bishop generalization to Riemann surfaces of the Mergelyan approximation theorem: Let \( (K_n)_{n=1}^{\infty} \) be a sequence of compact subsets \( K_n \) of a Riemann surface \( R \) such that \( K_n \cap K_m = \phi \) (\( n \neq m \)), \( \gamma_n \) a curve in \( R - \bigcup_{k=1}^{\infty} K_k \) except its end points connecting a boundary point of \( K_n \) to a boundary point in \( K_{n+1} \) such that \( \gamma_n \cap \gamma_m = \phi \) (\( n \neq m \)). Assume that \( R - F \) has no relatively compact component where \( F = \bigcup_{n=1}^{\infty} (K_n \cup \gamma_n) \). To each function \( \phi \) continuous on \( F \) and analytic in the interior of \( F \) and each positive number \( \varepsilon \) there exists an analytic function \( \Phi \) on \( R \) such that \( \sup_{F_n \cup \gamma_n} |\Phi - \phi| < \varepsilon/n \) (\( n = 1, 2, \cdots \)).

7. Let \( W_m \) be as in no. 5 for each \( m \) and \( Z_m \) be a finite Riemann surface obtained from \( W_m \) by attaching an annulus to each boundary component of \( W_m \). By using a sequence \( (L_m)_{n=1}^{\infty} \) of rectangular strips \( L_m \) we construct a Riemann surface \( R((Z_n), (L_m)) \) defined in (2). Let \( \gamma_m \) be a curve in \( R((Z_n), (L_n)) - \bigcup_{n=1}^{\infty} \overline{W}_n \) except its end points connecting a boundary point of \( \overline{W}_m \) and a boundary point of \( \overline{W}_{m+1} \) for each \( m \) such that \( \gamma_n \cap \gamma_m = \phi \) (\( n \neq m \)). Then \( F = \bigcup_{n=1}^{\infty} (\overline{W}_n \cup \gamma_n) \).
is qualified to be an $F$ in no. 6 with respect to the Riemann surface $R((Z_n), (L_n))$. Consider continuous functions $f_0$ and $g_0$ on $F$ such that $f_0|_{W_m} = f_m$, $g_0|_{W_m} = g_m$ ($m = 1, 2, \cdots$), $|f_0|, |g_0| \leq 1$ and $|f_0| + |g_0| \geq \delta$ on $F$. By no. 6 there exist analytic functions $f$ and $g$ on $R((Z_n), (L_n))$ such that
\[
(6) \quad \sup_{\overline{W}_m \cup T_m} |f - f_0| + \sup_{\overline{W}_m \cup T_m} |g - g_0| < \delta/4m \quad (m = 1, 2, \cdots).
\]

8. Let $W$ be a connected neighborhood of $F$ in $R((Z_n), (L_n))$ such that $|f|, |g| \leq 2, |f| + |g| \geq \delta/2$ on $W$. The surface $W$ is the B. Cole example (see Gamelin [3, pp. 49–52]). For each $m$, let $S_m$ be a finite surface with an analytic border $\partial S_m$ such that $\overline{W}_m \subset S_m \subset W \cap Z_m$ and $Y_m$ a rectangular strip $\{0 \leq \text{Re} z \leq 2, 0 < \text{Im} z < b_m\}$ in the sense of conformal equivalence such that the left side of $Y_m$ is a part of $\partial S_m$, the right side of $Y_m$ is a part of $\partial S_{m+1}$, $Y_m \cap \{\Phi \in S_m \cup Y_m\} = \phi$, $\Phi_{ij}(S_m \cup Y_m) \subset W$, and $\Phi_{ij}(S_m \cup Y_m) \supset F$. Consider the surface $R((S_n), (Y_n))$ defined in (2), which may also be called the B. Cole example since it is a neighborhood of $F$ contained in $W$. Let $\sigma'_n$ and $\sigma''_n$ be as in no. 1 with $c_n = b_n/2$, for example. We finally consider
\[
(7) \quad R = R((S_n), (Y_n), (\eta_n)) = R((S_n), (Y_n)) - \bigcup_{n=1}^{\infty} (\sigma'_n \cup \sigma''_n)
\]
defined in (1). By proposition in no. 1, $R = R((S_n), (Y_n), (\eta_n))$ can be made to a hyperbolic regular Riemann surface of Parreau-Widom type if the sequence $(\eta_n)$ is so chosen that it converges to zero sufficiently rapidly.

9. Since the surface $R$ given by (7) now so made that it is of Parreau-Widom type, it has many nice properties concerning the class $H^\infty(R)$. For example, $H^\infty(R)$ separates points in $R$ and the natural injection $R \to \mathcal{M}(R)$ of $R$ into the maximal ideal space $\mathcal{M}(R)$ of $H^\infty(R)$ is bicontinuous and the image in $\mathcal{M}(R)$, identified with $R$, is open in $\mathcal{M}(R)$ (Stanton [9]). The proof of our theorem is over if we show that $R$ is not dense in $\mathcal{M}(R)$.

Observe that functions $f$ and $g$ in no. 3 may be viewed as in $H^\infty(R)$ and, by (6), satisfy $|f|, |g| \leq 2, |f| + |g| \geq \delta/2$ on $R$, and
\[
(8) \quad \sup_{\overline{W}_m} |f - f_m| + \sup_{\overline{W}_m} |g - g_m| < 1/m \quad (m = 1, 2, \cdots),
\]
where $\bigcup_{i=1}^{\infty} \overline{W}_n \subset R$. Suppose that the indefinite equation $f\phi + g\psi = 1$ on $R$ has solutions $\phi$ and $\psi$ in $H^\infty(R)$. Set $f_m\phi + g_m\psi = \lambda_m$ on $W_m$. Observe that $\sup_{\overline{W}_m} |1 - \lambda_m| < (\sup_{\overline{W}_m} |\phi| + \sup_{\overline{W}_m} |\psi|)/m$ as a consequence of (8). Therefore $f_m(\phi/\lambda_m) + g_m(\psi/\lambda_m) = 1$ on $W_m$ with $\phi/\lambda_m$ and $\psi/\lambda_m$
in $H^\infty(W_m)$. By no. 5, we then have $\sup_{\overline{W}_m} |\phi/\lambda_m| + \sup_{\overline{W}_m} |\psi/\lambda_m| \geq m$ $(m = 1, 2, \cdots)$, which contradicts $\sup_{\overline{W}_m} |1 - \lambda_m| \to 0$ $(m \to \infty)$. This shows that there exists a maximal ideal $M_0$ in $\mathcal{M}(R)$ containing $f$ and $g$. Then $f(M_0) = g(M_0) = 0$ and the assumption $\inf (|f| + |g|) \geq \delta/2 > 0$ imply that $M_0$ is not in the closure of $R$ in $\mathcal{M}(R)$ so that $R$ is not dense in $\mathcal{M}(R)$.

REFERENCES


Received January 13, 1981. This work was partly supported by Grant-in-Aid for Scientific Research, No. 454027, Japanese Ministry of Education, Science and Culture.

NAGOYA INSTITUTE OF TECHNOLOGY
GOKISO, SHOWA, NAGOYA 466
JAPAN
Abdul Aziz, On the zeros of composite polynomials ........................................... 1
Salomon Benzaquen and Enrique M. Cabaña, The expected measure of 
the level sets of a regular stationary Gaussian process ................................. 9
Claudio D’Antoni, Roberto Longo and László Zsidó, A spectral mapping 
theorem for locally compact groups of operators ........................................ 17
Ronald Dotzel, Semifree finite group actions on homotopy spheres ............. 25
Daniel H. Gottlieb, The Lefschetz number and Borsuk-Ulam theorems ....... 29
Shui-Hung Hou, On property (Q) and other semicontinuity properties of 
multifunctions ................................................................................................. 39
Kevin Mor McCrimmon, Compatible Peirce decompositions of Jordan 
triple systems ................................................................................................. 57
Mitsuru Nakai, Corona problem for Riemann surfaces of Parreau-Widom 
type ................................................................................................................. 103
Jack Ray Porter and R. Grant Woods, Extensions of Hausdorff spaces ........ 111
Milton Rosenberg, Quasi-isometric dilations of operator-valued measures 
and Grothendieck’s inequality ........................................................................ 135
Joseph L. Taylor, A bigger Brauer group ...................................................... 163
Thomas Vogel, Symmetric unbounded liquid bridges .................................. 205
Steve Wright, The splitting of operator algebras. II ..................................... 243