

Pacific Journal of Mathematics

A DICHOTOMY FOR A CLASS OF POSITIVE DEFINITE FUNCTIONS

ALBERTO ALESINA AND LEONEDE DE MICHELE

A DICHOTOMY FOR A CLASS OF POSITIVE DEFINITE FUNCTIONS

ALBERTO ALESINA AND LEONEDE DE MICHELE

We study under which conditions certain positive definite functions on discrete free groups, are weakly associated with the left regular representation.

1. **Introduction.** In this paper we study some properties of a class of positive definite functions on free groups introduced in [3]. These functions resemble in many respects Riesz products on abelian groups and have been used to investigate properties of the Fourier-Stieltjes algebra of a free group. Let G be a discrete group, and F a free subset of G , namely a set with no relations among its elements. For every $x \in [F]$, the group generated by F , the length of x , with respect to F , is the number of factors in $F \cup F^{-1}$ which are needed to write x as a reduced word in the free generators of F . We denote by $|x|$ the length of x . We recall the following definition given in [3].

DEFINITION. A Haagerup function on G is a function u such that:

- (i) u is zero on $G \setminus [F]$ and $|u(x)| \leq 1$ for every $x \in [F]$,
- (ii) $u(1) = 1$,
- (iii) $\overline{u(x)} = u(x^{-1})$,
- (iv) $u(xy) = u(x)u(y)$ if $|xy| = |x| + |y|$.

The following result, which is similar to Zygmund's dichotomy theorem [11] on Riesz products, was proved in [3]:

THEOREM. *If*

$$\sum_{x \in F} |u(x)|^2 = +\infty$$

then u belongs to the orthogonal complement, in the Fourier-Stieltjes algebra, of $B_\lambda(G)$, the space of all coefficients of the representations weakly contained in the left regular representation of G .

If

$$\sum_{x \in F} |u(x)|^2 < 1/2$$

then $u \in A(G)$, the space of coefficients of the regular representation.

In this paper we fill the gap between the above conditions, proving that, if $\sum_{x \in F} |u(x)|^2 < +\infty$, then u is not in the orthogonal complement of $A(G)$ in the Fourier-Stieltjes algebra of G .

Moreover a direct computation shows that, unlike classical case, no necessary and sufficient condition can be given in terms of the L^2 -norm of $u|_F$ in order that the function u belongs to $B_\lambda(G)$. Therefore the above result is, in some sense, the best possible. Finally we prove that, as for Riesz products, the spectrum of some Haagerup functions is strictly larger than the range.

2. Let $B(G)$ be the Fourier-Stieltjes algebra of G , consisting of all linear combinations of positive definite functions. Following [6] we recall that $B(G)$ is a Banach algebra under the norm which makes it the dual space of the group C^* -algebra $C^*(G)$ whose universal W^* -enveloping algebra is denoted by $W^*(G)$.

The Fourier algebra $A(G)$, and $B_\lambda(G)$, are closed ideals of $B(G)$. The space $B_\lambda(G)$ may be identified with the dual space of $C_\lambda^*(G)$, the completion of $L^1(G)$ with respect to the convolution norm. It follows from [5, Cor. 3, p. 42] and [9] that $B_\lambda(G)$ is complemented in $B(G)$, i.e., $B(G) = B_\lambda(G) \oplus B_\lambda^\perp(G)$ where $B_\lambda^\perp(G)$ is a closed subspace of $B(G)$, invariant under translation by elements of G . We shall say that a positive definite function u is orthogonal to $B_\lambda(G)$, if $u \in B_\lambda^\perp(G)$. Similarly $A(G)$ is complemented in $B(G)$ and $A^\perp(G)$ will denote its orthogonal complement (see also [10], p. 22 and Prop. 1, p. 33). Finally we recall that every positive definite function ϕ defines [4, p. 256] a Hilbert seminorm on the space of finitely supported functions:

$$(1) \quad \|f\|_\phi^2 = \langle f^* * f, \phi \rangle = \sum_{x, z \in G} \phi(x^{-1}z) \overline{f(x)} f(z).$$

A more convenient form for this seminorm, if ϕ is a Haagerup function, is provided by the following lemma:

LEMMA 1. *Let F be a free subset of G and u a Haagerup function such that $u(x) \neq 0$ for $x \in F$.*

Then for every finitely supported function f , with $\text{supp } f \subseteq [F]$:

$$(2) \quad \|f\|_u^2 = |\langle u, f \rangle|^2 + \sum_{0 < |z|} (|u(z)|^{-2} - |u(z^\sim)|^{-2}) |\langle u, f \chi_{A(z)} \rangle|^2$$

where

$$|z^\sim| = |z| - 1 \quad \text{and} \quad |z^{\sim^{-1}}z| = 1 \\ A(z) = \{x \in [F] : |z^{-1}x| = |x| - |z|\}$$

and $\chi_{A(z)}$ is the characteristic function of $A(z)$.

Proof. As the right hand side of (2) can be written in the form:

$$\sum_{x, y \in [F]} v(x, y) \overline{f(x)} f(y)$$

we only need to prove that $v(x, y) = u(x^{-1}y)$.

Let $|x^{-1}y| = |x| + |y|$, then x and y can not belong to the same $A(z)$ for every $|z| \geq 1$. Since the term $\overline{f(x)}f(y)$ appears in $|\langle u, f\chi_{A(z)} \rangle|^2$ if and only if $x, y \in A(z)$, then it follows that $v(x, y) = u(x^{-1})u(y) = u(x^{-1}y)$. More generally, suppose x and y satisfy the following condition: $x = z_0x_0, y = z_0y_0$ with $|z_0| \geq 1$ and $|x| = |z_0| + |x_0|, |y| = |z_0| + |y_0|, |x^{-1}y| = |x_0| + |y_0|$. Let $B_{z_0} = \{z \in F : 1 \leq |z| \leq |z_0|, |z^{-1}z_0| = |z_0| - |z|\}$; then, by the same argument as above, we obtain:

$$\begin{aligned} v(x, y) &= u(x^{-1})u(y) + \sum_{z \in B_{z_0}} (|u(z)|^{-2} - |u(z^{-1})|^{-2})u(x^{-1})u(y) \\ &= u(x^{-1})u(y)|u(z_0)|^{-2} = u(x^{-1}y). \end{aligned}$$

REMARKS. 1. The above formula gives a new, direct proof that Haagerup functions are positive definite.

2. If $u(x) = 0$ for some $x \in F$, a similar but more complicated formula holds.

LEMMA 2. Let F be a free subset of $G, \delta_1 = \chi_{\{1\}}$ and u an Haagerup function such that:

$$0 < |u(x)| < 1 \text{ for every } x \in F.$$

If

$$\sum_{x \in F} |u(x)|^2 < +\infty$$

then for every finitely supported function f with $f(1) = 0$:

$$(3) \quad \|f - \delta_1\|_u^2 > c$$

for some positive constant c .

Proof.

$$\|f - \delta_1\|_u^2 = 1 - 2 \operatorname{Re} \langle u, f \rangle + \|f\|_u^2$$

by (2)
$$\geq |1 - \langle u, f \rangle|^2 + \sum_{|z|=1} (|u(z)|^{-2} - 1) |\langle u, f\chi_{A(z)} \rangle|^2$$

$$(4) \quad \geq |1 - \langle u, f \rangle|^2 + c' \sum_{|z|=1} |\langle u, f\chi_{A(z)} \rangle|^2 |u(z)|^{-2},$$

where

$$c' = \operatorname{Inf}_{|z|=1} (1 - |u(z)|^2)$$

is greater than zero.

Let

$$A = \{z : |z| = 1 \text{ and } |\langle u, f\chi_{A(z)} \rangle| \geq (1/4) |\langle u, f \rangle| |u(z)|^2 \|u\chi_F\|_2^{-2}\}$$

then

$$\sum_{z \in A} |\langle u, f\chi_{A(z)} \rangle| \geq (1/2) |\langle u, f \rangle| .$$

Therefore:

$$\begin{aligned} (5) \quad \sum_{|z|=1} |\langle u, f\chi_{A(z)} \rangle|^2 |u(z)|^{-2} &\geq \sum_{z \in A} |\langle u, f\chi_{A(z)} \rangle|^2 |u(z)|^{-2} \\ &\geq (1/2) |\langle u, f \rangle| \|u\chi_F\|_2^{-2} \sum_{z \in A} |\langle u, f\chi_{A(z)} \rangle| \\ &\geq (1/4) |\langle u, f \rangle|^2 \|u\chi_F\|_2^{-2} . \end{aligned}$$

From (4) and (5) we obtain

$$\|f - \delta_1\|_u^2 \geq |1 - \langle u, f \rangle|^2 + c'' |\langle u, f \rangle|^2 \geq c ,$$

where

$$c = \min(1/2, c''/2) .$$

THEOREM. *Let F be a free subset of G , u an Haagerup function such that*

$$0 < |u(x)| < 1: x \in [F]$$

and

$$\sum_{x \in F} |u(x)|^2 < +\infty .$$

Then u does not belong to $A^+(G)$.

Proof. Let Φ_u be the positive linear functional on $C^*(G)$ canonically associated with u . We first prove that condition (3) in Lemma 2 implies Φ_u is not orthogonal to Φ_{δ_1} .

Assume the contrary: then there exists an hermitian projection P in $W^*(G)$ such that

$$\langle \Phi_u, P \rangle = 1 \quad \text{and} \quad \langle \Phi_{\delta_1}, P \rangle = 0 \quad [4, 12.3.1.(i)] .$$

By Kaplansky's density theorem [5, p. 43], for any $\varepsilon > 0$ we can find an hermitian element $P' \in C^*(G)$ such that

$$\|P'\|_{C^*} \leq 1, \quad |\langle P', \Phi_u \rangle - 1| < \varepsilon, \quad |\langle P', \Phi_{\delta_1} \rangle| < \varepsilon .$$

However P' can be approximated with a finitely supported function f on $[F]$ such that $f(1) = 0$ and:

$$(6) \quad \|f\|_{C^*} \leq 1 + \varepsilon$$

$$(7) \quad |\langle u, f \rangle - 1| < 3\varepsilon, \quad |\langle u, f^* \rangle - 1| < 3\varepsilon .$$

For such a function, (6) and (7) imply:

$$\begin{aligned} \|f - \delta_1\|_u &= \langle u, f^{**} * f \rangle - \langle u, f \rangle - \langle u, f^* \rangle + 1 \\ &\leq \langle u, f^{**} * f \rangle - 1 + 6\varepsilon \\ &\leq (1 + \varepsilon)^2 - 1 + 6\varepsilon \leq 9\varepsilon, \end{aligned}$$

but this contradicts condition (3). By Lemma 2, Φ_u is not orthogonal to Φ_{δ_1} and this implies

$$(8) \quad A_{\pi_u} \cap A(G) \neq \{0\}$$

where A_{π_u} is the closed, translation invariant subspace of $B(G)$ generated by u (see for example [1]).

Since $A(G)^\perp$ is closed and translation invariant, $u \in A(G)^\perp$ implies $A_{\pi_u} \subseteq A(G)^\perp$ contradicting (8).

REMARK. It is easy to see that if $u(x) = 1$ for some $x \in F$, then $u \in A(G)^\perp$; moreover if $u(x_1) = u(x_2) = 1$ for some $x_1 \neq x_2$ in F , then $u \in B_\lambda(G)^\perp$. It will be proved in the following section that, if $u(x') = 1$, then $u \in B_\lambda(G)$ if and only if $u(x) = 0$ for every $x \in F, x \neq x'$.

3. In this section we prove two further properties of Haagerup functions; the second one shows another analogy with the classical Riesz products.

From [8, Cor. 3.2] a function $\phi(x) = \exp(-t|x|)$ on the free group G with finitely many generators a_1, \dots, a_N , is in $B_\lambda(G)$ if and only if

$$(9) \quad \|\phi\chi_F\|_2^2 = \sum_{i=1}^N |\phi(a_i)|^2 \leq N(2N - 1)^{-1}.$$

A restriction argument shows that the condition in [3, Th. 2] is in some sense the best possible in the case of infinitely many generators.

One may wonder if an L^2 -condition similar to (9) still holds for general Haagerup functions. The following proposition shows that this is not the case, even for the free group F_2 on two generators a and b .

PROPOSITION 1. *Let u be a Haagerup function on F_2 and*

$$\beta = |u(a)|^2 + 3|u(a)|^2|u(b)|^2 + |u(b)|^2.$$

Then $u \in B_\lambda(G)$ if and only if $\beta \leq 1$. Moreover $u \in A(G)$ if $\beta < 1$.

Proof. By [8, Th. 3.1(2)] it is enough to evaluate $\|u\chi_n\|_2$, where χ_n is the characteristic function of the set

$$E_n = \{x \in G: |x| = n\}.$$

We shall only sketch the lengthy computation involved. Computing

the number of elements in E_n respectively of the form

$$\begin{aligned} & \alpha^{\pm \varepsilon_1} b^{\pm \sigma_1} \dots \alpha^{\pm \varepsilon_s} b^{\pm \sigma_s} \\ & b^{\pm \sigma_1} \alpha^{\pm \varepsilon_1} \dots \alpha^{\pm \varepsilon_s} b^{\pm \sigma_{s+1}} \\ & \alpha^{\pm \varepsilon_1} b^{\pm \sigma_1} \dots b^{\pm \sigma_{s-1}} \alpha^{\pm \varepsilon_s} \end{aligned}$$

with

$$\sum \varepsilon_i = k, \quad \sum \sigma_i = n - k, \quad \varepsilon_i, \sigma_i \geq 1,$$

one obtains

$$\|u\mathcal{X}_n\|_2^2 = \sum_{k=0}^n R(n, k) |u(a)|^{2k} |u(b)|^{2(n-k)}$$

where, for n large enough:

$$\begin{aligned} R(n, k) &= \sum_{s=1}^{\min(k, n-k+1)} 2^{2s-1} \binom{k-1}{s-1} \left\{ 4 \binom{n-k}{s} + \binom{n-k-1}{s-2} \right\} \\ &= \sum_s \xi(s, k, n). \end{aligned}$$

By Stirling's formula

$$\gamma g(s, k, n) \leq \xi(s, k, n) \leq n^\delta g(s, k, n)$$

where $\gamma, \delta > 0$ are independent of n and k , and $g(s, k, n) = k^k (n-k)^{n-k} (s/2)^{-2s} (k-s)^{s-k} (n-k-s)^{k+s-n}$. Letting s be continuous, $g(s, k, n)$ takes its maximum value $g(s', k, n)$ at the point $s' = (2/3)(n - (n^2 - 3kn + 3k^2)^{1/2})$. Trivially

$$\gamma g(s', k, n) \leq R(n, k) \leq n^{\delta'} g(s', k, n)$$

and if we put $k = \alpha n$,

$$\gamma' (\max P(\alpha))^n \leq \|u\mathcal{X}_n\|_2^2 \leq n^{\delta''} (\max P(\alpha))^n$$

where

$$\begin{aligned} P(\alpha) &= \alpha^\alpha (1-\alpha)^{1-\alpha} \left\{ \alpha - \frac{2}{3} (1 - (1 - 3\alpha + 3\alpha^2)^{1/2}) \right\}^{-\alpha} \\ &\times \left\{ 1 - \alpha - \frac{2}{3} (1 - (1 - 3\alpha + 3\alpha^2)^{1/2}) \right\}^{\alpha-1} |u(a)|^{2\alpha} |u(b)|^{2(1-\alpha)}. \end{aligned}$$

It turns out that the maximum of $P(\alpha)$ is attained at

$$\alpha' = (1/2)(1 + (r-1)(r^2 + 14r + 1)^{-1/2}), \quad r = |u(a)|^2 |u(b)|^{-2}$$

and we finally obtain:

$$(10) \quad \gamma'' \leq \|u\mathcal{X}_n\|_2^2 \{(1/2) |u(b)|^2 (1 + r + (r^2 + 14r + 1)^{1/2})\}^{-n} \leq n^{\delta'''}.$$

Because the two following relations are equivalent

$$\begin{aligned} (1/2)|u(b)|^2(1+r+(r^2+14r+1)^{1/2}) &\leq 1 \\ |u(a)|^2 + 3|u(a)|^2|u(b)|^2 + |u(b)|^2 &\leq 1 \end{aligned}$$

it follows from (10) and [8, Th. 3.1(2)] that $u \in A(G)$ for $\beta < 1$ and $u \notin B_\lambda(G)$ if $\beta > 1$. If $\beta = 1$, set $u_t = e^{-t|x|}u$. For every $t > 0$ $\|u_t\| = 1$ and $u_t \in A(G)$ by the previous result. Then $u \in B_\lambda(G)$ because $u_t(x) \rightarrow u(x)$ as $t \rightarrow 0$ for every $x \in G$.

PROPOSITION 2. *Let u an Haagerup function which assumes the constant value A on the infinite free set F . Then if $|A| < 1$, A is not isolated in the Gelfand spectrum of u .*

Proof. Let \hat{u} be the Gelfand transform of u , and \mathcal{M} the maximal ideal space of $B(G)$. Suppose A isolated in the spectrum of u , then the set $H = \{y \in \mathcal{M} : \hat{u}(y) = A\}$ is an open compact set in \mathcal{M} . By Gelfand's operational calculus [7, § 14], there would exist a function $v \in B(G)$, such that v is identically one on H and zero on $\mathcal{M} \setminus H$. Then uv is supported by F and there this function assumes the constant value A . Because any function of $B(G)$ supported on a free set must vanish at infinity, see, for example, [2], we have a contradiction.

REFERENCES

1. G. Arzac, *Sur l'espace de Banach engendré par les coefficients d'une représentation unitaire*, Publ. Dép. Math. (Lyon), **13** (1976), 1-101.
2. L. De Michele and P. M. Soardi, *A noncommutative extension of Helson's translation lemma*, Boll. Un. Mat. Ital., (4) **9** (1974), 800-806.
3. L. De Michele and A. Figà-Talamanca, *Positive definite functions on free groups*, Amer. J. Math., **102** (1980), 503-509.
4. J. Dixmier, *Les C^* -algèbres et Leurs Représentations*, Gauthier-Villars, Paris, 1964.
5. ———, *Les Algèbres d'Opérateurs dans l'Espace Hilbertien (Algèbres de von Neumann)*, Gauthier-Villars, Paris, 1957.
6. P. Eymard, *L'algèbre de Fourier d'une groupe localement compact*, Bull. Soc. Mat. France, **92** (1964), 181-236.
7. I. M. Gelfand, D. A. Raikov, and G. E. Shilov, *Commutative Normed Rings*, Chelsea, New York, 1964.
8. U. Haagerup, *An example of a non-nuclear C^* -algebra, which has the metric approximation property*, Invent. Math., **50** (1979), 279-293.
9. M. Takesaki, *On the conjugate space of an operator algebra*, Tôhoku Math. J., **10** (1958), 194-203.
10. M. E. Walter, *W^* -algebras and nonabelian harmonic analysis*, J. Functional Analysis, **11** (1972), 17-38.
11. A. Zygmund, *On the lacunary trigonometric series*, Trans. Amer. Math. Soc., **34** (1932), 435-446.

Received April 16, 1981.

UNIVERSITY OF MILAN
MILAN, ITALY

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

DONALD BABBITT (Managing Editor)

University of California
Los Angeles, CA 90024

HUGO ROSSI

University of Utah
Salt Lake City, UT 84112

C. C. MOORE and ARTHUR AGUS

University of California
Berkeley, CA 94720

J. DUGUNDJI

Department of Mathematics
University of Southern California
Los Angeles, CA 90007

R. FINN and J. MILGRAM

Stanford University
Stanford, CA 94305

ASSOCIATE EDITORS

R. ARENS

E. F. BECKENBACH

B. H. NEUMANN

F. WOLF

K. YOSHIDA

SUPPORTING INSTITUTIONS

UNIVERSITY OF ARIZONA
UNIVERSITY OF BRITISH COLUMBIA
CALIFORNIA INSTITUTE OF TECHNOLOGY
UNIVERSITY OF CALIFORNIA
MONTANA STATE UNIVERSITY
UNIVERSITY OF NEVADA, RENO
NEW MEXICO STATE UNIVERSITY
OREGON STATE UNIVERSITY

UNIVERSITY OF OREGON
UNIVERSITY OF SOUTHERN CALIFORNIA
STANFORD UNIVERSITY
UNIVERSITY OF AAWAH
UNIVERSITY OF TOKYO
UNIVERSITY OF UTAH
WASHINGTON STATE UNIVERSITY
UNIVERSITY OF WASHINGTON

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies,

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be in typed form or offset-reproduced, (not dittoed), double spaced with large margins. Please do not use built up fractions in the text of the manuscript. However, you may use them in the displayed equations. Underline Greek letters in red, German in green, and script in blue. The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. Please propose a heading for the odd numbered pages of less than 35 characters. Manuscripts, in triplicate, may be sent to any one of the editors. Please classify according to the scheme of Math. Reviews, Index to Vol. 39. Supply name and address of author to whom proofs should be sent. All other communications should be addressed to the managing editor, or Elaine Barth, University of California, Los Angeles, California, 90024.

50 reprints to each author are provided free for each article, only if page charges have been substantially paid. Additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is issued monthly as of January 1966, Regular subscription rate: \$114.00 a year (6 Vol., 12 issues). Special rate: \$57.00 a year to individual members of supporting institution.

Subscriptions, orders for numbers issued in the last three calendar years, and changes of address should be sent to Pacific Journal of Mathematics, P.O. Box 969, Carmel Valley, CA 93924, U.S.A. Old back numbers obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.).

8-8, 3-chome, Takadanobaba, Shinjuku-ku, Tokyo 160, Japan.

Copyright © 1982 by Pacific Journal of Mathematics
Manufactured and first issued in Japan

Alberto Alesina and Leonede De Michele, A dichotomy for a class of positive definite functions	251
Kahtan Alzubaidy, Rank ₂ p -groups, $p > 3$, and Chern classes	259
James Arney and Edward A. Bender, Random mappings with constraints on coalescence and number of origins	269
Bruce C. Berndt, An arithmetic Poisson formula	295
Julius Rubin Blum and J. I. Reich, Pointwise ergodic theorems in l.c.a. groups	301
Jonathan Borwein, A note on ε -subgradients and maximal monotonicity	307
Andrew Michael Brunner, Edward James Mayland, Jr. and Jonathan Simon, Knot groups in S^4 with nontrivial homology	315
Luis A. Caffarelli, Avner Friedman and Alessandro Torelli, The two-obstacle problem for the biharmonic operator	325
Aleksander Całka, On local isometries of finitely compact metric spaces	337
William S. Cohn, Carleson measures for functions orthogonal to invariant subspaces	347
Roger Fenn and Denis Karmen Sjerve, Duality and cohomology for one-relator groups	365
Gen Hua Shi, On the least number of fixed points for infinite complexes	377
George Golightly, Shadow and inverse-shadow inner products for a class of linear transformations	389
Joachim Georg Hartung, An extension of Sion's minimax theorem with an application to a method for constrained games	401
Vikram Jha and Michael Joseph Kallaher, On the Lorimer-Rahilly and Johnson-Walker translation planes	409
Kenneth Richard Johnson, Unitary analogs of generalized Ramanujan sums	429
Peter Dexter Johnson, Jr. and R. N. Mohapatra, Best possible results in a class of inequalities	433
Dieter Jungnickel and Sharad S. Sane, On extensions of nets	437
Johan Henricus Bernardus Kemperman and Morris Skibinsky, On the characterization of an interesting property of the arcsin distribution	457
Karl Andrew Kosler, On hereditary rings and Noetherian V -rings	467
William A. Lampe, Congruence lattices of algebras of fixed similarity type. II	475
M. N. Mishra, N. N. Nayak and Swadeenananda Pattanayak, Strong result for real zeros of random polynomials	509
Sidney Allen Morris and Peter Robert Nickolas, Locally invariant topologies on free groups	523
Richard Cole Penney, A Fourier transform theorem on nilmanifolds and nil-theta functions	539
Andrei Shkalikov, Estimates of meromorphic functions and summability theorems	569
László Székelyhidi, Note on exponential polynomials	583
William Thomas Watkins, Homeomorphic classification of certain inverse limit spaces with open bonding maps	589
David G. Wright, Countable decompositions of E^n	603
Takayuki Kawada, Correction to: "Sample functions of Pólya processes"	611
Z. A. Chanturia, Errata: "On the absolute convergence of Fourier series of the classes $H^\omega \cap V[v]$ "	611