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KNOT GROUPS IN S^4 WITH NONTRIVIAL HOMOLOGY[1]

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In this paper we exhibit smooth 2-manifolds F^2 in the 4-sphere S^4 having the property that the second homology of the group $\pi_1(S^4-F^2)$ is nontrivial. In particular, we obtain tori for which $H_2(\pi_1)\cong Z_2$ and, by forming connected sums, surfaces of genus n for which $H_2(\pi_1)$ is the direct sum of n copies of Z_2. Corollaries include: (1) There are knotted surfaces in S^4 that cannot be constructed by forming connected sums of unknotted surfaces and knotted 2-spheres. (2) The class of groups that occur as knot groups of surfaces in S^4 is not contained in the class of high dimensional knot groups of S^n in S^{n+2}.

If F is a compact manifold ($\partial F = \phi$) in the n-sphere $S^n (n \geqq 4)$ then, using Alexander duality and the fact that $H_2(\pi_1(S^n - F))$ is a homomorphic image of $H_2(S^n - F)$, it is easy to show that $H_2(\pi_1(S^n - F))$ is no larger than $H^{n-3}(F)$. In the case where F is a 2-sphere in S^4, this is Kervaire's proof [6] that $H_2(\pi_1(S^4 - F))=0$. Since the property of vanishing second homology is so important in characterizing knot groups of spheres in spheres [6], it is interesting to ask [7, Problem 4.29] [14, Conjecture 4.13] whether it is shared by other manifolds F in S^4. The answer we obtain is "sometimes".

For example, if F^2 is a closed, orientable 2-manifold embedded in S^4 in a standard way (i.e., contained in the equatorial 3-sphere), then $\pi_1(S^4 - F^2) \cong Z$, which has trivial second homology. If we form the connected sum (analogous to composing knots $S^1 \subset S^3$) of such a surface F^2 with a knotted 2-sphere S^2, then the group of the knotted surface $F^2 \# S^2$ in S^4 is just $\pi_1(S^4 - S^2)$; as noted above, this has trivial homology.

On the other hand, in § 2, we shall exhibit smooth tori (of genus 1) F^2 in S^4 such that $H_2(\pi_1(S^4 - F^2)) \cong Z_2$. Such a torus cannot be expressed as the connected sum of an unknotted torus and a knotted 2-sphere. Furthermore, $\pi_1(S^4 - F^2)$ cannot occur [6] as the knot group of some $S^n \subset S^{n+2}$. By spinning, we can generate knotted embeddings of the n-torus $S^1 \times \cdots \times S^1$ in S^{n+2} having the same "unusual" knot groups.

In § 3, we establish a connected-sum lemma, $H_2(\pi_1(S^4-F_1^2\#F_2^2))\cong H_2(\pi_1(S^4 - F_1^2)) \oplus H_2(\pi_1(S^4 - F_2^2))$. By composing the tori found in § 2, we can therefore construct surfaces of any genus n, for which

[1] A preliminary report on this paper appeared as [1].

the second homology of the knot group is $Z_2 \oplus \cdots \oplus Z_2$ (n summands). Thus, using the upperbound $H^1(F)$ mentioned above, we conclude that the groups that occur as knot groups of surfaces of genus n in S^4 are a *proper* subset of the groups that arise from surfaces of genus $2n+1$.

It seems plausible that the number $2n+1$ (last sentence above) can be pushed closer to n. For surfaces of genus 1, we have been unable to find knot groups with second homology larger than Z_2, and we are left with the question: *Are there tori in S^4 whose knot groups have second homology equal to (even close to) the theoretical upperbound $Z \oplus Z$?*[2] In this connection, it may be noted that the example given in [12] of a homomorphic image, G, of a knot group $(S^1 \subset S^3)$ with $H_2(G) \neq 0$ actually has $H_2(G) \cong Z_2$; the groups G one obtains by killing the longitude of a knot with Property R [11] have $H_2(G) \cong Z$ [4].

1. Preliminaries. The spaces and subspaces we discuss are smooth or polyhedral. All homology groups are taken with integer coefficients. If G is a group and $x, y \in G$, then $[x, y]$ denotes $x^{-1}y^{-1}xy$; if $A, B \subseteq G$ then $[A, B]$ denotes the smallest normal subgroup of G containing $\{[a, b]\colon a \in A, b \in B\}$.

There are several (equivalent) definitions of the second homology of a group.

DEFINITION 1.1. If X is a connected CW-complex with $\pi_1(X) \cong G$ and $\pi_n(X) = 0$ $(n \geqq 2)$ then for each p, $H_p(G)$ is defined to be $H_p(X)$.

DEFINITION 1.2. If Y is connected CW-complex with $\pi_1(Y) \cong G$, and $\sum_2(Y)$ denotes the subgroup of $H_2(Y)$ generated by all singular 2-cycles representable by maps of a 2-sphere into Y, then $H_2(G) = H_2(Y)/\sum_2(Y)$. (Informally, $H_2(G) = H_2(Y)/\pi_2(Y)$.)

DEFINITION 1.3. If F is a free group, $\theta\colon F \to G$ an epimorphism, and $R = \ker\theta$, then $H_2(G) = R \cap [F, F]/[F, R]$.

The equivalence of 1.1 and 1.2 is clear, once one shows that 1.1 is unambiguous, since a space X (as in 1.1) can be built from Y (as in 1.2) by adjoining cells of dimension $\geqq 3$. The equivalence of 1.2 and 1.3 is shown in [5]. For computing $H_2(G)$, it may be convenient to view G as a quotient of some group A that (is not free but still) has trivial second homology. The following lemma of J. Stallings [13] provides the necessary instructions.

[2] See concluding Remark.

LEMMA 1.4. *If A is a group and N is a normal subgroup of A then there is a (natural) exact sequence*

$$H_2(A) \longrightarrow H_2(A/N) \longrightarrow N/[A, N] \longrightarrow H_1(A) \longrightarrow H_1(A/N) \longrightarrow 0 .$$

LEMMA 1.4.1. *If A is a group with $H_2(A) = 0$, N is a normal subgroup of A such that $N \subseteq [A, A]$, and $G = A/N$, then $H_2(G) \cong N/[A, N]$.*

Proof. This is just a special case of Lemma 1.4.

LEMMA 1.5. *Suppose a group G has a presentation of the form $\langle a, b; b = w^{-1}aw\rangle$, where w is some word in a and b. Then $H_2(G) = 0$.*

Proof. Let Y be a 2-complex formed by attaching one disk to a wedge of two circles, such that $\pi_1(Y) \cong G$. By counting cells, we see the Euler characteristic of Y is 0. Since $\beta_0(Y) = \beta_1(Y) = 1$, we conclude $\beta_2(Y) = 0$ and so, since Y is 2-dimensional, $H_2(Y) = 0$. According to Definition 1.2, $H_2(G) = 0$.

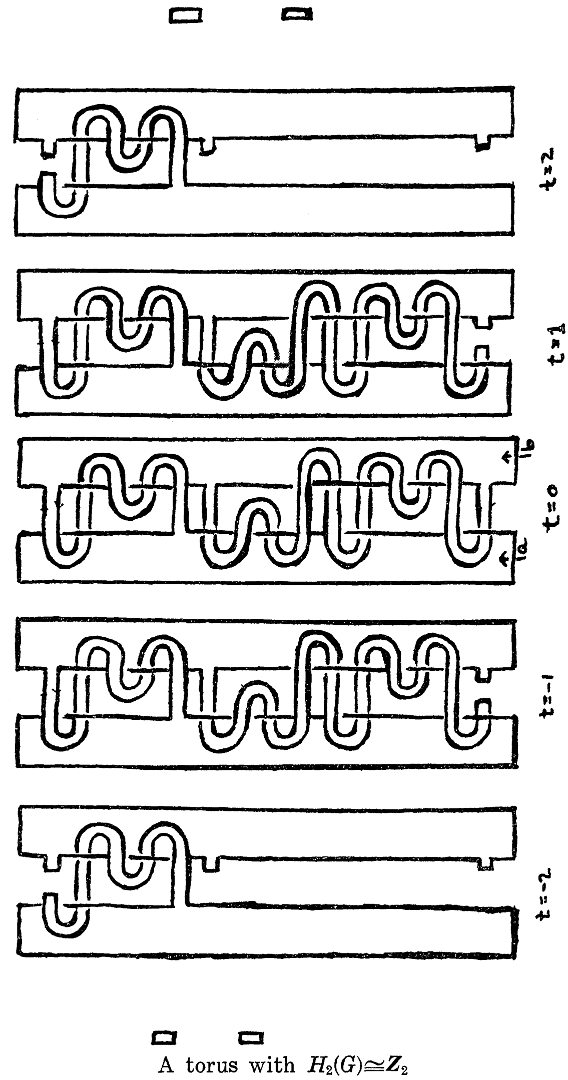
LEMMA 1.6. *Suppose a group G has a presentation of the form $\langle a, b; b = w^{-1}aw, [b, y] = 1\rangle$, for some words w, y in a and b. Then $H_2(G)$ is isomorphic to the cyclic subgroup generated by $[b, y]$ in the group $C = \langle a, b; b = w^{-1}aw, [a, [b, y]] = 1, [b, [b, y]] = 1\rangle$.*

Proof. Let $A = \langle a, b; b = w^{-1}aw\rangle$ and let N be the normal subgroup of A generated by $[b, y]$. By Lemma 1.5, $H_2(A) = 0$. By Lemma 1.4.1, we then have $H_2(G) \cong N/[A, N]$. The subgroup $[A, N]$ is the kernel of the obvious map of A onto C, so $H_2(G)$ is isomorphic to the image of N under this map; this image is precisely the cyclic subgroup of C generated by $[b, y]$.

2. Examples of tori in S^4. Our first example is illustrated in Figure 1, in the form of successive cross-sections (as in § 6 of [3]). We originally obtained this torus T by the methods of [16], so T is a symmetric ribbon surface. We can, at this point, either compute $\pi_1(S^4 - T)$ from Figure 1 as in [3], or start with a suitable presentation of the group and invoke [16]. In either case, we have the following.

PROPOSITION 2.1. *If T is the torus in Figure 1 then the group $G = \pi_1(S^4 - T)$ has a presentation*

$$\langle a, b; b = a^{-1}b^2ab^{-2}a, b = [ba^{-1}, a^{-1}b]^{-1}b[ba^{-1}, a^{-1}b]\rangle .$$



A torus with $H_2(G) \cong \boldsymbol{Z}_2$

FIGURE 1

THEOREM 2.2. *If G is the group in 2.1 then $H_2(G) \cong Z_2$.*

Proof. Let λ denote $[ba^{-1}, a^{-1}b]$, w denote $b^{-1}a^{-1}b^2ab^{-2}a$, $A = \langle a, b; w = 1 \rangle$ and $C = \langle a, b; w = [a, [b, \lambda]] = [b, [b, \lambda]] = 1 \rangle$. By Lemma 1.6, $H_2(G)$ is isomorphic to the cyclic subgroup of C generated by $[b, \lambda]$.

First note that in A, hence in C, $b^{-1}\lambda b = \lambda^{-1}$. (To see that $b^{-1}\lambda b\lambda = 1$ in A, first cyclically reduce $b^{-1}\lambda b\lambda$; then replace a subword, $a^{-1}b^2ab^{-2}a$, of this with "b"; then note that the word so obtained is a cyclic permutation of w^{-1}.) Thus $[b, \lambda] = \lambda^2$ and $[b, [b, \lambda]] = \lambda^4$ in A.

In C, since $[b, [b, \lambda]] = 1$, we have $\lambda^4 = 1$, i.e., $[b, \lambda]^2 = 1$. We thus have $H_2(G) \cong 0$ or Z_2; to establish the latter, we need to show λ^2 (i.e., $[b, \lambda]) \neq 1$ in C. Since $\lambda \in [C, C]$, we can compute the order of λ in C by computing its order in $[C, C]$.

Claim 2.3. $[C, C]$ has a presentation $\langle B_0, B_{-1}; [B_0, [B_0, B_{-1}]^2] = [B_{-1}, [B_0, B_{-1}]^2] = [B_0, B_{-1}]^4 = 1\rangle$, where $\lambda^2 = [B_0, B_{-1}]^2$.

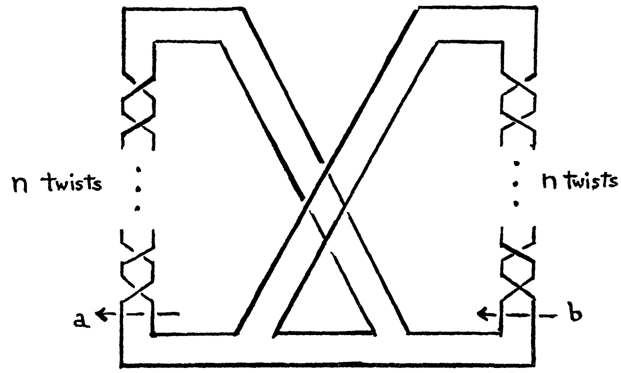
Proof of 2.3. To establish 2.3, we can use the Reidemeister-Schreier process [9, § 2.3], with coset representatives $\{a^n\}_{n\in Z}$ and rewriting function $\rho(b) = \rho(a) = a$, applied to the presentation $C \cong \langle a, b; w = [a, \lambda^2] = \lambda^4 = 1\rangle$. The presentation initially obtained will have infinitely many generators $B_n(=a^n(ba^{-1})a^{-n}, n \in Z)$, but almost all the generators and relations can be eliminated, leaving 2.3. Alternatively, we can argue as follows.

Let $D = \langle u, v; [u, [u, v]^2] = [v, [u, v]^2] = [u, v]^4 = 1\rangle$. The function $\theta(u) = v$, $\theta(v) = vu$ sends $[u, v]$ to $[u, v]^{-1}$ and therefore defines an automorphism of D. Extend D to a group $\tilde{D} = \langle D, b; b^{-1}gb=\theta(g)$, all $g \in D\rangle$. We then have $D = [\tilde{D}, \tilde{D}]$, and $\tilde{D} \cong \langle u, v, b; b^{-1}ub = v$, $b^{-1}vb = vu$, $[u, v]^4 = [u, [u, v]^2] = [v, [u, v]^2] = 1\rangle$. Use $v = b^{-1}ub$ to eliminate the generator v, introduce a new generator $a = u^{-1}b$, and use $u = ba^{-1}$ to eliminate the generator u. Since, as noted earlier, the relation $w = 1$ implies $b^{-1}\lambda b = \lambda^{-1}$, it is easy to show that $\tilde{D}$ is exactly C. We know $D = [\tilde{D}, \tilde{D}]$, and if we identify u with B_0, v with B_{-1}, we obtain 2.3.

We now map $[C, C]$ onto the group $\mathscr{D}_8 = \langle B_0, B_{-1}; B_0^2 = B_{-1}^2 = (B_0B_{-1})^8 = 1\rangle$ by setting $B_0^2 = B_{-1}^2 = 1$. Under this map, $\lambda^2 \to (B_0B_{-1})^4$. Since the order of B_0B_{-1} in $\mathscr{D}_8$ is exactly 8 [2, §§ 4.3, 4.4], we conclude $\lambda^2 \neq 1$ in C. This completes the proof of Theorem 2.2.

REMARK 2.4. It can be shown that the group $A = \langle a, b; b = a^{-1}b^2ab^{-2}a\rangle$, sometimes called the Fibonacci group, is a Z_2-extension of the group K of the "figure-8" knot [8, § V.2]. By erasing the lower band in Figure 1, we can see a symmetric ribbon 2-sphere with knot group A. The elements b^2 and $\lambda = [ba^{-1}, a^{-1}b]$ are, respectively, the meridian and longitude for K. The fact that K admits on outer automorphism α (conjugation by b in A) with certain properties (e.g., $\alpha(\lambda) = \lambda^{-1}$) can be used as the basis for an alternate proof that $H_2(G) \cong Z_2$. This analysis is the motivation for our next examples, and, in fact, the group G_1 below is isomorphic to the group G of Theorem 2.2.

We originally built the groups H_n (below) as Z_2-extensions of the knot groups $\mathscr{K}_n$ of the knots $K(n, n)$ shown in Figure 2. By [10, p. 229–230], $\mathscr{K}_n \cong \langle a, b, t; t^{-1}a^nbt = a^n, t^{-1}b^nt = a^{-1}b^n\rangle$. The



$K(n, n)$
FIGURE 2

function $\theta(t) = t$, $\theta(b) = t^{-1}b^ntb^{-n}$ defines an automorphism of $\mathscr{K}_n$ such that $\theta^2(g) = t^{-1}gt$ (all $g \in \mathscr{K}_n$). Let $H_n = \langle \mathscr{K}_n, s; s^2 = t, s^{-1}gs = \theta(g)$ (all $g \in \mathscr{K}_n)\rangle$, and $\lambda = [s^{-1}b^ns, b^n]$ (=the longitude of $K(n, n)$). We can show, using arguments similar to [10, proof of Cor. 4.7] that for n odd, centralizing $[b, \lambda]$ in H_n does not kill $[b, \lambda]$. It follows that for n odd, $H_2(G_n) = Z_2$, where $G_n = H_n/[b, \lambda]$. The proof below is somewhat removed from its knot theoretic origins, but the notation is consistent with the preceeding remarks.

THEOREM 2.5. *There exists an infinite family* $\{G_n\}$ *of groups such that*

(i) *For each* n, *there is a smooth torus* $T_n \cong S^1 \times S^1 \subsetneqq S^4$ *such that* $\pi_1(S^4 - T_n) \cong G_n$.

(ii) $G_m \ncong G_n$ $(m \neq n)$.

(iii) $H_2(G_n) \cong Z_2$ (n odd).

Proof. (Remark: Our proof that $H_2(G_n) \neq 0$ requires n to be odd, though another argument might make the assumption unnecessary.) Let $G_n = \langle b, s; s^{-2}b^ns^2 = s^{-1}bsb^n, [s, \lambda] = 1\rangle$, where $\lambda = [s^{-1}b^ns, b^n]$.

Claim 2.6. G_n has a Wirtinger presentation

$$\langle x, s; x = (s^{-1}xs^{-1})^ns(s^{-1}xs^{-1})^{-n},\ s = \lambda^{-1}s\lambda\rangle$$

where $x = b^nsb^{-n}$ (and λ now is expressed as a word in x and s).

Proof of 2.6. Rewrite the relation $s^{-2}b^ns^2 = s^{-1}bsb^n$ as $b = s^{-1}b^ns^2b^{-n}s^{-1}$. Introduce the new generator x and replace the first relation with $b = s^{-1}x^2s^{-1}$. Use the latter to eliminate the generator b.

Claim 2.7. For each n, G_n is the group of a smooth torus in S^4.

Proof of 2.7. This follows from 2.6 and the methods of [16]. Figure 1 illustrates how to weave bands between two unknotted curves, following the instructions of a Wirtinger presentation of a group, to obtain a surface with that knot group.

Claim 2.8. For $m \neq n$, $G_m \not\cong G_n$.

Proof of 2.8. These groups are distinguished by their Alexander polynomials ($\Delta(t) = nt^2 + t - n$).

Claim 2.9. For each n, $H_2(G_n) \cong 0$ or Z_2.

Proof of 2.9. Let $H_n = \langle b, s; s^{-2}b^n s^2 = s^{-1}bsb^n\rangle$ and let $\lambda = [s^{-1}b^n s, b^n]$ in H_n. Note that $s^{-1}\lambda s = [s^{-2}b^n s^2, s^{-1}b^n s] =$ (substitute) $[s^{-1}bsb^n, s^{-1}b^n s] = \lambda^{-1}$.

We observe that G_n is obtained from H_n by killing $[s, \lambda]$ and so, by Claim 2.6 and Lemma 1.6, $H_2(G_n)$ is isomorphic to the cyclic subgroup of $C_n = H_n/[H_n, [s, \lambda]]$ generated by $[s, \lambda]$. Since $[s, \lambda] = \lambda^2$ in H_n, we have $[s, [s, \lambda]] = \lambda^4$. Thus, in C_n, $[s, \lambda]^2 = \lambda^4 = 1$, so $[s, \lambda]$ has order 1 or 2 in C_n.

Claim 2.10. $H_2(G_n) \cong Z_2$ for n odd.

Proof of 2.10. From the proof of 2.9, we have $\lambda^4 = 1$ in C_n and need to show $\lambda^2 \neq 1$. We shall construct a homomorphic image D_ν of C_n in which λ^2 is central but nontrivial.

Let F denote the free nilpotent group of class 2 $\langle u, v; [[X, Y], Z]\rangle$. By a theorem of Gruenberg [9, § 6.5], F is residually a finite 2-group. Thus, since $[u, v]^2 \neq 1$ in F, there is, for some integer m, a group $\hat{F}$ in the variety of groups satisfying the laws $[[X, Y], Z] = 1$ and $X^{2^m} = 1$ that is a homomorphic image of F, and in which $[u, v]$ has order 2^r for some $r \geqq 2$. Since $\hat{F}$ is nilpotent of class 2, the cyclic subgroup generated by $[u, v]$ is central, hence normal, and we can pass to a quotient F^* in which $[u, v]^4 = 1$ (but $[u, v]^2 \neq 1$). Since F^* is nilpotent and generated by (the images of) u and v, any commutator $[g, h]$ equals some power of $[u, v]$, so $[g, h]^4 = 1$. Thus we may choose F^* to be the free group of rank 2 in the variety defined by the laws $X^{2^m} = [[X, Y], Z] = [X, Y]^4 = 1$.

For any integer ν, the free group $\langle x, y\rangle$ has an automorphism τ given by $\tau(x) = y$, $\tau(y) = y^\nu x$. Since F^* is a reduced free group (i.e., (free group)/(verbal subgroup)), τ induces an automorphism τ^* of F^*. Let D_ν be the extension of F^*, $D_\nu = \langle u, v, t; t^{-1}ut = v, t^{-1}vt = v^\nu u$, relations for $F^*(u, v)\rangle$. By eliminating $v (= t^{-1}ut)$, we obtain $D_\nu = \langle u, t; t^{-2}ut^2 = t^{-1}u^\nu tu$, relations for $F^*(u, t^{-1}ut)\rangle$. Note that in

D_ν, $[u, t^{-1}ut]$ has order exactly 4. We now restrict ν so that $\nu n \equiv 1$ modulo (2^m).

The group $C_n = H_n/[H_n, [s, \lambda]]$ has a presentation $\langle b, s; s^{-2}b^n s^2 = s^{-1}bsb^n, [b, \lambda^2] = \lambda^4 = 1\rangle$. Add the relation $b^{2^m} = 1$ to obtain a homomorph $\hat{C}_n$ of C_n. Introduce a new generator $r = b^n$. By choice of ν, we then have $r^\nu = b$; using this to eliminate b, we obtain $\hat{C}_n \cong \langle r, s; r^{2^m} = 1, \ s^{-2}rs^2 = s^{-1}r^\nu sr, [r, \lambda^2] = \lambda^4 = 1\rangle$, where $\lambda = [s^{-1}rs, r]$. The mapping $r \to u, s \to t$ defines an epimorphism of $\hat{C}_n$ onto D_ν. Since λ^2 is central and has order exactly 2 in D_ν, this completes the proof of 2.10.

3. Connected sums.

As with classical knots, one can compose knotted surfaces T_0, T_1 in 4-space (assuming T_0, T_1 are separated by a flat 3-plane or 3-sphere) by connecting T_0 and T_1 with a straight arc α and using α as a guide for an annulus from T_0 to T_1. We denote the surface so obtained by $T_0 \# T_1$. The group $\pi_1(S^4 - T_0 \# T_1)$ is of the form $G_0 *_{\mu_0=\mu_1} G_1$, where $G_i = \pi_1(S^4 - T_i)$ and μ_i is a meridian of T_i (in particular, μ_i generates $G_i/[G_i, G_i]$). The following lemma implies that second homology of groups is additive under this type of composition.

LEMMA 3.1. *Let G and H be groups, $g \in G, h \in H$, and suppose g has infinite order in $G/[G, G]$ and h has infinite order in H. Let $\mathscr{G}$ denote $G *_{g=h} H$. Then $H_2(\mathscr{G}) \cong H_2(G) \oplus H_2(H)$.*

Proof. Let X_G, X_H be connected, aspherical CW-complexes with fundamental groups G, H. Adjoin a cylinder $S^1 \times [0, 1]$ to the disjoint union of X_G and X_H using attaching maps of $S^1 \times \{0\} \to X_G$, $S^1 \times \{1\} \to X_H$ that trace out g, h. The space W so obtained has $\pi_1(W) \cong \mathscr{G}$. Furthermore, since g and h are of infinite order, it follows from [15, Theorem 5] that W is aspherical. According to Definition 1.1, $H_2(\mathscr{G}) \cong H_2(W)$, $H_2(G) \cong H_2(X_G)$, and $H_2(H) \cong H_2(X_H)$. Since, by hypothesis, $\langle g\rangle \to G/[G, G]$ is injective, the Mayer-Vietoris sequence for $(W, X_G \cup S^1 \times [0, 1), \ X_H \cup S^1 \times (0, 1])$ states that $H_2(W) \cong H_2(X_G) \oplus H_2(X_H)$.

THEOREM 3.2. *If T_0, T_1 are surfaces in S^4 with knot groups G_0, G_1 respectively, then $H_2(\pi_1(S^4 - T_0 \# T_1)) \cong H_2(G_0) \oplus H_2(G_1)$.*

COROLLARY 3.3. *The tori exhibited in § 2 are not compositions of unknotted tori with knotted 2-spheres.*

COROLLARY 3.4. *For each $n \geqq 1$, there exists a closed orientable* *surface of genus n, F_n, in S^4 such that* $H_2(\pi_1(S^4 - F_n)) \cong \underbrace{Z_2 \oplus \cdots \oplus Z_2}_{n}$.

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REMARK. We have learned that T. Maeda ("On the groups with Wirtinger presentations", Math. Seminar Notes, Kwansei Gakuin Univ., Sept. 1977) also has obtained an example of a group with nontrivial second homology (Z_2) that occurs as $\pi_1(S^4 - F^2)$ for some surface F^2. More recently, using methods similar to ours, C. Gordon has obtained tori in S^4 with $H_2(G) = Z_n$ for any desired $n \geqq 0$. Finally, R. Litherland has found tori realizing all the groups $Z_p \oplus Z_q$ $(p, q \geqq 0)$.

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