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KNOT GROUPS IN S^4 WITH NONTRIVIAL HOMOLOGY[1]

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In this paper we exhibit smooth 2-manifolds F^2 in the 4-sphere S^4 having the property that the second homology of the group $\pi_1(S^4-F^2)$ is nontrivial. In particular, we obtain tori for which $H_2(\pi_1)\cong Z_2$ and, by forming connected sums, surfaces of genus n for which $H_2(\pi_1)$ is the direct sum of n copies of Z_2. Corollaries include: (1) There are knotted surfaces in S^4 that cannot be constructed by forming connected sums of unknotted surfaces and knotted 2-spheres. (2) The class of groups that occur as knot groups of surfaces in S^4 is not contained in the class of high dimensional knot groups of S^n in S^{n+2}.

If F is a compact manifold ($\partial F = \phi$) in the n-sphere $S^n (n \geqq 4)$ then, using Alexander duality and the fact that $H_2(\pi_1(S^n - F))$ is a homomorphic image of $H_2(S^n - F)$, it is easy to show that $H_2(\pi_1(S^n - F))$ is no larger than $H^{n-3}(F)$. In the case where F is a 2-sphere in S^4, this is Kervaire's proof [6] that $H_2(\pi_1(S^4 - F))=0$. Since the property of vanishing second homology is so important in characterizing knot groups of spheres in spheres [6], it is interesting to ask [7, Problem 4.29] [14, Conjecture 4.13] whether it is shared by other manifolds F in S^4. The answer we obtain is "sometimes".

For example, if F^2 is a closed, orientable 2-manifold embedded in S^4 in a standard way (i.e., contained in the equatorial 3-sphere), then $\pi_1(S^4 - F^2) \cong Z$, which has trivial second homology. If we form the connected sum (analogous to composing knots $S^1 \subset S^3$) of such a surface F^2 with a knotted 2-sphere S^2, then the group of the knotted surface $F^2 \# S^2$ in S^4 is just $\pi_1(S^4 - S^2)$; as noted above, this has trivial homology.

On the other hand, in § 2, we shall exhibit smooth tori (of genus 1) F^2 in S^4 such that $H_2(\pi_1(S^4 - F^2)) \cong Z_2$. Such a torus cannot be expressed as the connected sum of an unknotted torus and a knotted 2-sphere. Furthermore, $\pi_1(S^4 - F^2)$ cannot occur [6] as the knot group of some $S^n \subset S^{n+2}$. By spinning, we can generate knotted embeddings of the n-torus $S^1 \times \cdots \times S^1$ in S^{n+2} having the same "unusual" knot groups.

In § 3, we establish a connected-sum lemma, $H_2(\pi_1(S^4-F_1^2\# F_2^2))\cong H_2(\pi_1(S^4 - F_1^2)) \oplus H_2(\pi_1(S^4 - F_2^2))$. By composing the tori found in § 2, we can therefore construct surfaces of any genus n, for which

[1] A preliminary report on this paper appeared as [1].

the second homology of the knot group is $Z_2 \oplus \cdots \oplus Z_2$ (n summands). Thus, using the upperbound $H^1(F)$ mentioned above, we conclude that the groups that occur as knot groups of surfaces of genus n in S^4 are a *proper* subset of the groups that arise from surfaces of genus $2n + 1$.

It seems plausible that the number $2n + 1$ (last sentence above) can be pushed closer to n. For surfaces of genus 1, we have been unable to find knot groups with second homology larger than Z_2, and we are left with the question: *Are there tori in S^4 whose knot groups have second homology equal to (even close to) the theoretical upperbound $Z \oplus Z$?*[2] In this connection, it may be noted that the example given in [12] of a homomorphic image, G, of a knot group ($S^1 \subset S^3$) with $H_2(G) \neq 0$ actually has $H_2(G) \cong Z_2$; the groups G one obtains by killing the longitude of a knot with Property R [11] have $H_2(G) \cong Z$ [4].

1. Preliminaries. The spaces and subspaces we discuss are smooth or polyhedral. All homology groups are taken with integer coefficients. If G is a group and $x, y \in G$, then $[x, y]$ denotes $x^{-1}y^{-1}xy$; if $A, B \subseteq G$ then $[A, B]$ denotes the smallest normal subgroup of G containing $\{[a, b]\colon a \in A, b \in B\}$.

There are several (equivalent) definitions of the second homology of a group.

DEFINITION 1.1. If X is a connected CW-complex with $\pi_1(X) \cong G$ and $\pi_n(X) = 0$ ($n \geqq 2$) then for each p, $H_p(G)$ is defined to be $H_p(X)$.

DEFINITION 1.2. If Y is connected CW-complex with $\pi_1(Y) \cong G$, and $\sum_2(Y)$ denotes the subgroup of $H_2(Y)$ generated by all singular 2-cycles representable by maps of a 2-sphere into Y, then $H_2(G) = H_2(Y)/\sum_2(Y)$. (Informally, $H_2(G) = H_2(Y)/\pi_2(Y)$.)

DEFINITION 1.3. If F is a free group, $\theta\colon F \to G$ an epimorphism, and $R = \ker \theta$, then $H_2(G) = R \cap [F, F]/[F, R]$.

The equivalence of 1.1 and 1.2 is clear, once one shows that 1.1 is unambiguous, since a space X (as in 1.1) can be built from Y (as in 1.2) by adjoining cells of dimension $\geqq 3$. The equivalence of 1.2 and 1.3 is shown in [5]. For computing $H_2(G)$, it may be convenient to view G as a quotient of some group A that (is not free but still) has trivial second homology. The following lemma of J. Stallings [13] provides the necessary instructions.

[2] See concluding Remark.

LEMMA 1.4. *If A is a group and N is a normal subgroup of A then there is a (natural) exact sequence*

$$H_2(A) \longrightarrow H_2(A/N) \longrightarrow N/[A, N] \longrightarrow H_1(A) \longrightarrow H_1(A/N) \longrightarrow 0 .$$

LEMMA 1.4.1. *If A is a group with $H_2(A) = 0$, N is a normal subgroup of A such that $N \subseteq [A, A]$, and $G = A/N$, then $H_2(G) \cong N/[A, N]$.*

Proof. This is just a special case of Lemma 1.4.

LEMMA 1.5. *Suppose a group G has a presentation of the form $\langle a, b; b = w^{-1}aw \rangle$, where w is some word in a and b. Then $H_2(G) = 0$.*

Proof. Let Y be a 2-complex formed by attaching one disk to a wedge of two circles, such that $\pi_1(Y) \cong G$. By counting cells, we see the Euler characteristic of Y is 0. Since $\beta_0(Y) = \beta_1(Y) = 1$, we conclude $\beta_2(Y) = 0$ and so, since Y is 2-dimensional, $H_2(Y) = 0$. According to Definition 1.2, $H_2(G) = 0$.

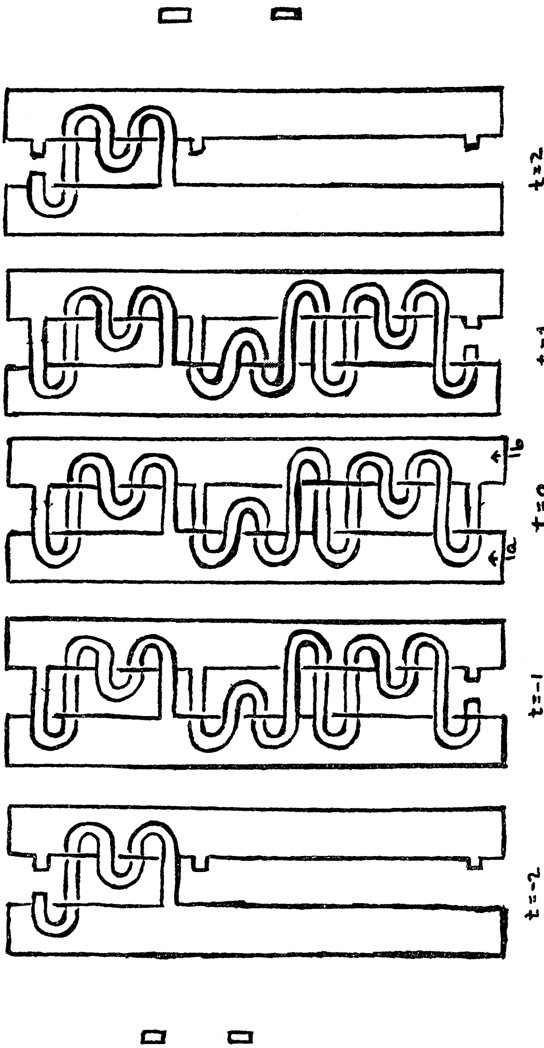
LEMMA 1.6. *Suppose a group G has a presentation of the form $\langle a, b; b = w^{-1}aw, [b, y] = 1 \rangle$, for some words w, y in a and b. Then $H_2(G)$ is isomorphic to the cyclic subgroup generated by $[b, y]$ in the group $C = \langle a, b; b = w^{-1}aw, [a, [b, y]] = 1, [b, [b, y]] = 1 \rangle$.*

Proof. Let $A = \langle a, b; b = w^{-1}aw \rangle$ and let N be the normal subgroup of A generated by $[b, y]$. By Lemma 1.5, $H_2(A) = 0$. By Lemma 1.4.1, we then have $H_2(G) \cong N/[A, N]$. The subgroup $[A, N]$ is the kernel of the obvious map of A onto C, so $H_2(G)$ is isomorphic to the image of N under this map; this image is precisely the cyclic subgroup of C generated by $[b, y]$.

2. Examples of tori in S^4. Our first example is illustrated in Figure 1, in the form of successive cross-sections (as in § 6 of [3]). We originally obtained this torus T by the methods of [16], so T is a symmetric ribbon surface. We can, at this point, either compute $\pi_1(S^4 - T)$ from Figure 1 as in [3], or start with a suitable presentation of the group and invoke [16]. In either case, we have the following.

PROPOSITION 2.1. *If T is the torus in Figure 1 then the group $G = \pi_1(S^4 - T)$ has a presentation*

$$\langle a, b; b = a^{-1}b^2ab^{-2}a, b = [ba^{-1}, a^{-1}b]^{-1}b[ba^{-1}, a^{-1}b] \rangle .$$



A torus with $H_2(G) \cong Z_2$
FIGURE 1

THEOREM 2.2. *If G is the group in* 2.1 *then* $H_2(G) \cong Z_2$.

Proof. Let λ denote $[ba^{-1}, a^{-1}b]$, w denote $b^{-1}a^{-1}b^2ab^{-2}a$, $A = \langle a, b; w = 1\rangle$ and $C = \langle a, b; w = [a, [b, \lambda]] = [b, [b, \lambda]] = 1\rangle$. By Lemma 1.6, $H_2(G)$ is isomorphic to the cyclic subgroup of C generated by $[b, \lambda]$.

First note that in A, hence in C, $b^{-1}\lambda b = \lambda^{-1}$. (To see that $b^{-1}\lambda b\lambda = 1$ in A, first cyclically reduce $b^{-1}\lambda b\lambda$; then replace a subword, $a^{-1}b^2ab^{-2}a$, of this with "b"; then note that the word so obtained is a cyclic permutation of w^{-1}.) Thus $[b, \lambda] = \lambda^2$ and $[b, [b, \lambda]] = \lambda^4$ in A.

In C, since $[b, [b, \lambda]] = 1$, we have $\lambda^4 = 1$, i.e., $[b, \lambda]^2 = 1$. We thus have $H_2(G) \cong 0$ or Z_2; to establish the latter, we need to show λ^2 (i.e., $[b, \lambda]) \neq 1$ in C. Since $\lambda \in [C, C]$, we can compute the order of λ in C by computing its order in $[C, C]$.

Claim 2.3. $[C, C]$ has a presentation $\langle B_0, B_{-1}; [B_0, [B_0, B_{-1}]^2] = [B_{-1}, [B_0, B_{-1}]^2] = [B_0, B_{-1}]^4 = 1\rangle$, where $\lambda^2 = [B_0, B_{-1}]^2$.

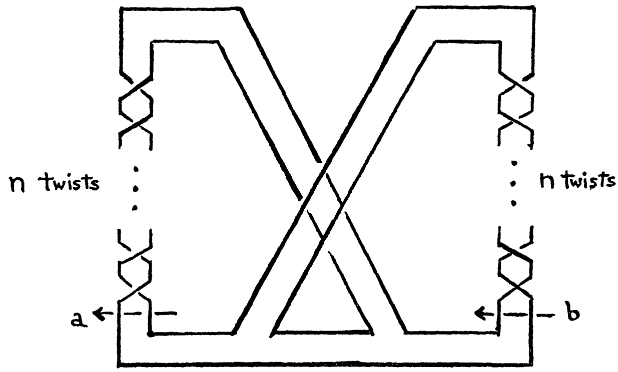
Proof of 2.3. To establish 2.3, we can use the Reidemeister-Schreier process [9, § 2.3], with coset representatives $\{a^n\}_{n \in Z}$ and rewriting function $\rho(b) = \rho(a) = a$, applied to the presentation $C \cong \langle a, b; w = [a, \lambda^2] = \lambda^4 = 1\rangle$. The presentation initially obtained will have infinitely many generators $B_n(= a^n(ba^{-1})a^{-n}, n \in Z)$, but almost all the generators and relations can be eliminated, leaving 2.3. Alternatively, we can argue as follows.

Let $D = \langle u, v; [u, [u, v]^2] = [v, [u, v]^2] = [u, v]^4 = 1\rangle$. The function $\theta(u) = v$, $\theta(v) = vu$ sends $[u, v]$ to $[u, v]^{-1}$ and therefore defines an automorphism of D. Extend D to a group $\tilde{D} = \langle D, b; b^{-1}gb = \theta(g)$, all $g \in D\rangle$. We then have $D = [\tilde{D}, \tilde{D}]$, and $\tilde{D} \cong \langle u, v, b; b^{-1}ub = v$, $b^{-1}vb = vu$, $[u, v]^4 = [u, [u, v]^2] = [v, [u, v]^2] = 1\rangle$. Use $v = b^{-1}ub$ to eliminate the generator v, introduce a new generator $a = u^{-1}b$, and use $u = ba^{-1}$ to eliminate the generator u. Since, as noted earlier, the relation $w = 1$ implies $b^{-1}\lambda b = \lambda^{-1}$, it is easy to show that $\tilde{D}$ is exactly C. We know $D = [\tilde{D}, \tilde{D}]$, and if we identify u with B_0, v with B_{-1}, we obtain 2.3.

We now map $[C, C]$ onto the group $\mathscr{D}_8 = \langle B_0, B_{-1}; B_0^2 = B_{-1}^2 = (B_0B_{-1})^8 = 1\rangle$ by setting $B_0^2 = B_{-1}^2 = 1$. Under this map, $\lambda^2 \to (B_0B_{-1})^4$. Since the order of B_0B_{-1} in $\mathscr{D}_8$ is exactly 8 [2, §§ 4.3, 4.4], we conclude $\lambda^2 \neq 1$ in C. This completes the proof of Theorem 2.2.

REMARK 2.4. It can be shown that the group $A = \langle a, b; b = a^{-1}b^2ab^{-2}a\rangle$, sometimes called the Fibonacci group, is a Z_2-extension of the group K of the "figure-8" knot [8, § V.2]. By erasing the lower band in Figure 1, we can see a symmetric ribbon 2-sphere with knot group A. The elements b^2 and $\lambda = [ba^{-1}, a^{-1}b]$ are, respectively, the meridian and longitude for K. The fact that K admits on outer automorphism α (conjugation by b in A) with certain properties (e.g., $\alpha(\lambda) = \lambda^{-1}$) can be used as the basis for an alternate proof that $H_2(G) \cong Z_2$. This analysis is the motivation for our next examples, and, in fact, the group G_1 below is isomorphic to the group G of Theorem 2.2.

We originally built the groups H_n (below) as Z_2-extensions of the knot groups $\mathscr{K}_n$ of the knots $K(n, n)$ shown in Figure 2. By [10, p. 229–230], $\mathscr{K}_n \cong \langle a, b, t; t^{-1}a^nbt = a^n, t^{-1}b^nt = a^{-1}b^n\rangle$. The



$K(n, n)$
FIGURE 2

function $\theta(t) = t$, $\theta(b) = t^{-1}b^n t b^{-n}$ defines an automorphism of $\mathscr{K}_n$ such that $\theta^2(g) = t^{-1}gt$ (all $g \in \mathscr{K}_n$). Let $H_n = \langle \mathscr{K}_n, s; s^2 = t, s^{-1}gs = \theta(g)$ (all $g \in \mathscr{K}_n)\rangle$, and $\lambda = [s^{-1}b^n s, b^n]$ (=the longitude of $K(n, n)$). We can show, using arguments similar to [10, proof of Cor. 4.7] that for n odd, centralizing $[b, \lambda]$ in H_n does not kill $[b, \lambda]$. It follows that for n odd, $H_2(G_n) = Z_2$, where $G_n = H_n/[b, \lambda]$. The proof below is somewhat removed from its knot theoretic origins, but the notation is consistent with the preceeding remarks.

THEOREM 2.5. *There exists an infinite family* $\{G_n\}$ *of groups such that*

(i) *For each* n, *there is a smooth torus* $T_n \cong S^1 \times S^1 \subseteq S^4$ *such that* $\pi_1(S^4 - T_n) \cong G_n$.

(ii) $G_m \not\cong G_n$ $(m \neq n)$.

(iii) $H_2(G_n) \cong Z_2$ (n odd).

Proof. (Remark: Our proof that $H_2(G_n) \neq 0$ requires n to be odd, though another argument might make the assumption unnecessary.) Let $G_n = \langle b, s; s^{-2}b^n s^2 = s^{-1}bsb^n, [s, \lambda] = 1\rangle$, where $\lambda = [s^{-1}b^n s, b^n]$.

Claim 2.6. G_n has a Wirtinger presentation

$$\langle x, s; x = (s^{-1}xs^{-1})^n s(s^{-1}xs^{-1})^{-n},\ s = \lambda^{-1}s\lambda\rangle$$

where $x = b^n s b^{-n}$ (and λ now is expressed as a word in x and s).

Proof of 2.6. Rewrite the relation $s^{-2}b^n s^2 = s^{-1}bsb^n$ as $b = s^{-1}b^n s^2 b^{-n} s^{-1}$. Introduce the new generator x and replace the first relation with $b = s^{-1}x^2 s^{-1}$. Use the latter to eliminate the generator b.

Claim 2.7. For each n, G_n is the group of a smooth torus in S^4.

Proof of 2.7. This follows from 2.6 and the methods of [16]. Figure 1 illustrates how to weave bands between two unknotted curves, following the instructions of a Wirtinger presentation of a group, to obtain a surface with that knot group.

Claim 2.8. For $m \neq n$, $G_m \not\cong G_n$.

Proof of 2.8. These groups are distinguished by their Alexander polynomials ($\Delta(t) = nt^2 + t - n$).

Claim 2.9. For each n, $H_2(G_n) \cong 0$ or Z_2.

Proof of 2.9. Let $H_n = \langle b, s; s^{-2}b^n s^2 = s^{-1}bsb^n\rangle$ and let $\lambda = [s^{-1}b^n s, b^n]$ in H_n. Note that $s^{-1}\lambda s = [s^{-2}b^n s^2, s^{-1}b^n s] =$ (substitute) $[s^{-1}bsb^n, s^{-1}b^n s] = \lambda^{-1}$.

We observe that G_n is obtained from H_n by killing $[s, \lambda]$ and so, by Claim 2.6 and Lemma 1.6, $H_2(G_n)$ is isomorphic to the cyclic subgroup of $C_n = H_n/[H_n, [s, \lambda]]$ generated by $[s, \lambda]$. Since $[s, \lambda] = \lambda^2$ in H_n, we have $[s, [s, \lambda]] = \lambda^4$. Thus, in C_n, $[s, \lambda]^2 = \lambda^4 = 1$, so $[s, \lambda]$ has order 1 or 2 in C_n.

Claim 2.10. $H_2(G_n) \cong Z_2$ for n odd.

Proof of 2.10. From the proof of 2.9, we have $\lambda^4 = 1$ in C_n and need to show $\lambda^2 \neq 1$. We shall construct a homomorphic image D_ν of C_n in which λ^2 is central but nontrivial.

Let F denote the free nilpotent group of class 2 $\langle u, v; [[X, Y], Z]\rangle$. By a theorem of Gruenberg [9, § 6.5], F is residually a finite 2-group. Thus, since $[u, v]^2 \neq 1$ in F, there is, for some integer m, a group $\hat{F}$ in the variety of groups satisfying the laws $[[X, Y], Z] = 1$ and $X^{2^m} = 1$ that is a homomorphic image of F, and in which $[u, v]$ has order 2^r for some $r \geqq 2$. Since $\hat{F}$ is nilpotent of class 2, the cyclic subgroup generated by $[u, v]$ is central, hence normal, and we can pass to a quotient F^* in which $[u, v]^4 = 1$ (but $[u, v]^2 \neq 1$). Since F^* is nilpotent and generated by (the images of) u and v, any commutator $[g, h]$ equals some power of $[u, v]$, so $[g, h]^4 = 1$. Thus we may choose F^* to be the free group of rank 2 in the variety defined by the laws $X^{2^m} = [[X, Y], Z] = [X, Y]^4 = 1$.

For any integer ν, the free group $\langle x, y\rangle$ has an automorphism τ given by $\tau(x) = y$, $\tau(y) = y^\nu x$. Since F^* is a reduced free group (i.e., (free group)/(verbal subgroup)), τ induces an automorphism τ^* of F^*. Let D_ν be the extension of F^*, $D_\nu = \langle u, v, t; t^{-1}ut = v, t^{-1}vt = v^\nu u$, relations for $F^*(u, v)\rangle$. By eliminating $v (= t^{-1}ut)$, we obtain $D_\nu = \langle u, t; t^{-2}ut^2 = t^{-1}u^\nu tu$, relations for $F^*(u, t^{-1}ut)\rangle$. Note that in

D_ν, $[u, t^{-1}ut]$ has order exactly 4. We now restrict ν so that $\nu n \equiv 1$ modulo (2^m).

The group $C_n = H_n/[H_n, [s, \lambda]]$ has a presentation $\langle b, s; s^{-2}b^n s^2 = s^{-1}bsb^n, [b, \lambda^2] = \lambda^4 = 1\rangle$. Add the relation $b^{2^m} = 1$ to obtain a homomorph $\hat{C}_n$ of C_n. Introduce a new generator $r = b^n$. By choice of ν, we then have $r^\nu = b$; using this to eliminate b, we obtain $\hat{C}_n \cong \langle r, s; r^{2^m} = 1, \ s^{-2}rs^2 = s^{-1}r^\nu sr, [r, \lambda^2] = \lambda^4 = 1\rangle$, where $\lambda = [s^{-1}rs, r]$. The mapping $r \to u, s \to t$ defines an epimorphism of $\hat{C}_n$ onto D_ν. Since λ^2 is central and has order exactly 2 in D_ν, this completes the proof of 2.10.

3. Connected sums. As with classical knots, one can compose knotted surfaces T_0, T_1 in 4-space (assuming T_0, T_1 are separated by a flat 3-plane or 3-sphere) by connecting T_0 and T_1 with a straight arc α and using α as a guide for an annulus from T_0 to T_1. We denote the surface so obtained by $T_0 \# T_1$. The group $\pi_1(S^4 - T_0 \# T_1)$ is of the form $G_0 *_{\mu_0=\mu_1} G_1$, where $G_i = \pi_1(S^4 - T_i)$ and μ_i is a meridian of T_i (in particular, μ_i generates $G_i/[G_i, G_i]$). The following lemma implies that second homology of groups is additive under this type of composition.

LEMMA 3.1. *Let G and H be groups, $g \in G$, $h \in H$, and suppose g has infinite order in $G/[G, G]$ and h has infinite order in H. Let $\mathscr{G}$ denote $G *_{g=h} H$. Then $H_2(\mathscr{G}) \cong H_2(G) \oplus H_2(H)$.*

Proof. Let X_G, X_H be connected, aspherical CW-complexes with fundamental groups G, H. Adjoin a cylinder $S^1 \times [0, 1]$ to the disjoint union of X_G and X_H using attaching maps of $S^1 \times \{0\} \to X_G$, $S^1 \times \{1\} \to X_H$ that trace out g, h. The space W so obtained has $\pi_1(W) \cong \mathscr{G}$. Furthermore, since g and h are of infinite order, it follows from [15, Theorem 5] that W is aspherical. According to Definition 1.1, $H_2(\mathscr{G}) \cong H_2(W)$, $H_2(G) \cong H_2(X_G)$, and $H_2(H) \cong H_2(X_H)$. Since, by hypothesis, $\langle g\rangle \to G/[G, G]$ is injective, the Mayer-Vietoris sequence for $(W, X_G \cup S^1 \times [0, 1), \ X_H \cup S^1 \times (0, 1])$ states that $H_2(W) \cong H_2(X_G) \oplus H_2(X_H)$.

THEOREM 3.2. *If T_0, T_1 are surfaces in S^4 with knot groups G_0, G_1 respectively, then $H_2(\pi_1(S^4 - T_0 \# T_1)) \cong H_2(G_0) \oplus H_2(G_1)$.*

COROLLARY 3.3. *The tori exhibited in § 2 are not compositions of unknotted tori with knotted 2-spheres.*

COROLLARY 3.4. *For each $n \geqq 1$, there exists a closed orientable* *surface of genus n*, F_n, *in* S^4 *such that* $H_2(\pi_1(S^4-F_n))\cong \underbrace{Z_2\oplus\cdots\oplus Z_2}_{n}$.

Acknowledgments. We wish to thank Dennis Johnson and Dennis Roseman for helpful conversations. We also wish to thank the referee for several helpful suggestions.

REMARK. We have learned that T. Maeda ("On the groups with Wirtinger presentations", Math. Seminar Notes, Kwansei Gakuin Univ., Sept. 1977) also has obtained an example of a group with nontrivial second homology (Z_2) that occurs as $\pi_1(S^4-F^2)$ for some surface F^2. More recently, using methods similar to ours, C. Gordon has obtained tori in S^4 with $H_2(G)=Z_n$ for any desired $n\geqq 0$. Finally, R. Litherland has found tori realizing all the groups $Z_p\oplus Z_q$ (p, $q\geqq 0$).

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