

Pacific Journal of Mathematics

**ON LOCAL ISOMETRIES OF FINITELY COMPACT METRIC
SPACES**

ALEKSANDER CAŁKA

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By local isometries we mean mappings which locally preserve distances. Local isometries which do not increase distances are called nonexpansive local isometries. A few of the main results are:

1. Let f be a local isometry (nonexpansive local isometry) of a finitely compact metric space (M, ρ) into itself. If for each (some) $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then there exists a unique decomposition of M into disjoint open sets, $M = M'_0 \cup M'_1 \cup \dots$, such that (i) f maps M'_0 injectively into itself, and (ii) $f(M'_{i+1}) \subset M'_i$ for each $i = 0, 1, \dots$. Moreover, f maps M'_0 homeomorphically (isometrically) onto itself.
2. Let f be a nonexpansive local isometry (local isometry) of a connected (convex) finitely compact metric space (M, ρ) into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then f is an isometry onto.

1. Introduction. Let f be a mapping of a metric space (M, ρ) into a metric space (N, σ) . We will call f a *local isometry* if for each $z \in M$ there is a neighborhood U_z of z such that $\sigma(f(x), f(y)) = \rho(x, y)$ for all $x, y \in U_z$. If f is a local isometry and also a nonexpansive mapping (i.e., $\sigma(f(x), f(y)) \leq \rho(x, y)$ for all $x, y \in M$), we will say that f is a *nonexpansive local isometry*.

A metric space (M, ρ) is said to be *finitely compact* [2] if each bounded and closed subset of M is compact.

The purpose of this paper is to extend the results of the author's paper [4] to those local isometries f of a finitely compact metric space (M, ρ) into itself which have the property that for each $z \in M$ the sequence $\{f^n(z)\}$ is bounded. In § 2 we give some more notation and preliminary lemmas. Section 3 contains the main results. Roughly speaking, the main theorem is: Let f be a local isometry (nonexpansive local isometry) of a finitely compact metric space (M, ρ) into itself. If for each (for some) $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then there exists a unique decomposition of M into disjoint open sets, $M = M'_0 \cup M'_1 \cup \dots$, such that (i) f maps M'_0 injectively into itself, (ii) $f(M'_i) \subset M'_{i-1}$ for each $i \geq 1$. Moreover, f maps M'_0 homeomorphically (isometrically) onto itself.

It should be noted that open surjective local isometries were studied by Busemann [2], [3], Kirk [5], [6], [7] and Szenthe [8], [9], [10], in the special case where (M, ρ) is a G -space (Busemann [2] called them "locally isometric mappings"). In [5] Kirk proved that

if an open local isometry f of a G -space (M, ρ) onto itself has a fixed point, then f is an isometry (from which it follows that if the isometries of (M, ρ) onto itself form a transitive group, then each open surjective local isometry is an isometry). Later Kirk [6] proved that if an open local isometry f of a G -space (M, ρ) onto itself has the property that for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then f is an isometry.

In § 4 and § 5 of the present paper, by using the results of § 3, we extend the above results of Kirk to the case of general local isometries of finitely compact metric spaces.

2. Preliminaries.

(2.1) DEFINITION. Let ρ_i , $i = 0, 1$, be metrics on a set M . We shall say that ρ_1 is *locally identical* with ρ_0 if the identity mapping, id_M , of M is a local isometry of (M, ρ_0) into (M, ρ_1) . We shall say that ρ_1 and ρ_0 are *locally identical* if ρ_i is locally identical with ρ_j , for all $i, j = 0, 1$.

(2.2) DEFINITION. Let f be a mapping of a metric space (M, ρ) into itself. Then the function ρ_f defined by

$$\rho_f(x, y) = \sup_{n \geq 0} \rho(f^n(x), f^n(y)) \quad \text{for all } x, y \in M,$$

(where $f^0 = \text{id}_M$, $f^{n+1} = f \circ f^n$) is called the *induced metric* on M .

(2.3) REMARKS. (i) Let ρ_i , $i = 0, 1$, be metrics on a set M such that ρ_1 and ρ_0 are locally identical. Then ρ_1 and ρ_0 are topologically equivalent. If (M, ρ_0) is finitely compact and $\rho_1 \geq \rho_0$, then (M, ρ_1) is also finitely compact. If f is a local isometry of (M, ρ_0) into itself, then f is also a local isometry of (M, ρ_1) into itself.

(ii) Let f be a mapping of a metric space (M, ρ) into itself such that for each $z \in M$ the sequence $\{f^n(z)\}$ is bounded. Then for each $x, y \in M$, $\rho_f(x, y) < \infty$, and hence the induced metric, ρ_f , is a metric on the set M such that

- (1) $\rho_f \geq \rho$,
- (2) f is a nonexpansive mapping of the metric space (M, ρ_f) into itself, and
- (3) $\rho_f = \rho$ if and only if f is a nonexpansive mapping of (M, ρ) into itself.

In [4] we proved the following theorem ((4.3) of [4]).

(2.4) THEOREM. *Let f be a local isometry of a compact metric*

space (M, ρ) into itself. Then there exists a unique decomposition of M into disjoint open sets,

$$M = M_0^f \cup \cdots \cup M_n^f \quad (0 \leq n),$$

such that (i) $f(M_0^f) = M_0^f$, (ii) $f(M_i^f) \subset M_{i-1}^f$ and $M_i^f \neq \emptyset$ for each i , $1 \leq i \leq n$. Moreover, the induced metric ρ_f is a metric on M such that ρ_f and ρ are locally identical and f is a nonexpansive local isometry of (M, ρ_f) into itself which maps M_0^f isometrically onto itself.

From this theorem we have

(2.5) COROLLARY. Let f be a one-to-one local isometry of a compact metric space (M, ρ) into itself. Then $f(M) = M$.

Proof. If f is one-to-one, then by (2.4), $M = M_0^f$ and hence $f(M) = M$.

REMARK. If f is a local isometry of a compact metric space (M, ρ) into itself and if N is a compact subset of M such that $f(N) \subset N$, then the restriction of f to N , f/N , is also a local isometry. For convenience, $N = N_0^f \cup \cdots \cup N_{n(N)}^f$ will denote the decomposition of N defined by (2.4) for f/N .

(2.6) PROPOSITION. Let f be a local isometry of a compact metric space (M, ρ) into itself. If N is a compact subset of M such that $f(N) \subset N$, then

$$N_i^f = N \cap M_i^f \quad \text{for each } i = 0, \dots, n(N),$$

where $n(N) = \max \{i \geq 0: N \cap M_i^f \neq \emptyset\}$.

Proof. By (2.4), it is sufficient only to show that $f(N \cap M_0^f) = N \cap M_0^f$. However, it follows from (2.4) that f maps $N \cap M_0^f$ isometrically into itself. Hence, by (2.5), $f(N \cap M_0^f) = N \cap M_0^f$ as desired.

We will need the following.

(2.7) LEMMA. Let f be a local isometry of a metric space (N, ρ) into itself. If N is a compact subset of M , then there exists a number $\delta > 0$ such that for each $z \in N$,

$$(4) \quad \rho(f(x), f(y)) = \rho(x, y),$$

for all $x, y \in S_\rho(z, \delta) = \{p \in M: \rho(z, p) < \delta\}$.

The straightforward verification of (2.7) is omitted.

The convexity in this paper is to be understood in the sense of Menger (cf. [1, p. 40]). A subset N of a metric space (M, ρ) is, accordingly, convex if for each two distinct points $x, y \in N$, there exists a point $z \in N$, $z \neq x, y$, such that $\rho(x, y) = \rho(x, z) + \rho(z, y)$.

Also, we will use

(2.8) LEMMA. *If f is a local isometry of a convex and complete metric space (M, ρ) into itself, then f is a nonexpansive local isometry.*

Proof. Let x and y be given points of M such that $x \neq y$. Since M is convex and complete, by a theorem of Menger (cf. [1, p. 41]) there exists a metric segment $L \subset M$ whose extremities are x and y ; that is, a subset isometric to an interval of length $\rho(x, y)$. Since L is compact, it follows that there exists a finite sequence z_0, z_1, \dots, z_k of points of L such that $z_0 = x$, $z_k = y$ and

$$\rho(f(z_i), f(z_{i+1})) = \rho(z_i, z_{i+1}) \quad \text{for each } i = 0, \dots, k-1$$

and

$$\rho(x, y) = \sum_{i=0}^{k-1} \rho(z_i, z_{i+1}).$$

Thus,

$$\rho(f(x), f(y)) \leq \sum_{i=0}^{k-1} \rho(f(z_i), f(z_{i+1})) = \sum_{i=0}^{k-1} \rho(z_i, z_{i+1}) = \rho(x, y).$$

This proves that f is a nonexpansive mapping, and hence a nonexpansive local isometry.

3. Local isometries and decomposition theorems. We shall now prove the following extension of (2.4).

(3.1) THEOREM. *Let f be a local isometry of a finitely compact metric space (M, ρ) into itself. If for each $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then there exists a unique decomposition of M into disjoint open sets,*

$$(5) \quad M = M_0^f \cup M_1^f \cup \dots,$$

such that

$$(6) \quad f \text{ maps } M_0^f \text{ injectively into itself,}$$

$$(7) \quad f(M_i^f) \subset M_{i-1}^f \quad \text{for each } i = 1, 2, \dots.$$

Moreover, the induced metric, ρ_f , is a metric on M such that ρ_f and ρ are locally identical, (M, ρ_f) is a finitely compact metric space and f is a nonexpansive local isometry of (M, ρ_f) into itself which maps M_i^f isometrically onto itself.

Proof. In the proof, for each $A \subset M$ and $\delta > 0$, $S_\rho(A, \delta)$ is the δ -ball in M about A and $\text{cl } A$ ($\text{Int } A$) is the closure (interior) of A . For each $z \in M$ we denote: $c(z) = \text{cl } \{f^n(z) : n \geq 0\}$.

We first define a sequence A_n , $n = 0, 1, \dots$, of compact subsets of M such that

$$(8) \quad f(A_n) \subset A_n \quad \text{for each } n = 0, 1, \dots,$$

$$(9) \quad A_n \subset \text{Int } A_{n+1} \quad \text{for each } n = 0, 1, \dots,$$

$$(10) \quad \bigcup_{n=0}^{\infty} A_n = M.$$

For each $z \in M$, let $\delta_z > 0$ be a number defined by (2.7) for the compact set $c(z)$ and let $V_z = S_\rho(c(z), \delta_z)$. Thus, for each $z \in M$, V_z is an open and bounded subset of M and using (4) and the fact that $f(c(z)) \subset c(z)$, we have $f(V_z) \subset V_z$. Since (M, ρ) has a countable base of neighborhoods, there exists a sequence z_n , $n = 0, 1, \dots$, of points of M such that $\bigcup_{n=0}^{\infty} V_{z_n} = M$. Define the sets A_n , $n = 0, 1, \dots$, inductively, as follows: $A_0 = \text{cl } V_{z_0}$ and $A_{n+1} = \bigcup_{i=0}^{k(n)} \text{cl } V_{z_i}$, where $k(n)$ is an integer such that $k(n) > n$ and $A_n \subset \bigcup_{i=0}^{k(n)} V_{z_i}$. Clearly, the sets A_n , $n = 0, 1, \dots$, satisfy conditions (8), (9) and (10), and are compact.

It follows now from (2.4), that for each $n \geq 0$, there exists a sequence $(A_n)_i^f$, $i = 0, 1, \dots$, of disjoint subsets of A_n such that

$$(11) \quad (A_n)_i^f \cap \text{Int } A_n \text{ is open, for each } i = 0, 1, \dots,$$

$$(12) \quad \bigcup_{i=0}^{\infty} (A_n)_i^f = A_n,$$

$$(13) \quad f \text{ maps } (A_n)_i^f \text{ injectively into itself,}$$

$$(14) \quad f((A_n)_i^f) \subset (A_n)_{i-1}^f, \quad \text{for each } i = 1, 2, \dots$$

By (2.6), we have

$$(15) \quad (A_n)_i^f = A_n \cap (A_{n+1})_i^f, \quad \text{for all } n, i = 0, 1, \dots$$

Now, for each $i = 0, 1, \dots$, we define the set M_i^f as follows:

$$M_i^f = \bigcup_{n=0}^{\infty} (A_n)_i^f.$$

Then, by (15) and the fact that $(A_n)_i^f$, $i \geq 0$, are disjoint, the sets M_i^f , $i \geq 0$, are disjoint. By (9) and (15),

$$(A_n)_i^f \subset (A_{n+1})_i^f \cap \text{Int } A_{n+1} \subset (A_{n+1})_i^f ,$$

hence,

$$M_i^f = \bigcup_{n=0}^{\infty} ((A_{n+1})_i^f \cap \text{Int } A_{n+1}) , \quad \text{for each } i = 0, 1, \dots ,$$

and therefore, by (11), the sets M_i^f , $i \geq 0$, are open. By (10) and (12),

$$\bigcup_{i=0}^{\infty} M_i^f = \bigcup_{i,n=0}^{\infty} (A_n)_i^f = \bigcup_{n=0}^{\infty} A_n = M ,$$

and it follows from (13), (14) and (15) that the sets M_i^f , $i \geq 0$, satisfy conditions (6) and (7). This proves the existence of the desired decomposition of M .

In order to prove the uniqueness, it is sufficient only to show that for each decomposition of M into disjoint open sets, $M = \bigcup_{i=0}^{\infty} M_i$, conditions (6) and (7) imply

$$(16) \quad M_0 = \{z \in M : f(c(z)) = c(z)\} .$$

Let us assume, $M = \bigcup_{i=0}^{\infty} M_i$ is a decomposition of M into disjoint open sets, satisfying conditions (6) and (7). If $z \in M_0$, then (6) implies that the restriction of f to $c(z)$ is a one-to-one local isometry of $c(z)$ into itself. Since $c(z)$ is compact, it follows from (2.5) that $f(c(z)) = c(z)$. Conversely, if $z \notin M_0$, then $z \in M_n$ for some $n \geq 1$. Using (7) and the fact that M_i , $i \geq 0$, are disjoint and open, we obtain

$$f(c(z)) \subset c(f(z)) \subset M_0 \cup \dots \cup M_{n-1} ,$$

hence $z \in c(z) \setminus c(f(z))$, i.e., $c(z) \neq c(f(z))$. Therefore (16) follows as desired.

Finally, by (ii) of (2.3), the induced metric, ρ_f , is a metric on M and it follows from (8), (9), (10) and (2.4) that ρ_f and ρ are locally identical (cf. also (1)). Hence, by (1) and (i) of (2.3), the metric space (M, ρ_f) is finitely compact and, by (2), f is a nonexpansive local isometry of (M, ρ_f) into itself. It follows from (2.4) and (15) and the definition of M_i^f that f maps M_i^f isometrically onto itself with respect to the metric ρ_f . This completes the proof.

(3.2) REMARK. Let f be a nonexpansive mapping of a metric space (M, ρ) into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then for each $x \in M$ the sequence $\{f^n(x)\}$ is bounded.

Indeed, since f is nonexpansive, then for all $x, z \in M$ and each $i = 0, 1, \dots$, we have

$$\rho(f^i(x), \{f^n(z)\}) \leq \rho(f^i(x), f^i(z)) \leq \rho(x, z) ,$$

hence, if $\{f^n(z)\}$ is bounded, then also $\{f^n(x)\}$ is bounded.

The following theorem is an immediate consequence of (3.1), (3.2) and (3).

(3.3) THEOREM. *Let f be a nonexpansive local isometry of a finitely compact metric space (M, ρ) into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then there exists a unique decomposition of M into disjoint open sets,*

$$M = M_0^f \cup M_1^f \cup \dots,$$

such that (i) f maps M_0^f injectively into itself, (ii) $f(M_i^f) \subset M_{i-1}^f$ for each $i = 1, 2, \dots$. Moreover, f maps M_0^f isometrically onto itself.

We have the following corollaries

(3.4) COROLLARY. *Let f be a local isometry of a finitely compact metric space (M, ρ) into itself. If for each $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then the following are equivalent:*

- (i) f is one-to-one,
- (ii) f is a homeomorphism of M onto itself,
- (iii) f is an isometry with respect to the induced metric ρ_f .

Proof. The proof follows from (3.1), since each of (i)–(iii) is equivalent to $M_0^f = M$.

(3.5) COROLLARY. *Let f be a nonexpansive local isometry of a finitely compact metric space (M, ρ) into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then the following are equivalent:*

- (i) f is one-to-one,
- (ii) f is a homeomorphism of M onto itself,
- (iii) f is an isometry onto.

Proof. This follows from (3.3) (or from (3.4) and (3)).

4. Some consequences. As an immediate consequence of (3.1), we get

(4.1) THEOREM. *Let f be a local isometry of a connected finitely compact metric space (M, ρ) into itself. If for each $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then the induced metric, ρ_f , is a metric on M such that ρ_f and ρ are locally identical, (M, ρ_f) is a finitely compact metric space and f is an isometry of (M, ρ_f) onto itself. In particular, f is a homeomorphism of M onto itself.*

As an immediate consequence of (3.3), we get

(4.2) THEOREM. *Let f be a nonexpansive local isometry of a connected finitely compact metric space (M, ρ) into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then f is an isometry onto.*

The corresponding statement concerning local isometries of convex finitely compact metric spaces is stated next.

(4.3) THEOREM. *Let f be a local isometry of a convex finitely compact metric space (M, ρ) into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then f is an isometry onto.*

Proof. Since (M, ρ) is convex and complete, by (2.8), f is a nonexpansive local isometry. Hence, our assertion follows from (4.2).

Finally, we note the following special cases of (4.2) and (4.3).

(4.4) COROLLARY. *Let f be a nonexpansive local isometry of a connected finitely compact metric space (M, ρ) into itself. If f has a fixed (periodic) point, then f is an isometry onto.*

(4.5) COROLLARY. *Let f be a local isometry of a convex finitely compact metric space (M, ρ) into itself. If f has a fixed (periodic) point, then f is an isometry onto.*

REMARK. Theorems (4.2) and (4.3) extend the result of [6]; Corollaries (4.4) and (4.5) extend Theorem 1 of [5] to the case of general local isometries of finitely compact metric spaces.

5. A condition on (M, ρ) under which local isometries are isometries. In this section, by using (3.3), we extend Theorem 3 of [5]. First, we shall prove

(5.1) PROPOSITION. *Let f be a nonexpansive local isometry of a finitely compact metric space (M, ρ) into itself. If (M, ρ) has a transitive group of isometries, then there exists a sequence N_n , $n = 0, 1, \dots$, of open and closed subsets of M such that $M = \bigcup_{n=0}^{\infty} N_n$ and for each $n \geq 0$, f maps N_n isometrically onto an open closed subset of M .*

Proof. Let $z \in M$. Then, by assumption, there exists an isometry g_z of (M, ρ) onto itself such that $g_z(f(z)) = z$. Since $g_z \circ f$ is a nonexpansive local isometry, it follows from (3.3) that there is an open and closed set N_z such that $z \in N_z$ and $g_z \circ f$ maps N_z isometrically onto itself. Hence $g_z^{-1}(N_z)$ is open and closed, and f maps N_z iso-

metrically onto $g_z^{-1}(N_z)$. Since (M, ρ) is separable, our assertion follows.

The next two results follow immediately from (5.1) and (2.8) (or, in a direct fashion, from (4.4) and (4.5)).

(5.2) THEOREM. *If a connected finitely compact metric space (M, ρ) has a transitive group of isometries, then each nonexpansive local isometry of (M, ρ) into itself is an isometry onto.*

(5.3) THEOREM. *If a convex finitely compact metric space (M, ρ) has a transitive group of isometries, then each local isometry of (M, ρ) into itself is an isometry onto.*

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