ON LOCAL ISOMETRIES OF FINITELY COMPACT METRIC SPACES

ALEKSANDER CAŁKA
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By local isometries we mean mappings which locally preserve distances. Local isometries which do not increase distances are called nonexpansive local isometries. A few of the main results are:

1. Let $f$ be a local isometry (nonexpansive local isometry) of a finitely compact metric space $(M, \rho)$ into itself. If for each (some) $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then there exists a unique decomposition of $M$ into disjoint open sets, $M = M'_0 \cup M'_1 \cup \cdots$, such that (i) $f$ maps $M'_0$ injectively into itself, and (ii) $f(M'_i) \subseteq M'_{i+1}$ for each $i = 0, 1, \ldots$. Moreover, $f$ maps $M'_0$ homeomorphically (isometrically) onto itself.

2. Let $f$ be a nonexpansive local isometry (local isometry) of a connected (convex) finitely compact metric space $(M, \rho)$ into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then $f$ is an isometry onto.

1. Introduction. Let $f$ be a mapping of a metric space $(M, \rho)$ into a metric space $(N, \sigma)$. We will call $f$ a local isometry if for each $z \in M$ there is a neighborhood $U_z$ of $z$ such that $\sigma(f(x), f(y)) = \rho(x, y)$ for all $x, y \in U_z$. If $f$ is a local isometry and also a nonexpansive mapping (i.e., $\sigma(f(x), f(y)) \leq \rho(x, y)$ for all $x, y \in M$), we will say that $f$ is a nonexpansive local isometry.

A metric space $(M, \rho)$ is said to be finitely compact [2] if each bounded and closed subset of $M$ is compact.

The purpose of this paper is to extend the results of the author's paper [4] to those local isometries $f$ of a finitely compact metric space $(M, \rho)$ into itself which have the property that for each $z \in M$ the sequence $\{f^n(z)\}$ is bounded. In §2 we give some more notation and preliminary lemmas. Section 3 contains the main results. Roughly speaking, the main theorem is: Let $f$ be a local isometry (nonexpansive local isometry) of a finitely compact metric space $(M, \rho)$ into itself. If for each (for some) $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then there exists a unique decomposition of $M$ into disjoint open sets, $M = M'_0 \cup M'_1 \cup \cdots$, such that (i) $f$ maps $M'_0$ injectively into itself, (ii) $f(M'_i) \subseteq M'_{i+1}$ for each $i \geq 1$. Moreover, $f$ maps $M'_0$ homeomorphically (isometrically) onto itself.

It should be noted that open surjective local isometries were studied by Busemann [2], [3], Kirk [5], [6], [7] and Szenthe [8], [9], [10], in the special case where $(M, \rho)$ is a G-space (Busemann [2] called them "locally isometric mappings"). In [5] Kirk proved that
if an open local isometry \( f \) of a \( G \)-space \((M, \rho)\) onto itself has a fixed point, then \( f \) is an isometry (from which it follows that if the isometries of \((M, \rho)\) onto itself form a transitive group, then each open surjective local isometry is an isometry). Later Kirk [6] proved that if an open local isometry \( f \) of a \( G \)-space \((M, \rho)\) onto itself has the property that for some \( z \in M \) the sequence \( \{f^n(z)\} \) is bounded, then \( f \) is an isometry.

In §4 and §5 of the present paper, by using the results of §3, we extend the above results of Kirk to the case of general local isometries of finitely compact metric spaces.

2. Preliminaries.

(2.1) DEFINITION. Let \( \rho_i \), \( i = 0, 1 \), be metrics on a set \( M \). We shall say that \( \rho_0 \) is locally identical with \( \rho_1 \) if the identity mapping, \( \text{id}_M \), of \( M \) is a local isometry of \((M, \rho_0)\) into \((M, \rho_1)\). We shall say that \( \rho_i \) and \( \rho_j \) are locally identical if \( \rho_i \) is locally identical with \( \rho_j \), for all \( i, j = 0, 1 \).

(2.2) DEFINITION. Let \( f \) be a mapping of a metric space \((M, \rho)\) into itself. Then the function \( \rho_f \) defined by

\[
\rho_f(x, y) = \sup_{n \geq 0} \rho(f^n(x), f^n(y)) \quad \text{for all } x, y \in M,
\]

(where \( f^0 = \text{id}_M \), \( f^{n+1} = f \circ f^n \)) is called the induced metric on \( M \).

(2.3) REMARKS. (i) Let \( \rho_i \), \( i = 0, 1 \), be metrics on a set \( M \) such that \( \rho_i \) and \( \rho_j \) are locally identical. Then \( \rho_i \) and \( \rho_j \) are topologically equivalent. If \((M, \rho_0)\) is finitely compact and \( \rho_i \cong \rho_j \), then \((M, \rho_i)\) is also finitely compact. If \( f \) is a local isometry of \((M, \rho_0)\) into itself, then \( f \) is also a local isometry of \((M, \rho_i)\) into itself.

(ii) Let \( f \) be a mapping of a metric space \((M, \rho)\) into itself such that for each \( z \in M \) the sequence \( \{f^n(z)\} \) is bounded. Then for each \( x, y \in M \), \( \rho_f(x, y) < \infty \), and hence the induced metric, \( \rho_f \), is a metric on the set \( M \) such that

\[
(1) \quad \rho_f \cong \rho,
\]

\[
(2) \quad f \text{ is a nonexpansive mapping of the metric space } (M, \rho_f) \text{ into itself}, \quad \text{and}
\]

\[
(3) \quad \rho_f = \rho \text{ if and only if } f \text{ is a nonexpansive mapping of } (M, \rho) \text{ into itself}.
\]

In [4] we proved the following theorem ((4.3) of [4]).

(2.4) THEOREM. Let \( f \) be a local isometry of a compact metric
space \((M, \rho)\) into itself. Then there exists a unique decomposition of \(M\) into disjoint open sets,

\[ M = M'_0 \cup \cdots \cup M'_n \quad (0 \leq n), \]
such that (i) \(f(M'_i) = M'_i\), (ii) \(f(M'_i) \subset M'_{i-1}\) and \(M'_i \neq \emptyset\) for each \(i\), \(1 \leq i \leq n\). Moreover, the induced metric \(\rho_f\) is a metric on \(M\) such that \(\rho_f\) and \(\rho\) are locally identical and \(f\) is a nonexpansive local isometry of \((M, \rho_f)\) into itself which maps \(M'_i\) isometrically onto itself.

From this theorem we have

(2.5) COROLLARY. Let \(f\) be a one-to-one local isometry of a compact metric space \((M, \rho)\) into itself. Then \(f(M) = M\).

Proof. If \(f\) is one-to-one, then by (2.4), \(M = M'_0\) and hence \(f(M) = M\).

REMARK. If \(f\) is a local isometry of a compact metric space \((M, \rho)\) into itself and if \(N\) is a compact subset of \(M\) such that \(f(N) \subset N\), then the restriction of \(f\) to \(N\), \(f|_N\), is also a local isometry. For convenience, \(N = N'_0 \cup \cdots \cup N'_{n(N)}\) will denote the decomposition of \(N\) defined by (2.4) for \(f|_N\).

(2.6) PROPOSITION. Let \(f\) be a local isometry of a compact metric space \((M, \rho)\) into itself. If \(N\) is a compact subset of \(M\) such that \(f(N) \subset N\), then

\[ N'_i = N \cap M'_i \quad \text{for each} \quad i = 0, \cdots, n(N), \]

where \(n(N) = \max\{i \geq 0 : N \cap M'_i \neq \emptyset\}\).

Proof. By (2.4), it is sufficient only to show that \(f(N \cap M'_i) = N \cap M'_i\). However, it follows from (2.4) that \(f\) maps \(N \cap M'_i\) isometrically into itself. Hence, by (2.5), \(f(N \cap M'_i) = N \cap M'_i\) as desired.

We will need the following.

(2.7) LEMMA. Let \(f\) be a local isometry of a metric space \((N, \rho)\) into itself. If \(N\) is a compact subset of \(M\), then there exists a number \(\delta > 0\) such that for each \(z \in N\),

\[ \rho(f(x), f(y)) = \rho(x, y), \]

for all \(x, y \in S_{\rho}(z, \delta) = \{p \in M : \rho(z, p) < \delta\}\).
The straightforward verification of (2.7) is omitted.

The convexity in this paper is to be understood in the sense of Menger (cf. [1, p. 40]). A subset $N$ of a metric space $(M, \rho)$ is, accordingly, convex if for each two distinct points $x, y \in N$, there exists a point $z \in N$, $z \neq x, y$, such that $\rho(x, y) = \rho(x, z) + \rho(z, y)$.

Also, we will use

(2.8) **Lemma.** If $f$ is a local isometry of a convex and complete metric space $(M, \rho)$ into itself, then $f$ is a nonexpansive local isometry.

**Proof.** Let $x$ and $y$ be given points of $M$ such that $x \neq y$. Since $M$ is convex and complete, by a theorem of Menger (cf. [1, p. 41]) there exists a metric segment $L \subset M$ whose extremities are $x$ and $y$; that is, a subset isometric to an interval of length $\rho(x, y)$. Since $L$ is compact, it follows that there exists a finite sequence $z_0, z_1, \ldots, z_k$ of points of $L$ such that $z_0 = x$, $z_k = y$ and

$$\rho(f(x), f(z_{i+1})) = \rho(z_i, z_{i+1}) \text{ for each } i = 0, \ldots, k - 1,$$

and

$$\rho(x, y) = \sum_{i=0}^{k-1} \rho(z_i, z_{i+1}).$$

Thus,

$$\rho(f(x), f(y)) \leq \sum_{i=0}^{k-1} \rho(f(z_i), f(z_{i+1})) = \sum_{i=0}^{k-1} \rho(z_i, z_{i+1}) = \rho(x, y).$$

This proves that $f$ is a nonexpansive mapping, and hence a nonexpansive local isometry.

3. Local isometries and decomposition theorems. We shall now prove the following extension of (2.4).

(3.1) **Theorem.** Let $f$ be a local isometry of a finitely compact metric space $(M, \rho)$ into itself. If for each $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then there exists a unique decomposition of $M$ into disjoint open sets,

$$M = M'_0 \cup M'_1 \cup \cdots,$$

such that

(6) \hspace{1cm} $f$ maps $M'_i$ injectively into itself,

(7) \hspace{1cm} $f(M'_i) \subset M'_{i-1}$ for each $i = 1, 2, \ldots$. 
Moreover, the induced metric, \( \rho_f \), is a metric on \( M \) such that \( \rho_f \) and \( \rho \) are locally identical, \( (M, \rho_f) \) is a finitely compact metric space and \( f \) is a nonexpansive local isometry of \( (M, \rho_f) \) into itself which maps \( M \) isometrically onto itself.

**Proof.** In the proof, for each \( A \subset M \) and \( \delta > 0 \), \( S_\rho(A, \delta) \) is the \( \delta \)-ball in \( M \) about \( A \) and \( \text{cl} A \) (Int \( A \)) is the closure (interior) of \( A \). For each \( z \in M \) we denote: \( c(z) = \text{cl} \{f^n(z): n \geq 0\} \).

We first define a sequence \( A_n, n = 0, 1, \ldots \), of compact subsets of \( M \) such that

\[
\begin{align*}
(8) & \quad f(A_n) \subset A_n \quad \text{for each} \quad n = 0, 1, \ldots, \\
(9) & \quad A_n \subset \text{Int} A_{n+1} \quad \text{for each} \quad n = 0, 1, \ldots, \\
(10) & \quad \bigcup_{n=0}^\infty A_n = M. 
\end{align*}
\]

For each \( z \in M \), let \( \delta_z > 0 \) be a number defined by (2.7) for the compact set \( c(z) \) and let \( V_z = S_\rho(c(z), \delta_z) \). Thus, for each \( z \in M \), \( V_z \) is an open and bounded subset of \( M \) and using (4) and the fact that \( f(c(z)) \subset c(z) \), we have \( f(V_z) \subset V_z \). Since \( (M, \rho) \) has a countable base of neighborhoods, there exists a sequence \( z_n, n = 0, 1, \ldots \), of points of \( M \) such that \( \bigcup_{n=0}^\infty V_{z_n} = M \). Define the sets \( A_n, n = 0, 1, \ldots \), inductively, as follows: \( A_0 = \text{cl} V_{z_0} \) and \( A_{n+1} = \bigcup_{i=0}^{k(n)} \text{cl} V_{z_i} \), where \( k(n) \) is an integer such that \( k(n) > n \) and \( A_n \subset \bigcup_{i=0}^{k(n)} V_{z_i} \). Clearly, the sets \( A_n, n = 0, 1, \ldots \), satisfy conditions (8), (9) and (10), and are compact.

It follows now from (2.4), that for each \( n \geq 0 \), there exists a sequence \( (A_n)_{i}, i = 0, 1, \ldots \), of disjoint subsets of \( A_n \) such that

\[
\begin{align*}
(11) & \quad (A_n)_{i} \cap \text{Int} A_n \text{ is open, for each} \quad i = 0, 1, \ldots, \\
(12) & \quad \bigcup_{i=0}^\infty (A_n)_{i} = A_n, \\
(13) & \quad f \text{ maps } (A_n)_{i} \text{ injectively into itself,} \\
(14) & \quad f((A_n)_{i}) \subset (A_{n+1})_{i+1}, \quad \text{for each} \quad i = 1, 2, \ldots.
\end{align*}
\]

By (2.6), we have

\[
\begin{align*}
(15) & \quad (A_n)_{i} = A_n \cap (A_{n+1})_{i}, \quad \text{for all} \quad n, i = 0, 1, \ldots.
\end{align*}
\]

Now, for each \( i = 0, 1, \ldots \), we define the set \( M'_i \) as follows:

\[
M'_i = \bigcup_{n=0}^\infty (A_n)_{i}.
\]

Then, by (15) and the fact that \( (A_n)_{i}, i \geq 0 \), are disjoint, the sets \( M'_i, i \geq 0 \), are disjoint. By (9) and (15),
\[ (A_n)^i \subset (A_{n+1})^i \cap \text{Int } A_{n+1} \subset (A_{n+1})^i, \]
hence,
\[ M^i = \bigcup_{n=0}^{\infty} ((A_{n+1})^i \cap \text{Int } A_{n+1}), \text{ for each } i = 0, 1, \ldots, \]
and therefore, by (11), the sets \( M^i, i \geq 0, \) are open. By (10) and (12),
\[ \bigcup_{i=0}^{\infty} M^i = \bigcup_{i=n=0}^{\infty} (A_n)^i = \bigcup_{n=0}^{\infty} A_n = M, \]
and it follows from (13), (14) and (15) that the sets \( M^i, i \geq 0, \) satisfy conditions (6) and (7). This proves the existence of the desired decomposition of \( M. \)

In order to prove the uniqueness, it is sufficient only to show that for each decomposition of \( M \) into disjoint open sets, \( M = \bigcup_{i=0}^{\infty} M^i, \)
conditions (6) and (7) imply
\[ (16) \quad M_0 = \{ z \in M: f(c(z)) = c(z) \}. \]

Let us assume, \( M = \bigcup_{i=0}^{\infty} M^i \) is a decomposition of \( M \) into disjoint open sets, satisfying conditions (6) and (7). If \( z \in M_0, \) then (6) implies that the restriction of \( f \) to \( c(z) \) is a one-to-one local isometry of \( c(z) \) into itself. Since \( c(z) \) is compact, it follows from (2.5) that \( f(c(z)) = c(z). \) Conversely, if \( z \in M_0, \) then \( z \in M_n \) for some \( n \geq 1. \) Using (7) and the fact that \( M^i, i \geq 0, \) are disjoint and open, we obtain
\[ f(c(z)) \subset c(f(z)) \subset M_0 \cup \cdots \cup M_n \cup \cdots, \]
hence \( z \in c(z) \setminus c(f(z)), \) i.e., \( c(z) \neq c(f(z)). \) Therefore (16) follows as desired.

Finally, by (ii) of (2.3), the induced metric, \( \rho_f, \) is a metric on \( M \) and it follows from (8), (9), (10) and (2.4) that \( \rho_f \) and \( \rho \) are locally identical (cf. also (1)). Hence, by (1) and (i) of (2.3), the metric space \( (M, \rho_f) \) is finitely compact and, by (2), \( f \) is a nonexpansive local isometry of \( (M, \rho_f) \) into itself. It follows from (2.4) and (15) and the definition of \( M_0' \) that \( f \) maps \( M_0' \) isometrically onto itself with respect to the metric \( \rho_f. \) This completes the proof.

(3.2) Remark. Let \( f \) be a nonexpansive mapping of a metric space \( (M, \rho) \) into itself. If for some \( z \in M \) the sequence \( \{ f^n(z) \} \) is bounded, then for each \( x \in M \) the sequence \( \{ f^n(x) \} \) is bounded.

Indeed, since \( f \) is nonexpansive, then for all \( x, z \in M \) and each \( i = 0, 1, \ldots, \) we have
\[ \rho(f^i(x), f^i(z)) \leq \rho(f^i(x), f^i(z)) \leq \rho(x, z), \]
hence, if \( \{ f^n(z) \} \) is bounded, then also \( \{ f^n(x) \} \) is bounded.
The following theorem is an immediate consequence of (3.1), (3.2) and (3).

(3.3) Theorem. Let \( f \) be a nonexpansive local isometry of a finitely compact metric space \((M, \rho)\) into itself. If for some \( z \in M \) the sequence \( \{f^n(z)\} \) is bounded, then there exists a unique decomposition of \( M \) into disjoint open sets,

\[
M = M'_0 \cup M'_1 \cup \cdots ,
\]

such that (i) \( f \) maps \( M'_i \) injectively into itself, (ii) \( f(M'_i) \subset M'_{i-1} \) for each \( i = 1, 2, \ldots \). Moreover, \( f \) maps \( M'_0 \) isometrically onto itself.

We have the following corollaries

(3.4) Corollary. Let \( f \) be a local isometry of a finitely compact metric space \((M, \rho)\) into itself. If for each \( z \in M \) the sequence \( \{f^n(z)\} \) is bounded, then the following are equivalent:

(i) \( f \) is one-to-one,
(ii) \( f \) is a homeomorphism of \( M \) onto itself,
(iii) \( f \) is an isometry with respect to the induced metric \( \rho_f \).

Proof. The proof follows from (3.1), since each of (i)-(iii) is equivalent to \( M'_0 = M \).

(3.5) Corollary. Let \( f \) be a nonexpansive local isometry of a finitely compact metric space \((M, \rho)\) into itself. If for some \( z \in M \) the sequence \( \{f^n(z)\} \) is bounded, then the following are equivalent:

(i) \( f \) is one-to-one,
(ii) \( f \) is a homeomorphism of \( M \) onto itself,
(iii) \( f \) is an isometry onto.

Proof. This follows from (3.3) (or from (3.4) and (3)).

4. Some consequences. As an immediate consequence of (3.1), we get

(4.1) Theorem. Let \( f \) be a local isometry of a connected finitely compact metric space \((M, \rho)\) into itself. If for each \( z \in M \) the sequence \( \{f^n(z)\} \) is bounded, then the induced metric, \( \rho_f \), is a metric on \( M \) such that \( \rho_f \) and \( \rho \) are locally identical, \((M, \rho_f)\) is a finitely compact metric space and \( f \) is an isometry of \((M, \rho_f)\) onto itself. In particular, \( f \) is a homeomorphism of \( M \) onto itself.

As an immediate consequence of (3.3), we get
(4.2) Theorem. Let $f$ be a nonexpansive local isometry of a connected finitely compact metric space $(M, \rho)$ into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then $f$ is an isometry onto.

The corresponding statement concerning local isometries of convex finitely compact metric spaces is stated next.

(4.3) Theorem. Let $f$ be a local isometry of a convex finitely compact metric space $(M, \rho)$ into itself. If for some $z \in M$ the sequence $\{f^n(z)\}$ is bounded, then $f$ is an isometry onto.

Proof. Since $(M, \rho)$ is convex and complete, by (2.8), $f$ is a nonexpansive local isometry. Hence, our assertion follows from (4.2).

Finally, we note the following special cases of (4.2) and (4.3).

(4.4) Corollary. Let $f$ be a nonexpansive local isometry of a connected finitely compact metric space $(M, \rho)$ into itself. If $f$ has a fixed (periodic) point, then $f$ is an isometry onto.

(4.5) Corollary. Let $f$ be a local isometry of a convex finitely compact metric space $(M, \rho)$ into itself. If $f$ has a fixed (periodic) point, then $f$ is an isometry onto.

Remark. Theorems (4.2) and (4.3) extend the result of [6]; Corollaries (4.4) and (4.5) extend Theorem 1 of [5] to the case of general local isometries of finitely compact metric spaces.

5. A condition on $(M, \rho)$ under which local isometries are isometries. In this section, by using (3.3), we extend Theorem 3 of [5]. First, we shall prove

(5.1) Proposition. Let $f$ be a nonexpansive local isometry of a finitely compact metric space $(M, \rho)$ into itself. If $(M, \rho)$ has a transitive group of isometries, then there exists a sequence $N_n$, $n = 0, 1, \cdots$, of open and closed subsets of $M$ such that $M = \bigcup_{n=0}^{\infty} N_n$ and for each $n \geq 0$, $f$ maps $N_n$ isometrically onto an open closed subset of $M$.

Proof. Let $z \in M$. Then, by assumption, there exists an isometry $g_z$ of $(M, \rho)$ onto itself such that $g_z(f(z)) = z$. Since $g_z \circ f$ is a nonexpansive local isometry, it follows from (3.3) that there is an open and closed set $N_z$ such that $z \in N_z$ and $g_z \circ f$ maps $N_z$ isometrically onto itself. Hence $g_z^{-1}(N_z)$ is open and closed, and $f$ maps $N_z$ iso-
metrically onto \( g_1^{-1}(N_x) \). Since \((M, \rho)\) is separable, our assertion follows.

The next two results follow immediately from (5.1) and (2.8) (or, in a direct fashion, from (4.4) and (4.5)).

(5.2) Theorem. If a connected finitely compact metric space \((M, \rho)\) has a transitive group of isometries, then each nonexpansive local isometry of \((M, \rho)\) into itself is an isometry onto.

(5.3) Theorem. If a convex finitely compact metric space \((M, \rho)\) has a transitive group of isometries, then each local isometry of \((M, \rho)\) into itself is an isometry onto.

References


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