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## **BUNDLES OVER CONFIGURATION SPACES**

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## BUNDLES OVER CONFIGURATION SPACES

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Let  $F(\mathbf{R}^n, k)$  be the configuration space of ordered sets of  $k$  distinct points in  $\mathbf{R}^n$ .  $F(\mathbf{R}^n, k)$  is acted upon freely by the symmetric group on  $k$  letters,  $\Sigma_k$ . In this paper we calculate the order of the vector bundles

$$\xi_{n,k}: F(\mathbf{R}^n, k) \times_{\Sigma_k} \mathbf{R}^k \rightarrow F(\mathbf{R}^n, k)/\Sigma_k.$$

Applications to the study of iterated loop spaces of spheres are also discussed.

1. The study of the stable homotopy type of the spaces  $\Omega^n S^{n+r}$  has received much attention in recent years [2, 8, 13]. The starting point for this study was Snaith's stable descomposition [18]:

$$\Omega^n S^{n+r} \simeq_s \bigvee_{k \geq 0} F(\mathbf{R}^n, k)^+ \wedge_{\Sigma_k} S^{r(k)},$$

where  $F(\mathbf{R}^n, k)^+$  is the configuration space of  $k$  ordered distinct points in  $\mathbf{R}^n$  together with a disjoint basepoint,  $S^{r(k)}$  is the  $k$ -fold smash product of  $S^r$  with itself,  $\Sigma_k$  is the symmetric group of  $k$  letters, and where " $\simeq_s$ " denotes stable homotopy equivalence.

The space  $F(\mathbf{R}^n, k)^+ \wedge_{\Sigma_k} S^{r(k)}$  is clearly the Thom complex of the  $r$ -fold Whitney sum of the vector bundle

$$\xi_{n,k}: F(\mathbf{R}^n, k) \times_{\Sigma_k} \mathbf{R}^k \rightarrow F(\mathbf{R}^n, k)/\Sigma_k.$$

If  $M(\xi_{n,k})$  is the associated Thom spectrum, then Snaith's theorem gives an equivalence of spectra

$$\Sigma^\infty \Omega^n S^{n+r} \simeq \bigvee_{k \geq 0} \Sigma^{rk} M(r\xi_{n,k}),$$

where  $\Sigma^\infty$  is the stabilization functor which assigns to a space its associated suspension spectrum.

If  $\phi_{n,k}$  is the stable order of  $\xi_{n,k}$  (i.e.,  $\phi_{n,k}$  is the smallest integer such that  $\phi_{n,k} \xi_{n,k}$  is stably trivial) then we have the obvious periodicity

$$M((r + \phi_{n,k})\xi_{n,k}) \simeq \Sigma^{k\phi_{n,k}} M(r\xi_{n,k}).$$

This, together with Snaith's theorem gives clear interrelationships amongst the stable homotopy types of the spaces  $\Omega^n S^{n+r}$  as  $r$  varies.

The case  $n = 2$  is well understood by the work of F. Cohen, M. Mahowald, and R. J. Milgram [5], who proved that  $\phi_{2,k} = 2$  for all  $k$ . The resulting periodicity in the homotopy type of the associated Thom

spectra was used by M. Mahowald [13] and R. Cohen [8] to construct new infinite families in the stable homotopy ring  $\pi_*^s$ .

It is the purpose of this paper to compute the orders  $\phi_{n,k}$  for general  $n$  and  $k$ . Our main result can be stated as follows. Let

$$a_{n,k} = 2^{\rho(n-1)} \prod_{3 \leq p \leq k} p^{\lfloor (n-1)/2 \rfloor}$$

where  $p$  denotes an odd prime, and where  $\rho(m)$  is Adam’s vector field number:  $\rho(m) =$  the number of positive integers  $\leq m$  that are congruent to 0, 1, 2, or 4 mod 8.

**THEOREM 1.1.** *If  $n \not\equiv 0 \pmod{4}$ , then  $\phi_{n,k} = a_{n,k}$ . Furthermore, if  $n \equiv 0 \pmod{4}$ , then  $a_{n,k} \mid \phi_{n,k}$  and  $\phi_{n,k} \mid 2a_{n,k}$ .*

**REMARKS.** 1. The bundle  $\xi_{n,2}$  is easily seen to be stably isomorphic to the canonical line bundle over  $\mathbf{R}P^{n-1}$ , so the fact that  $\phi_{n,2} = 2^{\rho(n-1)}$  is the classical result of Adams [1].

2. Using the Atiyah-Hirzebruch spectral sequence converging to the KO-theory of  $F(\mathbf{R}^n, p)/\Sigma_p$ , S. W. Yang computed the order of  $\xi_{n,p}$ , and proved that  $a_{n,k} \mid \phi_{n,k}$  [20].

3. The conjecture that  $\phi_{n,k} = a_{n,k}$  was made by Yang, Mahowald, and F. Cohen.

The essential idea in the proof of 1.1 is to notice that the classifying map

$$f_{n,k}: F(\mathbf{R}^n, k)/\Sigma_k \rightarrow BO$$

of  $\xi_{n,k}$  factors as a composition of maps, one of which is the natural inclusion

$$i_n: \Omega_0^n S^n \hookrightarrow Q_0 S^0,$$

where  $QX = \lim_{m \rightarrow \infty} \Omega^m \Sigma^m X$ , and where  $\Omega_k^n S^n$  denotes the component of  $\Omega^n S^n$  containing maps of degree  $k$ . We then study the order of  $i_n$  localized at a prime  $p$ , using the results of F. Cohen, J. Moore, and J. Neisendorfer [6, 7, 15] and of Toda [19].

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**2. Proof of Theorem 1.1.** Our first object is to identify the classifying maps of the bundles  $\xi_{n,k}$ . This is done easily by recalling first that  $F(\mathbf{R}^\infty, k) = \lim_{n \rightarrow \infty} F(\mathbf{R}^n, k)$  is a contractible space, acted upon freely by  $\Sigma_k$ , and therefore  $F(\mathbf{R}^\infty, k)/\Sigma_k = B\Sigma_k$ . For a proof of this, see for instance [14].

Thus the bundle

$$\xi_{\infty,k}: F(\mathbf{R}^\infty, k) \times_{\Sigma_k} \mathbf{R}^k \rightarrow F(\mathbf{R}^\infty, k)/\Sigma_k = B\Sigma_k$$

is classified by the map

$$f_k: B\Sigma_k \rightarrow BO(k)$$

induced by the regular representation of  $\Sigma_k$  in  $O(k)$ . Moreover, since the bundle  $\xi_{n,k}$  is the pull-back of  $\xi_{\infty,k}$  under the inclusion  $F(\mathbf{R}^n, k)/\Sigma_k \subset F(\mathbf{R}^\infty, k)/\Sigma_k$ ,  $\xi_{n,k}$  is classified by the map

$$f_{n,k}: F(\mathbf{R}^n, k)/\Sigma_k \subset F(\mathbf{R}^\infty, k)/\Sigma_k = B\Sigma_k \xrightarrow{f_k} BO(k).$$

The stable order  $\phi_{n,k}$  of  $\xi_{n,k}$  is the order of the class determined by  $f_{n,k}$  in the abelian group  $[F(\mathbf{R}^n, k)/\Sigma_k, BO]$ . In order to determine  $\phi_{n,k}$  we first recall some of May's iterated loop space machinery [14].

Recall first the ‘‘approximations’’

$$\alpha_n: C_n X \rightarrow \Omega^n \Sigma^n X$$

of [14].  $C_n X$  is a filtered space which approximates  $\Omega^n \Sigma^n X$  in the sense that  $\alpha_n$  is a weak homotopy equivalence if  $X$  is connected. For  $X = S^0$ ,

$$C_n(S^0) \simeq \coprod_k F(\mathbf{R}^n, k)/\Sigma_k$$

and the map  $\alpha_n: \coprod_k F(\mathbf{R}^n, k)/\Sigma_k \rightarrow \Omega^n S^n$  takes  $F(\mathbf{R}^n, k)/\Sigma_k$  to  $\Omega_k^n S^n$ .

Now consider the map

$$\beta: \coprod_k BO(k) \rightarrow BO \times \mathbf{Z}$$

which includes  $BO(k)$  into  $BO \times \{k\}$  in the obvious manner. Let  $\eta: QS^0 \rightarrow BO \times \mathbf{Z}$  be the infinite loop map induced by the map  $S^0 \rightarrow BO \times \mathbf{Z}$  which sends 0 to the basepoint in  $BO \times \{0\}$  and 1 to the basepoint in  $BO \times \{1\}$ . We then have

**LEMMA 2.1.** *The following diagram homotopy commutes for all positive integers  $n$  and  $k$ .*

$$\begin{array}{ccccc}
 F(\mathbf{R}^n, k)/\Sigma_k \subset \coprod_j F(\mathbf{R}^n, j)/\Sigma_j & \longrightarrow & \coprod_j F(\mathbf{R}^\infty, j)/\Sigma_j & \xrightarrow{\coprod_j f_j} & \coprod_j BO(j) \\
 \downarrow \alpha_n & & \downarrow \alpha_\infty & & \downarrow \beta \\
 \Omega^n S^n & \xrightarrow{i_n} & QS^0 & \xrightarrow{\eta} & BO \times \mathbf{Z} \\
 \downarrow *[-k] & & \downarrow *[-k] & & \downarrow *[-k] \\
 \Omega^n S^n & \xrightarrow{i_n} & QS^0 & \xrightarrow{\eta} & BO \times \mathbf{Z}
 \end{array}$$

where  $*[-k]$  translates components by  $-k$ .

*Proof.* This follows directly from May’s iterated loop space machinery, and an explicit proof is found in [4].

Note that the classifying map  $f_{n,k}: F(\mathbf{R}^n, k)/\Sigma_k \rightarrow BO = BO \times \{0\} \subset BO \times \mathbf{Z}$  of  $\xi_{n,k}$  is the composition obtained by going along the top and then down the right-hand side of the diagram in Lemma 2.1. Now since  $\eta$  is a map of infinite loop spaces, and therefore like  $i_n$  is an  $H$ -map, Lemma 2.1 implies that the power of  $p$  in the prime factorization of  $\phi_{n,k}$  is bounded by the order of the localization at  $p$  of  $i_n \in [\Omega_0^n S^n, Q_0 S^0]$ .

**PROPOSITION 2.2.** *For a prime  $p$ , let  $i_{n,p}: \Omega_0^n S_{(p)}^n \rightarrow Q_0 S_{(p)}^0$  be the localization of  $i_n$ . Then in  $[\Omega_0^n S_{(p)}^n, Q_0 S_{(p)}^0]$  the order of  $i_{n,p}$  divides  $p^q$ , where*

$$q = \begin{cases} \left\lfloor \frac{n-1}{2} \right\rfloor & \text{if } p \text{ is odd} \\ \rho(n-1) & \text{if } p = 2 \text{ and } n \not\equiv 0 \pmod{4} \\ \rho(n-1) + 1 & \text{if } p = 2 \text{ and } n \equiv 0 \pmod{4}. \end{cases}$$

Notice that Theorem 1.1 is a corollary of Proposition 2.2 in view of Yang’s results [20] (see the second remark following the statement of Theorem 1.1), and the fact that if  $k < p$ ,  $F(\mathbf{R}^\infty, k)/\Sigma_k = B\Sigma_k$  is homology  $p$ -equivalent to a point.

*Proof of 2.2.* We prove Proposition 2.2 in several cases.

*Case 1.*  $p$  odd and  $n$  odd (say  $n = 2m + 1$ ).

Recent results of Selick [17], Cohen, Moore and Neisendorfer [6, 7], and Neisendorfer [15] imply that the identity element

$$1 \in \left[ \Omega_0^{2m+1} S_{(p)}^{2m+1}, \Omega_0^{2m+1} S_{(p)}^{2m+1} \right]$$

has order  $p^m$ . Since  $i_n$  is an  $H$ -map, the result follows in this case.

Case 2.  $p = 2, n$  odd.

To verify this case we shall use the Kahn-Priddy theorem [10]:

**THEOREM 2.3.** *There exist maps  $s: QRP^\infty \rightarrow Q_0S^0$  and  $j: Q_0S^0 \rightarrow QRP^\infty$  such that when localized at the prime 2,  $s \circ j$  is a homotopy equivalence.*

In [16], Segal gave a proof of this theorem in which he showed that when restricted to  $\Omega_0^n S^n \subset Q_0S^0$ ,  $j$  factors through a map  $j_n: \Omega_0^n S^n \rightarrow QRP^{n-1}$ . In [3], Caruso, Cohen, May, and Taylor also gave a proof of the Kahn-Priddy theorem, obtaining Segal's factorization, and in which explicit formulae for the maps  $j_n, j$ , and  $s$  are given.

In any case, using the proof and the formulae in [3] of this theorem, N. Kuhn verified that the maps  $j_n$  and  $j$  are one-fold loop maps [12]. The fact that  $j$  is an  $H$ -map actually follows immediately from Kahn's work in [11]. Using these results, we shall consider the following homotopy commutative diagram of spaces localized at 2.

$$\begin{array}{ccc}
 \Omega_0^n S^n & \xrightarrow{\quad} & Q_0S^0 \\
 \downarrow j_n & \nearrow i_n \quad j & \uparrow (s \circ j)^{-1} \\
 QRP^{n-1} \subset QRP^\infty & \xrightarrow{s} & Q_0S^0
 \end{array}$$

where  $(s \circ j)^{-1}$  is a homotopy inverse to  $s \circ j$ . Since  $s$  is an infinite loop map, and  $j$  deloops once,  $s \circ j$  and therefore  $(s \circ j)^{-1}$  are maps of loop spaces. Thus the order of  $i_n$  (localized at 2) divides the order of the identity of  $QRP^{n-1}$ , which Toda showed to be  $2^{\rho(n-1)}$  when  $n$  is odd [19]. This proves the proposition in this case.

Case 3.  $n = 2m$ .

Consider the following fibration of James [9].

$$S^{2m-1} \xrightarrow{e} \Omega S^{2m} \xrightarrow{h} \Omega S^{4m-1}$$

This fibration yields the classical EHP sequence in homotopy groups. Apply  $\Omega^{2m-1}$  to this fibration and consider the following diagram.

$$\begin{array}{ccccc}
 \Omega^{2m-1}S^{2m-1} & \xrightarrow{T} & \Omega^{2m-1}S^{2m-1} & & \\
 \downarrow e & & \downarrow e & & \\
 \Omega_0^{2m}S^{2m} & \xrightarrow{T} & \Omega_0^{2m}S^{2m} & \xrightarrow{i_{2m}} & Q_0S^0 \\
 \downarrow h & \nearrow [i, i]' & \downarrow h & & \\
 \Omega^{2m}S^{4m-1} & \xrightarrow{T} & \Omega^{2m}S^{4m-1} & & 
 \end{array}$$

where  $T$  is twice the identity map, and  $[i, i]' = \Omega^{2m}[i, i]$ , where  $[i, i]: S^{4m-1} \rightarrow S^{2m}$  is the Whitehead product of the identity with itself.

LEMMA 2.4. *In the above diagram we have*

- (a) *both squares commute,*
- (b) *the lower triangle commutes, and*
- (c)  $i_{2m} \circ [i, i]'$  *is null homotopic.*

*Proof.* The commutativity of the two squares is obvious, and the commutativity of the lower triangle follows from the standard fact that the Hopf invariant of  $[i, i]$  is 2. Similarly, the fact that  $i_{2m} \circ [i, i]' = 0$  follows from the standard fact that the Whitehead product  $[i, i]$  stabilizes to zero.

COROLLARY 2.5. *There exists a map  $g: \Omega_0^{2m}S^{2m} \rightarrow \Omega_0^{2m-1}S^{2m-1}$  so that  $T \simeq [i, i]' \circ h + e \circ g$ .*

*Proof.* By the lemma,  $h \circ (T - [i, i]' \circ h)$  is null homotopic, and therefore  $T - [i, i]' \circ h$  lifts to a map  $g: \Omega_0^{2m}S^{2m} \rightarrow \Omega_0^{2m-1}S^{2m-1}$  satisfying the required property.

We are now ready to prove the proposition in this final case. Localizing at 2, we have that

$$\begin{aligned}
 2^{\rho(2m-2)+1}i_{2m} &= 2^{\rho(2m-2)}(i_{2m} \circ T) \\
 &= 2^{\rho(2m-2)}(i_{2m} \circ [i, i]' \circ h + i_{2m} \circ e \circ g)
 \end{aligned}$$

by 2.5, and which equals  $2^{\rho(2m-2)}(i_{2m-1} \circ g)$  by 2.4 part c and the fact that  $i_{2m-1} = i_{2m} \circ e$ . But  $2^{\rho(2m-2)}i_{2m-1}$  is null homotopic by the result in case 2. We may therefore conclude that

$$2^{\rho(2m-2)+1}i_{2m} = 0.$$

Similarly, localized at  $p$  odd and using the result of case 1, we obtain that  $2p^{\lfloor (n-1)/2 \rfloor}i_{2m}$  is null homotopic, and therefore so is  $p^{\lfloor (n-1)/2 \rfloor}i_{2m}$ .

Thus we have proved the proposition when  $p$  is odd, and summarizing the results in  $p = 2$ , we have:

$$\begin{aligned} 2^{\rho(n-1)}i_n &= 0 && \text{if } n \text{ is odd,} \\ 2^{\rho(n-2)+1}i_n &= 0 && \text{if } n \text{ is even,} \\ \text{and } 2^{\rho(n)}i_n &= 0 && \text{if } n \text{ is even.} \end{aligned}$$

The last equation follows from the first since  $i_{2m}$  factors through  $i_{2m+1}$ .

Notice that if  $n \equiv 2 \pmod{8}$ ,  $\rho(n-1) = \rho(n-2) + 1$  and therefore  $2^{\rho(n-1)}i_n = 0$ . If  $n \equiv 6 \pmod{8}$ ,  $\rho(n-1) = \rho(n)$  so  $2^{\rho(n-1)}i_n = 0$ . Thus if  $n \not\equiv 0 \pmod{4}$ ,  $2^{\rho(n-1)}i_n$  is null homotopic. If  $n \equiv 0 \pmod{4}$ ,  $\rho(n-1) = \rho(n-2)$  so  $2^{\rho(n-1)+1}i_n = 0$ .

This completes the proof of Proposition 2.2, and therefore of Theorem 1.1.

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