

# Pacific Journal of Mathematics

**MEROMORPHIC FUNCTIONS THAT SHARE TWO FINITE  
VALUES WITH THEIR DERIVATIVE**

GARY GUNDERSEN

## MEROMORPHIC FUNCTIONS THAT SHARE TWO FINITE VALUES WITH THEIR DERIVATIVE

GARY G. GUNDERSEN

**It is shown that if a nonconstant meromorphic function  $f$  and its derivative  $f'$  share two finite values (counting multiplicities), then  $f(z) = ce^z$ .**

We say two meromorphic functions  $f(z)$  and  $g(z)$  *share* the finite complex value  $c$  if  $f(z) - c$  and  $g(z) - c$  have the same zeros. We will state whether a value is shared **CM** (counting multiplicities), **IM** (ignoring multiplicities), or by **DM** (by different multiplicities at one point or more). In this paper all functions will be assumed to be meromorphic in the whole complex plane, unless stated otherwise.

R. Nevanlinna [7, p. 109] proved that if  $f$  and  $g$  share five values **IM**, then either  $f = g$  or  $f$  and  $g$  are both constants. He also found [7, Chapter V] the particular form of all pairs  $f, g$  that share four values **CM** and all pairs  $f, g$  that share three values **CM**.

L. A. Rubel and C. C. Yang proved the following result:

**THEOREM A. [8]** *If a nonconstant entire function  $f$  and its derivative  $f'$  share two finite values **CM**, then  $f = f'$ .*

E. Mues and N. Steinmetz [6] have shown that “**CM**” can be replaced by “**IM**” in Theorem A (another proof of this result for nonzero shared values is in [3]).

On the other hand, the meromorphic function [6]

$$(1) \quad f(z) = \left( \frac{1}{2} - \frac{\sqrt{5}}{2} i \tan \left( \frac{\sqrt{5}}{4} iz \right) \right)^2$$

shares 0 by **DM** and 1 by **DM** with  $f'$ ; while the meromorphic function [3]

$$(2) \quad f(z) = \frac{2A}{1 - Be^{-2z}}, \quad A \neq 0, B \neq 0$$

shares 0 (lacunary) and  $A$  by **DM** with  $f'$ ; and  $f \neq f'$  in both (1) and (2).

The purpose of this paper is to prove

**THEOREM 1.** *If a nonconstant meromorphic function  $f$  and its derivative  $f'$  share two finite values **CM**, then  $f = f'$ .*

E. Mues has shown the author how he used sums of logarithmic derivatives to prove some results on meromorphic functions that share four values. By choosing the sums in clever ways he was able to effectively combine Nevanlinna theory with the shared value properties. For the proof of Theorem 1 we will use natural extensions of his ideas and combine these extensions with the relationship of  $f$  and  $f'$ . S. Bank and the author had earlier found a proof of Theorem 1 in the special case when  $\text{order}(f) < 2$ , but this proof cannot be used if  $\text{order}(f) \geq 2$ .

Examples (1) and (2) both have 0 as one of the shared values which is certainly a special case. Hence it is still not known whether there exists a meromorphic function  $f$  such that  $f$  and  $f'$  share two finite nonzero values, by **DM** for at least one value.

It should be mentioned that if a nonconstant meromorphic function  $f$  and its derivative  $f'$  share three finite values **IM**, then  $f = f'$  [3, 6].

We will assume that the reader is familiar with the standard notations and fundamental results of Nevanlinna theory as found in [5]. We define  $S(r, f)$  to be any quantity that satisfies

$$\frac{S(r, f)}{T(r, f)} \rightarrow 0$$

as  $r \rightarrow \infty$  outside a possible exceptional set of  $r$  of finite linear measure.

The next result will follow from Theorem 1.

**COROLLARY 1.** *If  $a$  and  $b$  are two distinct complex constants and  $w$  is a nonconstant entire function, then the algebraic differential equation*

$$(3) \quad f' = \frac{(a - be^w)f + ab(e^w - 1)}{(1 - e^w)f + ae^w - b}$$

*does not possess a nonconstant meromorphic solution  $f$ .*

We mention that it follows from the general theorems of F. Gackstatter and I. Laine [2] and N. Steinmetz [9] that there does not exist a nonconstant meromorphic solution  $f$  of equation (3) such that  $T(r, e^w) = S(r, f)$ .

The following result is obtained by combining Theorem 1 and [3, Theorem 1].

**COROLLARY 2.** *If  $h$  and  $g$  are nonconstant entire functions such that (i)  $h', g'$  share 0 **CM**, (ii)  $h'', g''$  share 0 **CM**, and (iii)  $a(1 + h/h') = b(1 + g/g')$  for distinct nonzero numbers  $a$  and  $b$ , then  $h'(z) \equiv Ce^{-z}$  and  $g'(z) \equiv Ke^{-z}$  for nonzero constants  $C, K$ .*

Entire functions  $H$  and  $G$  such that (i)  $H$  and  $G$  share  $0$  CM and (ii)  $H'$  and  $G'$  share  $0$  CM, are studied in [4].

I would like to thank E. Mues and S. Bank for some helpful discussions.

*Proof of Theorem 1.* Let  $a$  and  $b$  be the shared values. If  $ab = 0$ , then  $0$  is a Picard value of both  $f$  and  $f'$  and it follows that  $f = f'$  [3, Theorem 4(i)].

Now suppose that  $a \neq 0$  and  $b \neq 0$ . Then all  $a$ -points and  $b$ -points for both  $f$  and  $f'$  are simple. It is easy to see that  $f'$  cannot be a constant. We will first prove

LEMMA 1. *If in addition to the hypothesis of Theorem 1 we have  $N(r, f) = S(r, f)$ , then  $f = f'$ .*

*Proof.* Set  $\phi_1 = f'/(f - b) - f''/(f' - b)$ .

Then from Nevanlinna's fundamental estimate of the logarithmic derivative we obtain

$$\begin{aligned} m(r, \phi_1) &\leq m\left(r, \frac{f'}{f - b}\right) + m\left(r, \frac{f''}{f' - b}\right) + O(1) \\ &= S(r, f) + S(r, f'). \end{aligned}$$

Since  $T(r, f') \leq 2T(r, f) + S(r, f)$  this means that  $m(r, \phi_1) = S(r, f)$ . Since  $\phi_1$  is the logarithmic derivative of  $(f - b)(f' - b)^{-1}$ , it follows that  $N(r, \phi_1) = \bar{N}(r, f) = S(r, f)$ . Hence  $T(r, \phi_1) = S(r, f)$ .

Similarly, if

$$\phi_2 = \frac{f'}{f - a} - \frac{f''}{f' - a},$$

then  $T(r, \phi_2) = S(r, f)$ .

Suppose first that  $\phi_1 \neq 0$  (i.e.  $\phi_1(z) \not\equiv 0$ ) and  $\phi_2 \neq 0$ . Then from Nevanlinna's fundamental estimate and Jensen's Theorem we obtain

$$\begin{aligned} \frac{\phi_1}{f-a} &= \frac{f'}{(f-a)(f-b)} - \frac{f''}{f'(f'-b)} \cdot \frac{f'}{f-a}, \\ m\left(r, \frac{1}{f-a}\right) &\leq m\left(r, \frac{1}{\phi_1}\right) + m\left(r, \frac{f'}{(f-a)(f-b)}\right) \\ &\quad + m\left(r, \frac{f''}{f'(f'-b)}\right) + m\left(r, \frac{f'}{f-a}\right) + O(1) \\ &\leq T\left(r, \frac{1}{\phi_1}\right) + S(r, f) + S(r, f') + S(r, f) \\ &\leq T(r, \phi_1) + S(r, f) = S(r, f). \end{aligned}$$

Similarly, by using  $\phi_2/(f-b)$  we obtain  $m(r, 1/(f-b)) = S(r, f)$ .

Now if  $f \neq f'$ , then from the first fundamental theorem and the fundamental estimate we get that

$$\begin{aligned} 2T(r, f) &= T(r, f, a) + T(r, f, b) + O(1) \\ &= N(r, f, a) + N(r, f, b) + S(r, f) \\ &\leq N\left(r, \frac{f'}{f}, 1\right) + S(r, f) \leq T\left(r, \frac{f'}{f}, 1\right) + S(r, f) \\ &= m\left(r, \frac{f'}{f}\right) + N\left(r, \frac{f'}{f}\right) + S(r, f) \\ &= S(r, f) + \bar{N}(r, f, 0) \leq T(r, f) + S(r, f), \end{aligned}$$

which implies the contradiction  $T(r, f) = S(r, f)$ . Thus  $f = f'$ .

On the other hand, if  $\phi_1 = 0$  (i.e.  $\phi_1(z) \equiv 0$ ), then from integration of  $\phi_1$  we get

$$\frac{f-b}{f'-b} = C$$

where  $C$  is some nonzero constant. If  $C = 1$ , then  $f = f'$ . If  $C \neq 1$ , then  $a$  is a Picard value for both  $f$  and  $f'$ . This is impossible because  $a \neq 0$  [5, p. 60]. Similarly, if  $\phi_2 = 0$ , then  $f = f'$ . This proves Lemma 1.

Proceeding now with the proof of Theorem 1, we will assume that  $f \neq f'$ . Consider the following function:

$$(4) \quad \psi = \frac{f'}{f-a} + \frac{f'}{f-b} - \frac{f''-f'}{f'-f}.$$

$m(r, \psi) \leq S(r, f) + S(r, f' - f) \leq S(r, f)$ . Since  $\psi$  is the logarithmic derivative of

$$F = \frac{(f-a)(f-b)}{f'-f},$$

it follows that  $N(r, \psi) = \bar{N}(r, F)$ . Hence

$$(5) \quad T(r, \psi) = \bar{N}(r, F) + S(r, f).$$

Suppose that  $z_0$  is a simple pole of  $f$ . We will examine the value of  $\psi(z_0)$ .

Note that since  $f$  and  $f'$  share  $a$  and  $b$  **CM**, it follows that there is an entire function  $w(z)$  such that

$$(6) \quad \frac{(f' - a)(f - b)}{(f - a)(f' - b)} = e^w.$$

(6) can be rewritten as

$$(7) \quad (a - b)(f' - f) = (e^w - 1)(f - a)(f' - b).$$

From (7) we see that  $e^w - 1$  has a simple zero at  $z_0$  and the residue of  $f$  at  $z_0$  is  $(a - b)(w'(z_0))^{-1}$ . We emphasize here that the assumption that  $f$  has a simple pole implies that  $w' \neq 0$ . If

$$(8) \quad \frac{f'(z)}{f(z)} = -\frac{1}{z - z_0} + \sum_{n=0}^{\infty} B_n(z - z_0)^n$$

is the Laurent expansion about  $z_0$ , then for any fixed  $c \in \mathbb{C}$ , we find that  $A_0 = B_0 - cw'(z_0)(a - b)^{-1}$  in the Laurent expansion

$$\frac{f'(z)}{f(z) - c} = -\frac{1}{z - z_0} + \sum_{n=0}^{\infty} A_n(z - z_0)^n$$

about  $z_0$ . It is easily found that  $C_0 = 1$  in the Laurent expansion

$$\frac{f''(z) - f'(z)}{f'(z) - f(z)} = -\frac{2}{z - z_0} + \sum_{n=0}^{\infty} C_n(z - z_0)^n$$

about  $z_0$ . Substitution of these calculations into (4) gives

$$(9) \quad \psi(z_0) = 2B_0 - \frac{a + b}{a - b}w'(z_0) - 1.$$

We will further examine the constant  $B_0$ . To this end, set

$$(10) \quad f(z) = \frac{(a - b)/w'(z_0)}{z - z_0} + \sum_{n=0}^{\infty} D_n(z - z_0)^n,$$

and substitute the expansion (10) into (7) and equate the coefficient of  $(z - z_0)^{-1}$ . This yields

$$(11) \quad -\frac{(a-b)^2}{w'(z_0)} = (D_0 - a)(b-a) - (a-b)^2 \frac{w''(z_0) + (w'(z_0))^2}{2(w'(z_0))^2},$$

$$D_0 = \frac{a-b}{w'(z_0)} + \frac{1}{2}(a+b) + \frac{(b-a)w''(z_0)}{2(w'(z_0))^2}.$$

Now substitute the expansion (10) into (8), multiply by  $f$ , and equate the coefficient of  $(z - z_0)^{-1}$ . Using (11) we obtain

$$(12) \quad 0 = -D_0 + \frac{(a-b)B_0}{w'(z_0)},$$

$$B_0 = \frac{a+b}{2(a-b)}w'(z_0) + 1 - \frac{w''(z_0)}{2w'(z_0)}.$$

Substitution of (12) into (9) gives

$$(13) \quad \psi(z_0) = 1 - \frac{w''(z_0)}{w'(z_0)}.$$

We have two cases.

*Case 1.*  $\psi(z) \not\equiv 1 - w''(z)/w'(z)$ .

Note that if  $f$  has a pole of order  $k$  at  $z_1$  then from (7),  $e^w - 1$  has a zero of order  $k$  at  $z_1$ ; if  $k \geq 2$ , then  $w'$  has a zero of order  $k - 1$  at  $z_1$ . Combining this observation with (13) gives

$$(14) \quad N(r, f) \leq \bar{N}\left(r, \psi - 1 + \frac{w''}{w'}, 0\right) + N(r, w', 0).$$

From (6) we obtain

$$\begin{aligned} m(r, e^w) &\leq 2T(r, f') + 2T(r, f) + O(1) \\ &\leq 6T(r, f) + S(r, f). \end{aligned}$$

Since  $w'$  is the logarithmic derivative of  $e^w$ , this means that

$$(15) \quad m(r, w') = S(r, f).$$

Hence from (14), (5), and (15) we obtain

$$\begin{aligned}
 (16) \quad N(r, f) &\leq T\left(r, \psi - 1 + \frac{w''}{w'}, 0\right) + T(r, w', 0) \\
 &= T\left(r, \psi - 1 + \frac{w''}{w'}\right) + T(r, w') + O(1) \\
 &\leq T(r, \psi) + T\left(r, \frac{w''}{w'}\right) + S(r, f) \\
 &\leq \bar{N}(r, F) + S(r, w') + S(r, f) \leq \bar{N}(r, F) + S(r, f).
 \end{aligned}$$

We will now prove an inequality in the opposite direction. First differentiate (6) logarithmically to get

$$w' = \frac{f''}{f' - a} + \frac{f'}{f - b} - \frac{f'}{f - a} - \frac{f''}{f' - b}.$$

Multiplying by  $f'$  and using (15) gives

$$\begin{aligned}
 f' &= \frac{b - a}{w'} \cdot \left[ \frac{f'}{f - a} \cdot \frac{f'}{f - b} - \frac{f'f''}{(f' - a)(f' - b)} \right], \\
 m(r, f') &\leq m\left(r, \frac{1}{w'}\right) + m\left(r, \frac{f'}{f - a}\right) + m\left(r, \frac{f'}{f - b}\right) \\
 &\quad + m\left(r, \frac{f'f''}{(f' - a)(f' - b)}\right) + O(1) \\
 &\leq T(r, w') + S(r, f) + S(r, f') \leq S(r, f).
 \end{aligned}$$

Hence

$$(17) \quad T(r, f') = N(r, f') + S(r, f).$$

We note also that [5, p. 33]

$$\begin{aligned}
 (18) \quad m\left(r, \frac{1}{f - a}\right) + m\left(r, \frac{1}{f - b}\right) \\
 &\leq m\left(r, \frac{1}{f - a} + \frac{1}{f - b}\right) + O(1) \\
 &\leq m\left(r, \frac{f'}{f - a} + \frac{f'}{f - b}\right) + m\left(r, \frac{1}{f'}\right) + O(1) \\
 &\leq m\left(r, \frac{f'}{f - a}\right) + m\left(r, \frac{f'}{f - b}\right) + m\left(r, \frac{1}{f'}\right) + O(1) \\
 &= m\left(r, \frac{1}{f'}\right) + S(r, f).
 \end{aligned}$$



Now by the second fundamental theorem we have

$$2T(r, f') \leq \bar{N}(r, f') + N(r, f', 0) + N(r, f', a) \\ + N(r, f', b) + S(r, f').$$

Therefore, by using  $N(r, f, a) = N(r, f', a)$ ,  $N(r, f, b) = N(r, f', b)$ , (17), and (18), we obtain

$$2N(r, f') \leq \frac{1}{2}N(r, f') + T(r, f', 0) - m(r, f', 0) \\ + N(r, f, a) + N(r, f, b) + S(r, f) \\ \leq \frac{1}{2}N(r, f') + T(r, f') - m(r, f, a) - m(r, f, b) \\ + N(r, f, a) + N(r, f, b) + S(r, f) \\ = \frac{3}{2}N(r, f') - T(r, f, a) - T(r, f, b) \\ + 2N(r, f, a) + 2N(r, f, b) + S(r, f);$$

hence

$$(19) \quad \frac{1}{2}N(r, f') + 2T(r, f) \leq 2N(r, f, a) + 2N(r, f, b) + S(r, f).$$

Since a pole of  $f$  of order  $k \geq 2$  is a zero of  $w'$  of order  $k - 1$ , then by using (15) we get

$$N(r, f) \leq \bar{N}(r, f) + N(r, w', 0) \\ \leq \frac{1}{2}N(r, f') + T(r, w') + O(1) = \frac{1}{2}N(r, f') + S(r, f).$$

Hence from (19) we obtain

$$(20) \quad \frac{1}{2}N(r, f) + T(r, f) \leq N(r, f, a) + N(r, f, b) + S(r, f).$$

Then from (20) we can deduce that

$$\bar{N}(r, F) + \frac{1}{2}N(r, f) + T(r, f) \\ \leq \bar{N}(r, F) + N(r, f, a) + N(r, f, b) + S(r, f) \\ \leq N\left(r, \frac{f'}{f}, 1\right) + \bar{N}(r, w', 0) + N(r, f, 0) - \bar{N}(r, f, 0) + S(r, f) \\ \leq T\left(r, \frac{f'}{f}\right) + T(r, w') + N(r, f, 0) - \bar{N}(r, f, 0) + S(r, f) \\ \leq \bar{N}(r, f) + N(r, f, 0) + S(r, f) \leq N(r, f) + T(r, f) + S(r, f),$$

which gives

$$(21) \quad \bar{N}(r, F) \leq \frac{1}{2}N(r, f) + S(r, f).$$

But (16) and (21) together imply that

$$N(r, f) = S(r, f).$$

By Lemma 1,  $f = f'$ , which we have been assuming is not true.

*Case 2.*  $\psi(z) \equiv 1 - w''(z)/w'(z)$ .

Then integration of (4) gives

$$(22) \quad F = K \frac{e^z}{w'}$$

where  $K$  is a nonzero constant. If  $z_0$  is a simple pole of  $f$ , then from (7), the residue of  $f$  at  $z_0$  is  $(a - b)(w'(z_0))^{-1}$ . Thus from (22) we obtain  $e^{z_0} = (b - a)K^{-1}$ . On the other hand, a pole of order  $n \geq 2$  of  $f$  is a pole of order  $n - 1$  of  $F$ . Since 0 and  $\infty$  are Picard values for  $e^w$ , we have that  $T(r, e^w) = N(r, e^w, 1) + S(r, e^w)$  from the second fundamental theorem. Combining these observations with (7) and (22) we see that

$$\begin{aligned} T(r, e^w) &= N(r, e^w, 1) + S(r, e^w) \\ &= N\left(r, \frac{(f - a)(f' - b)}{f' - f}\right) + S(r, e^w) \\ &\leq \bar{N}\left(r, e^z, \frac{b - a}{K}\right) + N(r, w', 0) + N(r, F) + S(r, e^w) \\ &= \frac{r}{\pi} + 2N(r, w', 0) + S(r, e^w) \\ &\leq \frac{r}{\pi} + 2T(r, w') + S(r, e^w) \leq \frac{r}{\pi} + S(r, e^w). \end{aligned}$$

Hence outside a set of finite linear measure we have

$$(23) \quad T(r, e^w) \leq \frac{2r}{\pi}.$$

Now we will invoke the following lemma due to S. Bank.

**LEMMA 2.** [1, p. 68] *If  $g(r)$  and  $h(r)$  are monotone nondecreasing functions on  $(0, \infty)$  such that  $g(r) \leq h(r)$  for all  $r$  outside a set of finite linear measure, then for any given real number  $\lambda > 1$ , there exists  $r_0 > 0$  such that  $g(r) \leq h(\lambda r)$  for all  $r \geq r_0$ .*

Lemma 2 (with  $\lambda = 2$ ) applied to the inequality (23) gives

$$T(r, e^w) \leq \frac{4r}{\pi} \quad \text{for } r \geq r_0.$$

Hence the order of  $e^w$  is at most one. It follows that  $e^w = Ae^{Bz}$  for some nonzero constants  $A$  and  $B$ . Since  $w' = B$ , (22) reduces to

$$(24) \quad F = Ce^z$$

where  $C$  is a nonzero constant. Eliminating  $f'$  between (24) and (6) yields

$$(25) \quad f = a - Ce^z + \frac{a - b}{Ae^{Bz} - 1}.$$

Then

$$(26) \quad f' = -Ce^z + \frac{(b - a)ABe^{Bz}}{(Ae^{Bz} - 1)^2}.$$

Substitution of (26) and (25) into (24) gives

$$(27) \quad bCe^z(Ae^{Bz} - 1)^2 + C^2e^{2z}(Ae^{Bz} - 1)^2 + (a - b)^2Ae^{Bz} \\ + (b - a)Ce^z(Ae^{Bz} - 1) + C(a - b)ABe^{(B+1)z} = 0.$$

If the constant  $B$  is not real, then by equating the coefficient of  $e^z$  on the left side of (27) to zero, we get  $aC = 0$ , which is a contradiction. If  $B > 0$ , then the coefficient of  $e^{(2B+2)z}$  in (27) gives  $A^2C^2 = 0$ , which is a contradiction. If  $B < 0$ , then the coefficient of  $e^{2z}$  gives the contradiction  $C^2 = 0$ . Thus  $B = 0$  which is a contradiction.

Cases 1 and 2 have both led to a contradiction. Therefore  $f$  cannot have any simple poles.

If  $w' \neq 0$ , then from (15),

$$N(r, f) \leq N(r, w', 0) \leq T(r, w') + O(1) = S(r, f).$$

Then  $f = f'$  by Lemma 1, which contradicts the original assumption. If  $w' = 0$ , then  $e^w = C$  where  $C$  is a nonzero constant. If  $C = 1$  then  $f = f'$  from (7), a contradiction. If  $C \neq 1$ , then  $f$  has no poles and  $f = f'$  from Lemma 1, a contradiction.

Therefore, the original assumption that  $f \neq f'$  has yielded a contradiction, and the proof of Theorem 1 is now complete.

*Proof of Corollary 1.* Since equation (3) is merely a rewriting of equation (6), the result easily follows.

#### REFERENCES

1. S. Bank, *A general theorem concerning the growth of solutions of first-order algebraic differential equations*, *Compositio Math.*, **25** (1972), 61–70.
2. F. Gackstatter and I. Laine, *Zur theorie der gewöhnlichen differential-gleichungen im komplexen*, *Ann. Polon. Math.*, **38** (1980), 259–287.
3. G. G. Gundersen, *Meromorphic functions that share finite values with their derivative*, *J. Math. Anal. Appl.*, **75** (1980), 441–446.

4. \_\_\_\_\_, *When two entire functions and also their first derivatives have the same zeros*, Indiana Univ. Math. J., **30** (1981), 293–303.
5. W. K. Hayman, *Meromorphic Functions*, Clarendon Press, Oxford, 1964.
6. E. Mues and N. Steinmetz, *Meromorphe funktionen, die mit ihrer ableitung werte teilen*, Manuscripta Math., **29** (1979), 195–206.
7. R. Nevanlinna, *Le Théorème de Picard-Borel et la Theorie des Fonctions Méromorphes*, Gauthier-Villars, Paris, 1929.
8. L. A. Rubel and C. C. Yang, *Values shared by an entire function and its derivative*, in “Complex Analysis, Kentucky 1976” (Proc. Conf.), Lecture Notes in Mathematics, No. 599, 101–103, Springer-Verlag, Berlin/Heidelberg/New York, 1977.
9. N. Steinmetz, *Bermerkung zu einem Satz von Yosida*, in “Complex Analysis, Joensuu 1978” (Proc. Colloq.), Lecture Notes in Mathematics, No. 747, 369–377, Springer-Verlag, Berlin/Heidelberg/New York, 1979.

Received July 30, 1981.

UNIVERSITY OF NEW ORLEANS  
NEW ORLEANS, LA 70148



# PACIFIC JOURNAL OF MATHEMATICS

## EDITORS

DONALD BABBITT (Managing Editor)

University of California  
Los Angeles, CA 90024

HUGO ROSSI

University of Utah  
Salt Lake City, UT 84112

C. C. MOORE and ARTHUR OGUS

University of California  
Berkeley, CA 94720

J. DUGUNDJI

Department of Mathematics  
University of Southern California  
Los Angeles, CA 90089-1113

R. FINN and H. SAMELSON

Stanford University  
Stanford, CA 94305

## ASSOCIATE EDITORS

R. ARENS

E. F. BECKENBACH  
(1906–1982)

B. H. NEUMANN

F. WOLF

K. YOSHIDA

## SUPPORTING INSTITUTIONS

UNIVERSITY OF ARIZONA  
UNIVERSITY OF BRITISH COLUMBIA  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
UNIVERSITY OF CALIFORNIA  
MONTANA STATE UNIVERSITY  
UNIVERSITY OF NEVADA, RENO  
NEW MEXICO STATE UNIVERSITY  
OREGON STATE UNIVERSITY

UNIVERSITY OF OREGON  
UNIVERSITY OF SOUTHERN CALIFORNIA  
STANFORD UNIVERSITY  
UNIVERSITY OF HAWAII  
UNIVERSITY OF TOKYO  
UNIVERSITY OF UTAH  
WASHINGTON STATE UNIVERSITY  
UNIVERSITY OF WASHINGTON

---

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

---

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be in typed form or offset-reproduced (not dittoed), double spaced with large margins. Please do not use built up fractions in the text of the manuscript. However, you may use them in the displayed equations. Underline Greek letters in red, German in green, and script in blue. The first paragraph must be capable of being used separately as a synopsis of the entire paper. In particular it should contain no bibliographic references. Please propose a heading for the odd numbered pages of less than 35 characters. Manuscripts, in triplicate, may be sent to any one of the editors. Please classify according to the scheme of Math. Reviews, Index to Vol. 39. Supply name and address of author to whom proofs should be sent. All other communications should be addressed to the managing editor, or Elaine Barth, University of California, Los Angeles, California 90024.

There are page-charges associated with articles appearing in the Pacific Journal of Mathematics. These charges are expected to be paid by the author's University, Government Agency or Company. If the author or authors do not have access to such Institutional support these charges are waived. Single authors will receive 50 free reprints; joint authors will receive a total of 100 free reprints. Additional copies may be obtained at cost in multiples of 50.

---

The *Pacific Journal of Mathematics* is issued monthly as of January 1966. Regular subscription rate: \$132.00 a year (6 Vol., 12 issues). Special rate: \$66.00 a year to individual members of supporting institutions.

Subscriptions, orders for numbers issued in the last three calendar years, and changes of address should be sent to Pacific Journal of Mathematics, P.O. Box 969, Carmel Valley, CA 93924, U.S.A. Old back numbers obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

---

The Pacific Journal of Mathematics ISSN 0030-8730 is published monthly by the Pacific Journal of Mathematics at P.O. Box 969, Carmel Valley, CA 93924. Application to mail at Second-class postage rates is pending at Carmel Valley, California, and additional mailing offices. Postmaster: Send address changes to Pacific Journal of Mathematics, P. O. Box 969, Carmel Valley, CA 93924.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Copyright © 1983 by Pacific Journal of Mathematics

<b>Spiros Argyros</b> , On compact spaces without strictly positive measure .....	257
<b>Steven Robert Bell</b> , Regularity of the Bergman projection in certain nonpseudoconvex domains .....	273
<b>Carlos R. Borges and Gary Fred Gruenhagen</b> , Sup-characterization of stratifiable spaces .....	279
<b>Giuseppe Ceresa and Alberto Collino</b> , Some remarks on algebraic equivalence of cycles .....	285
<b>Charles Kam-Tai Chui and Maurice Hasson</b> , Degree of uniform approximation on disjoint intervals .....	291
<b>Gary Gundersen</b> , Meromorphic functions that share two finite values with their derivative .....	299
<b>Richard I. Hartley</b> , Lifting group homomorphisms .....	311
<b>Gerald William Johnson and David Lee Skoug</b> , Notes on the Feynman integral. III. The Schroedinger equation .....	321
<b>John Cronan Kieffer</b> , Some topologies on the set of discrete stationary channels .....	359
<b>Harald Luschgy and Wolfgang Thomsen</b> , Extreme points in the Hahn-Banach-Kantorovič setting .....	387
<b>Zbigniew Piotrowski, A. Rosłanowski and Brian M. Scott</b> , The pinched-cube topology .....	399
<b>Elias Saab and Paulette Saab</b> , A dual geometric characterization of Banach spaces not containing $l_1$ .....	415
<b>Walter Schachermayer</b> , Norm attaining operators on some classical Banach spaces .....	427
<b>Martin Scharlemann</b> , Essential tori in 4-manifold boundaries .....	439
<b>Jacques C. H. Simon</b> , Nonlinear representations of Poincaré group and global solutions of relativistic wave equations .....	449
<b>Adrian R. Wadsworth</b> , $p$ -Henselian field: $K$ -theory, Galois cohomology, and graded Witt rings .....	473