

Pacific Journal of Mathematics

A NOTE ON PRIMARY POWERS OF A PRIME IDEAL

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Let $X \subset P_k^n$ be an irreducible projective variety. Let $B = k[x_0, \dots, x_n]$ and let $P \subset B$ be the homogeneous prime ideal of X generated by $\text{ht}(P) + 1$ elements and let $A = B/P$ be the homogeneous coordinate ring of X . The following are equivalent: (1) $A_{(P)}$ is a complete intersection for all homogeneous prime ideals p in A of height 1; (2) P^2 is primary; (3) P^i is primary for all integers $i > 0$.

1. Introduction. In [RV, Theorem 3.3, p. 497], Robbiano and Valla proved the following: if $Y \subset X \subset P_k^n$ are projective schemes in the projective n -space over a field k , which are complete intersections in P_k^n and if Y is a positive dimensional irreducible, reduced normal subscheme of X with $(\text{sing } X) \cap Y \subseteq \text{sing } Y$, and if P is the prime ideal of Y in the homogeneous coordinate ring of X , then (a) P^2 is primary and (b) P^n is primary for every integer $n > 0$ if $\dim Y > \text{codim } X$. [RV, Example 2, p. 560] gave an example of a projectively Gorenstein projective smooth curve in P_k^7 with homogeneous prime ideal P such that P^2 is not primary. It is known that an almost complete intersection is not Gorenstein. We prove the following theorem: Let X be a projective irreducible variety in P_k^n which is an almost complete intersection, i.e. the prime ideal P of X in the polynomial ring $k[x_0, \dots, x_n]$ is generated by $(\text{codim } X) + 1$ elements. Let $B = k[x_0, \dots, x_n]$ and let $A = B/P = \bigoplus_{i \geq 0} A_i$ be the homogeneous coordinate ring of X , where A_i is the group of all homogeneous elements of degree i and $A_i A_j \subset A_{i+j}$. For a homogeneous ideal prime p in A let

$$A_{(p)} = \{a_i/s_i \mid a_i \in A_i, s_i \in A_i - p\}.$$

The following are equivalent:

(1) $A_{(p)}$ is a complete intersection for all homogeneous prime ideals p of height 1 in A .

(2) P^2 is primary.

(3) P^i is primary for all integers $i > 0$. Thus for an almost complete intersection X , “ X is free of codim 1 singularities”, is equivalent to “ P^i is primary for all integers $i > 0$ ”. Examples of projective varieties which are almost complete intersections are plentiful, for example, the Segre imbedding of $P_k^1 \times P_k^2$ into P_k^5 and the twist cubic in P_k^3 [Sz, p. 15–4]. The local case of (1) \Leftrightarrow (2) was proved in [K2, Theorem, p. 1]: Let B be a regular

local ring, $P \subset B$ a prime ideal which is an almost complete intersection, and let $A = B/P$. Then P^2 is primary if and only if A_q is a complete intersection for all prime ideals q of height 1 in A . Recently [H, Theorem 3.1] proves that (1)—(3) are equivalent in the affine case: Let B be a Cohen-Macaulay ring, and let $P \subset B$ be a prime ideal which is an almost complete intersection such that B_P is a regular local ring. Then the following are equivalent: (1) $P^2 = P^{(2)}$; (2) $P^n = P^{(n)}$ for $n > 0$ where $P^{(n)}$ is the n th symbolic power of P ; (3) B_Q is a complete intersection for all $Q \in \text{Spec}(B)$ with $Q \supset P$ and $\text{ht}(Q/P) = 1$; (4) $P_Q^{(n)} = P_Q^n$ for every Q in (3); and (5) $\text{gr}_P(B)$ is a domain, where $\text{gr}_P(B) = B/P \oplus P/P^2 \oplus \cdots$. The purpose of this note is two-fold. One is that the projective case can be easily derived from the local case by introducing a simple but interesting lemma of Seidenberg [S, Lemma 1, p. 618] which says that if A is a graded domain and q is a nonhomogeneous prime of height 1, then A_q is normal. The other is that we make some further comparisons between $A_{(p)}$ and the usual localization A_p for the homogeneous prime ideals p of A and show that $A_{(p)}$ is an almost complete intersection (complete intersection) if and only if A_p is an almost complete intersection (complete intersection). Equivalences of these two local rings on the properties of being regular and normal were proved in [K1] and those on the Cohen-Macaulay condition, the Gorenstein property, and the Buchsbaum singularity have been shown in [DE].

2. Notations and Definitions. Let $A = \bigoplus_{i \geq 0} A_i$ be a graded domain. Let p be a homogeneous ideal of A contained in $\bigoplus_{i > 0} A_i$, A_p is the usual localization of A with respect to the multiplicative set $A - p$, namely

$$A_p = \{a/s \mid a \in A, s \in A - p\},$$

$$A_{(p)} = \{a_i/s_i \mid a_i \in A_i \text{ and } s_i \in A_i - p\}.$$

If $I \subset A$ is a homogeneous ideal, then I_p will mean the extended ideal of I in A_p and

$$I_{(p)} = \{b_i/s_i \mid b_i \in I \cap A_i \text{ and } s_i \in A_i - p\}.$$

Let $f \in A_1$, and $f \neq 0$.

$$A_{(f)} = \{A_i/f^i \mid a_i \in A_i\} \text{ and } I_{(f)} = \{b_i/f^i \mid b_i \in I \cap A_i\}.$$

$\mu(I) \equiv$ minimal number of generators of I , $\mu(I_p) \equiv$ minimal number of generators of I_p in A_p and $\mu(I_{(p)}) \equiv$ minimal number of generators of $I_{(p)}$ in $A_{(p)}$. A_p ($A_{(p)}$) is a complete intersection if it is isomorphic to the

quotient ring of a regular local ring B modulo and ideal \mathfrak{A} generated by a regular B -sequence. A_p ($A_{(p)}$) is an almost complete intersection if it is isomorphic to the quotient ring of a regular local ring B modulo an ideal \mathfrak{A} minimally generated by $\text{ht}(\mathfrak{A}) + 1$ elements. In this case the first $\text{ht}(\mathfrak{A})$ elements can be taken to be a regular B -sequence.

Let $k[x_0, \dots, x_n]$ be a polynomial ring in indeterminates x_0, \dots, x_n and let $P \subset k[x_0, \dots, x_n]$ be an ideal. P is an almost complete intersection if $\mu(P) = \text{ht}(P) + 1$.

3. Notes.

LEMMA 1. Let $R = \bigoplus_{i \geq 0} R_i$ be a Noetherian graded ring with R_0 a field, and let $\mathfrak{m} = \bigoplus_{i > 0} R_i$ be the irrelevant maximal ideal. Let $\mathfrak{A} \subset R$ be a homogeneous ideal. Then (1) $\mathfrak{A} R_{\mathfrak{m}} \cap R = \mathfrak{A}$; (2) $\mu(\mathfrak{A}) = \mu(\mathfrak{A} R_{\mathfrak{m}})$; and (3) The minimal generating set of \mathfrak{A} can be chosen to consist of homogeneous elements.

Proof 1. It suffices to show that $\mathfrak{A} \cdot R_{\mathfrak{m}} \cap R \subset \mathfrak{A}$. Let $f \in \mathfrak{A} \cdot R_{\mathfrak{m}} \cap R$ and write $f = g/t$, where $g \in \mathfrak{A}$, $t \in R - \mathfrak{m}$. There exists $s \in R - \mathfrak{m}$ such that $s(ft - g) = 0$. Let $st = a_0 + a_1 + \dots + a_u$, $sg = g_c + g_{c+1} + \dots + g_d$ and $f = f_r + \dots + f_v$, where $a_i, g_i, f_i \in R_i$ and $a_0 \neq 0$, for some nonnegative integers c, d, r, u, v . Then $a_0 \cdot f_r = g_c \in \mathfrak{A}$ and $r = c$ because \mathfrak{A} is a homogeneous ideal. Thus $f_r \in \mathfrak{A}$. Also $a_0 f_{r+1} + a_1 f_r = g_{c+1} \in \mathfrak{A}$. It follows that $f_{r+1} \in \mathfrak{A}$. Inductively, we get $f_i \in \mathfrak{A}$ for $i = r, \dots, v$, and hence $f \in \mathfrak{A}$. Thus $\mathfrak{A} \cdot R_{\mathfrak{m}} \cap R = \mathfrak{A}$.

For (2). Since R is noetherian \mathfrak{A} is of finite type. It is known [B, Cor. 2, p. 248] that $\mu(\mathfrak{A}) = \dim_{R_0} \mathfrak{A}/\mathfrak{m}\mathfrak{A}$. As

$$\begin{aligned} \mathfrak{A} R_{\mathfrak{m}}/\mathfrak{A} \cdot \mathfrak{m} R_{\mathfrak{m}} &\cong \mathfrak{A} \cdot R_{\mathfrak{m}} \otimes_{R_{\mathfrak{m}}} R_{\mathfrak{m}}/\mathfrak{m} R_{\mathfrak{m}} \cong (\mathfrak{A} \otimes_R R_{\mathfrak{m}}) \otimes_{R_{\mathfrak{m}}} R_0 \\ &\cong \mathfrak{A} \otimes_R (R_{\mathfrak{m}} \otimes_{R_{\mathfrak{m}}} R_0) \cong \mathfrak{A} \otimes_R R_0 \cong \mathfrak{A}/\mathfrak{m}\mathfrak{A}, \end{aligned}$$

it follows that

$$\mu(\mathfrak{A} R_{\mathfrak{m}}) = \dim_{R_0} (\mathfrak{A} R_{\mathfrak{m}}/\mathfrak{A} \cdot \mathfrak{m} R_{\mathfrak{m}}) = \dim_{R_0} (\mathfrak{A}/\mathfrak{m}\mathfrak{A}) = \mu(\mathfrak{A}).$$

For (3). \mathfrak{A} is homogeneous. Therefore \mathfrak{A} has a set of generators consisting of homogeneous elements. Let $\{h_1, \dots, h_r\}$ be a minimal set of homogeneous generators. Then $r \geq \mu(\mathfrak{A})$. Let $\bar{h}_1, \dots, \bar{h}_r$ be the residue classes of h_1, \dots, h_r in the $\mu(\mathfrak{A})$ -dimensional vector space $\mathfrak{A}/\mathfrak{m} \cdot \mathfrak{A}$ over

R_0 . We select a vector basis, say $\{\bar{h}, \dots, \bar{h}_{\mu(\mathfrak{A})}\}$. Then, by [B, Cor. 2, p. 248], $\{h_1, \dots, h_{\mu(\mathfrak{A})}\}$ form a minimal set of generators of \mathfrak{A} . Therefore $\mu(\mathfrak{A}) = r$.

LEMMA 2. Let $A = \bigoplus_{i \geq 0} A_i$ be a noetherian graded domain and let $I \subset A$ be a graded ideal. Let p be a graded prime ideal of A contained in $\bigoplus_{i > 0} A_i$ such that there is $r_1 \in A_1 - p$. Then (1) $\mu(I_{(p)}) = \mu(I_p)$; (2) $\text{depth } A_{(p)} = \text{depth } A_p$; and (3) $\dim A_p = \dim A_{(p)}$;

Proof. It follows from [K1, Theorem 2, p.456] and [DE, Theorem 1] that $A_p = A_{(p)}[r_1]_{\mathfrak{m}[r_1]}$ and r_1 is transcendental over $A_{(p)}$, where \mathfrak{m} is the maximal ideal of $A_{(p)}$, and a similar argument yields $I_p = I_{(p)}[r_1]_{\mathfrak{m}[r_1]}$.

$$\mu(I_p) = \dim_{A_p/pA_p}(I_p/I_p \cdot pA_p),$$

$$\mu(I_{(p)}) = \dim_{A_{(p)}/\mathfrak{m}A_{(p)}}(I_{(p)}/I_{(p)} \cdot \mathfrak{m}A_{(p)}).$$

Since $A_p/pA_p = k(r_1)$, a transcendental field extension over $k = A_{(p)}/\mathfrak{m}A_{(p)}$, and

$$\begin{aligned} I_p/I_p \cdot pA_p &\cong I_p \otimes_{A_p} A_p/pA_p = (I_{(p)} \otimes_{A_{(p)}} A_{(p)}[r_1]_{\mathfrak{m}[r_1]}) \otimes_{A_p} A_p/pA_p \\ &\cong I_{(p)} \otimes_{A_{(p)}} (A_p \otimes_{A_p} A_p/pA_p) \cong I_{(p)} \otimes_{A_{(p)}} k(r_1) \\ &\cong (I_{(p)} \otimes_{A_{(p)}} k) \otimes_k k(r_1) \cong I_{(p)}/I_{(p)} \mathfrak{m}A_{(p)} \otimes_k k(r_1). \end{aligned}$$

Therefore (1) $\mu(I_p) = \mu(I_{(p)})$. For (2),

$$\begin{aligned} A_p \otimes_{A_{(p)}} k &\cong A_p \otimes_{A_{(p)}} A_{(p)}/\mathfrak{m}A_{(p)} \\ &= A_{(p)}[r_1]_{\mathfrak{m}[r_1]}/\mathfrak{m}A_{(p)}[r_1]_{\mathfrak{m}[r_1]} \cong k(r_1), \end{aligned}$$

which is of depth 0. $A_{(p)} \rightarrow A_p$ is a local flat homomorphism. It follows from [M, Cor. 1, p. 154] that $\text{depth } A_{(p)} = \text{depth } A_p$. (3) [DE, Corollary to Theorem 1].

PROPOSITION. Let $A = \bigoplus_{i \geq 0} A_i$ be a noetherian graded domain with A_0 a regular ring and let p be a graded prime ideal of A such that there is $r_1 \in A_1 - p$.

(1) $A_{(p)}$ is a complete intersection if and only if A_p is a complete intersection.

(2) $A_{(p)}$ is an almost complete intersection if and only if A_p is an almost complete intersection.

Proof. Since A is Noetherian, A is finitely generated A_0 -algebra. Let S be a polynomial ring over A_0 , say $S = A_0[x_0, \dots, x_n]$ and let $I \subset (x_0, \dots, x_n)$ be the homogeneous prime ideal of S such that $A = S/I$. Let $P \subset S$ be the inverse image of p . Then $A_p = (S/I)_{P/I} = S_p/I \cdot S_p$. Let L_1 be an element of S of homogeneous degree 1 such that r_1 is the image of L_1 in A_1 . Then

$$A_{(p)} = \left(S_{(L_1)} \right)_{P_{(L_1)}} / \left(I_{(L_1)} \right) \cdot \left(S_{(L_1)} \right) = S_{(P)}/I_{(P)} \cdot S_{(P)},$$

where $S_{(L_1)}$, $P_{(L_1)}$, $I_{(L_1)}$ are dehomogenized S , P and I with respect to L_1 , respectively. That A_0 is regular implies $S = A_0[x_1, \dots, x_n]$ is regular. Thus S_p is regular and so is $S_{(P)}$ by [K1, Theorem 2e, p. 457]. A_p and $A_{(p)}$ are thus quotient rings of regular local rings. $\text{ht}(I_{(P)}) = \text{ht}(I_P)$ because $\dim S_p = \dim S_{(P)}$ and $\dim A_p = \dim A_{(p)}$. (1) If A_p is a complete intersection then, by Lemma 2, $\mu(I_P) = \text{ht}(I_P) = \text{ht}(I_{(P)}) = \mu(I_{(P)})$. Therefore $A_{(p)}$ is a complete intersection, and conversely.

(2) If A_p is an almost complete intersection then $\mu(I_P) = \text{ht}(I_P) + 1 = \text{ht}(I_{(P)}) + 1 = \mu(I_{(P)})$. Therefore $A_{(p)}$ is also an almost complete intersection, and conversely.

Note. (1) Also follows from a more general result [A, Theorem 2, p. 1413]: Let $f: (B, \mathfrak{U}) \rightarrow (A, \mathfrak{m})$ be a flat local homomorphism of noetherian local rings. Then A is a complete intersection if and only if the same is true of B and of $A \otimes_B B/\mathfrak{m}$. But our proofs is simple and direct for the case involved.

THEOREM. *Let $B = k[x_0, x_1, \dots, x_n]$ be a polynomial ring over a field k in the indeterminates x_0, \dots, x_n . Let $P \subset B$ be a homogeneous prime ideal such that P is an almost complete intersection. Let $A = B/P$. Then the following are equivalent:*

(1) P^2 is P -primary.

(2) For all $p \in \text{proj}(A)$ with $\text{ht}(p) = 1$, the local ring $A_{(p)}$ is a complete intersection.

(3) P^i is P primary for all integers $i > 0$.

Proof. For (1) \leftrightarrow (2). Let $\text{ht}(P) = r$. Let $M = (x_0, \dots, x_n) \cdot B$. Then by Lemma 1, $\mu(P) = \mu(PB_M) = r + 1$. Since for each $Q \in \text{Spec}(B)$ and $Q \supset P$, $\mu(PB_Q) \leq r + 1$ and $\text{ht}(PB_Q) = \text{ht}(P) = r$. Thus in the regular local ring B_M , $P \cdot B_M$ is locally a complete intersection or an almost complete intersection.

P^2 is P -primary if and only if $P^2 \cdot B_M$ is $P \cdot B_M$ -primary. By [K2, Theorem 1, p. 15] $P^2 B_M$ is primary if and only if for each $q \in \text{Spec}(A_m)$ with $\text{ht}(q) = 1$, the local ring A_q is a complete intersection, where $\mathfrak{m} = M/P$. Since A is a noetherian graded domain and \mathfrak{m} is the irrelevant maximal ideal, and suppose q above is homogeneous, then by the above proposition $A_{(q)}$ is a complete intersection. Conversely, if $A_{(q)}$ is a complete intersection for all homogeneous ideals q of height 1 in $\text{proj}(A)$, then A_q is a complete intersection by the proposition. Let $q' \in \text{Spec}(A)$ and $q' \notin \text{proj}(A)$ and $\text{ht}(q') = 1$. Then by [S, Lemma 1, p. 618] $A_{q'}$ is a normal local domain. Since $\dim A_{q'} = 1$, then $A_{q'}$ is a regular local ring, in particular a complete intersection. Thus A_q is a complete intersection for all $q \in \text{Spec} A$ and $\text{ht}(q) = 1$. Since Lemma 1(2) implies P_M is an almost complete intersection and [K2, Theorem 1; p. 15] implies P_M^2 is P -primary, P^2 is P -primary.

For (1) \leftrightarrow (3). As B is a polynomial ring, B is Cohen-Macaulay and B_p is regular. It follows from [H, Theorem 3.1] as quoted in the introduction, that $P^2 = P^{(2)}$ if and only if $P^i = P^{(i)}$ for all integers $i > 0$. Since $P^{(i)} = P^i$ if and only if P^i is primary, (1) is equivalent to (3).

COROLLARY 1. *Let P, S be the same as in the theorem. Let V be the projective variety in the projective n space P_k^n defined by P . If V is free of singularities of codimension 1, in particular V is locally normal, then P^n is P -primary for $n > 0$.*

Proof. V is locally normal implies V is free of singularities of codimension 1. Let A be the homogeneous coordinate ring of V . Then $A_{(p)}$ is a regular local ring for each homogeneous prime ideal p of height 1. Thus by the theorem, P^n is P -primary for all integers $n > 0$.

COROLLARY 2. *Let $P \subset k[x_0, x_1, \dots, x_n]$ be the homogeneous prime ideal of a nonsingular irreducible projective curve which is also an almost complete intersection. Then P^n is primary for all integers $n > 0$.*

Proof. This follows from Corollary 1.

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