CYCLIC GROUPS OF AUTOMORPHISMS OF COMPACT NONORIENTABLE KLEIN SURFACES WITHOUT BOUNDARY

Emilio Bujalance
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We obtain the minimum genus of the compact non-orientable Klein
surfaces of genus \( p \geq 3 \) without boundary which have a given cyclic
group of automorphisms.

1. Introduction. Let \( X \) be a compact Klein surface [1]. Singerman
[8] showed that the order of a group of automorphisms of a surface \( X \)
without boundary of algebraic genus \( g \geq 2 \) is bounded above by \( 84(g - 1) \),
and May [7] proved that if \( X \) has nonempty boundary, this bound is
\( 12(g - 1) \).

These bounds may be considered as particular cases of the general
problem of finding the minimum genus of surfaces for which a given finite
group \( G \) is a group of automorphisms. The study of cyclic groups is a
necessary preliminary to this, since it leads to limitations on the orders of
elements within a general group. In this paper we consider the above
problem for the case of cyclic groups of automorphisms of compact
non-orientable Klein surfaces without boundary. The corresponding prob-
lem for compact orientable Klein surfaces without boundary was solved
by Harvey [5].

2. Compact non-orientable Klein surfaces without boundary. By a
non-Euclidean crystallographic (NEC) group, we shall mean a discrete
subgroup \( \Gamma \) of the group of isometries \( G \) of the non-Euclidean plane, with
compact quotient space, including those which reverse orientation, reflec-
tions and glide reflections. We say that \( \Gamma \) is a proper NEC group if it is
not a Fuchsian group. We shall denote by \( \Gamma^+ \) the Fuchsian group
\( \Gamma \cap G^+ \), where \( G^+ \) is the subgroup of \( G \) whose elements are the orientation-preserving isometries.

NEC groups are classified according to their signature. The signature
of an NEC group \( \Gamma \) is either of the form

\[
(\ast) \quad \left( g; +; [m_1, \ldots, m_r]; \left\{ (n_{i1}, \ldots, n_{it_i}) \}_{i=1, \ldots, k} \right\} \right)
\]
or

\[
(\ast\ast) \quad \left( g; -; [m_1, \ldots, m_r]; \left\{ (n_{i1}, \ldots, n_{it_i}) \}_{i=1, \ldots, k} \right\} \right);
\]
the numbers \( m_i \) are the periods and the brackets \((n_{i1}, \ldots, n_{is_i})\), the period cycles.

A group \( \Gamma \) with signature (*) has the presentation given by generators

\[
x_i, \quad i = 1, \ldots, \tau, \quad c_{ij}, \quad i = 1, \ldots, k, \quad j = 0, \ldots, s_i, \\
\quad e_i, \quad i = 1, \ldots, k, \quad a_j, b_j, j = 1, \ldots, g,
\]

and relations

\[
x_i^{m_i} = 1, \quad i = 1, \ldots, \tau, \quad c_{is_i} = e_i^{-1} c_{i0} e_i, \quad i = 1, \ldots, k, \\
c_{ij} = (c_{ij}^{-1} c_{ij})^{n_{ij}} = 1, \quad i = 1, \ldots, k, \quad j = 1, \ldots, s_i, \\
x_1 \cdots x_\tau e_1 \cdots e_k a_1 b_1^{-1} b_1^{-1} \cdots a_g b_g^{-1} b_g^{-1} = 1.
\]

A group \( \Gamma \) with signature (**) has the presentation given by generators

\[
x_i, \quad i = 1, \ldots, \tau, \quad c_{ij}, \quad i = 1, \ldots, k, \quad j = 0, \ldots, s_i, \\
\quad e_i, \quad i = 1, \ldots, k, \quad d_j, \quad j = 1, \ldots, g,
\]

and relations

\[
c_{is_i} = e_i^{-1} c_{i0} e_i, \quad i = 1, \ldots, k, \quad x_i^{m_i} = 1, \quad i = 1, \ldots, \tau, \\
c_{ij} = (c_{ij}^{-1} c_{ij})^{n_{ij}} = 1, \quad i = 1, \ldots, k, \quad j = 1, \ldots, s_i, \\
x_1 \cdots x_\tau e_1 \cdots e_k d_1^2 \cdots d_g^2 = 1.
\]

From now on, we will denote by \( x_i, e_i, c_{ij}, a_i, b_i, d_i \) the above generators associated to the NEC groups.

(2.1) Definition. We shall say that an NEC group \( \Gamma_g \) is the group of an orientable surface if \( \Gamma_g \) has the signature \((g; +; [-]; \{-}\)) where \([-]\) indicates that the signature has no periods and \(\{-\}\) indicates that the signature has no period cycles.

(2.2) Definition. An NEC group \( \Gamma_p \) is the group of a non-orientable surface if \( \Gamma_p \) has the following signature \((p; -; [-]; \{-}\)).

For a given \( \Gamma_p \) we have that the orbit space \( D/\Gamma_p \) (where \( D = \mathbb{C}^+ \)) is a non-orientable surface of genus \( p \). The canonical projection \( \pi: D \to D/\Gamma_p \) induces an analytic and anti-analytic structure on \( D/\Gamma_p \), which establishes a structure of compact non-orientable Klein surface without boundary of genus \( p \) in \( D/\Gamma_p \).
From now on, Klein surfaces appearing in this paper are supposed to be compact without boundary.

Singerman has shown in [8] the following

(2.3) **Proposition.** If $G$ is a group of automorphisms of a non-orientable Klein surface of genus $p \geq 3$, then $G$ is finite.

(2.4) **Theorem.** A necessary and sufficient condition for a finite group $G$ to be a group of automorphisms of a non-orientable Klein surface of genus $p \geq 3$ is that there exist a proper NEC group $\Gamma$ and a homomorphism $\theta$: $\Gamma \to G$ such that the kernel of $\theta$ is a surface group and $\theta(\Gamma^+) = G$.

As a consequence of this theorem, we have that if $G$ is a finite group of automorphisms of a non-orientable Klein surface of genus $p \geq 3$ then $G \cong \Gamma/\Gamma_p$, where $\Gamma$ is a proper NEC group and $\Gamma_p$ is the group of a non-orientable surface; thus

$$\text{order}(G) = |\Gamma_p|/|\Gamma| = 2\pi(p - 2)/|\Gamma|,$$

where $|\ |$ denotes the non-Euclidean area of a fundamental region of the group.

(2.5) **Theorem.** If $G$ is a finite group, $G$ is a group of automorphisms of a non-orientable Klein surface of genus $p \geq 3$.

*Proof.* Let us suppose that $G$ has $n$ generators $g_1, g_2, \ldots, g_n$. There exists a proper NEC group $\Gamma_{2n+1}$ that is the group of a non-orientable surface, and therefore it has the following generators and relations:

$$\{a_1, a_2, \ldots, a_{2n+1} | a_1^2 \cdot a_2^2 \cdots a_{2n+1}^2 = 1\}.$$

We establish a homomorphism $\theta$: $\Gamma_{2n+1} \to G$, by defining

$$\theta(a_1) = g_1, \quad \theta(a_3) = g_2 \cdots \theta(a_{2n-1}) = g_n, \quad \theta(a_{2n+1}) = 1,$$

$$\theta(a_2) = g_1^{-1}, \quad \theta(a_4) = g_2^{-1}, \quad \theta(a_{2n}) = g_n^{-1}.$$

$\theta$ is an epimorphism. $\ker \theta$ is a normal subgroup of $\Gamma_{2n+1}$ with finite index, and therefore, $\ker \theta$ is an NEC group.

As $\Gamma_{2n+1}$ has neither periods nor period-cycles, and $\ker \theta$ is a normal subgroup of $\Gamma_{2n+1}$, by [2] and [3], $\ker \theta$ has neither periods nor period-cycles, and thus it is a surface group.

Moreover, as $a_1 \cdot a_{2n+1}, a_3 \cdot a_{2n+1}, \ldots, a_{2n-1} \cdot a_{2n+1}$ belong to $\Gamma_{2n+1}^+$ and $\theta(a_1 \cdot a_{2n+1}) = g_1, \quad \theta(a_3 \cdot a_{2n+1}) = g_2, \ldots, \theta(a_{2n-1} \cdot a_{2n+1}) = g_n$, then $\theta(\Gamma_{2n+1}^+) = G.$
By (2.4) $G$ is a group of automorphisms of a non-orientable Klein surface of genus $p \geq 3$.


(3.1) Definition. A homomorphism $\theta$ of a proper NEC group $\Gamma$ into a finite group is a non-orientable surface-kernel homomorphism if $\ker \theta$ is the group of a surface and $\theta(\Gamma^+) = G$.

From [2], [3] and (2.4) we get

(3.2) Proposition. A homomorphism $\theta$ of a proper NEC group $\Gamma$ of signature $(g; \pm; [m_1, \ldots, m_r]; \{(n_{i_1}, \ldots, n_{i_{s_1}}) \cdots (n_{k_1}, \ldots, n_{k_{s_k}})\})$ into a finite group $G$ is a non-orientable surface-kernel homomorphism if and only if $\theta(c_{i_j})$ has order 2, $\theta(x_i)$ has order $m_i$, $\theta(c_{i_j-1} \cdot c_{i_j})$ has order $n_{i_j}$ and $\theta(\Gamma^+) = G$.

(3.3) Corollary. Let $G$ be a finite group with odd order. Then there is no proper NEC group $\Gamma$ with period cycles for which there exists a non-orientable surface-kernel homomorphism $\theta: \Gamma \to G$.

(3.4) Corollary. There does not exist any proper NEC group $\Gamma$ with period cycles having some non-empty period cycle for which there is a non-orientable surface-kernel homomorphism $\theta: \Gamma \to \mathbb{Z}_n$ with $n$ even.

Proof. If there were a non-orientable surface-kernel homomorphism $\theta: \Gamma \to \mathbb{Z}_n$, we would have that for every $c_{i_j} \in \Gamma$, $\theta(c_{i_j})$ would have order 2 in $\mathbb{Z}_n$; if $\Gamma$ has some non-empty period cycle, there would be two reflections $c_{i_j}$, $c_{i_j+1} \in \Gamma$ such that $(c_{i_j} \cdot c_{i_j+1})^{n_{c_{i_j}} = 1}$ and, by (3.2), the order of $\theta(c_{i_j} \cdot c_{i_j+1})$ would be $n_{i_j}$, but this is impossible because

$$\theta(c_{i_j} \cdot c_{i_j+1}) = \theta(c_{i_j}) + \theta(c_{i_j+1}) = \bar{n}/2 + \bar{n}/2 = \bar{n},$$

where $\bar{p}$ denotes the equivalence class of the element $p$ of $\mathbb{Z}_n$.

(3.5) Theorem. Let $\Gamma$ be a proper NEC group with signature

$$\left( g; \pm; [m_1, \ldots, m_r]; \left\{ (-)(-), \ldots, (-) \right\}^k \right)$$

and let $n$ be even. Then there exists a non-orientable surface-kernel homomorphism $\theta: \Gamma \to \mathbb{Z}_n$ if and only if:

(i) $m_i \neq n \forall i \in I, I = \{1, \ldots, \tau\}$;

(ii) if $g = 0$, $k = 1$, then $\text{l.c.m.}(m_1 \cdot \ldots \cdot m_\tau) = n$. 

Proof. If there is a non-orientable surface-kernel homomorphism \( \theta : \Gamma \to Z_n \), then, by (3.2), \( \theta(\Gamma^+) = Z_n \).

By Theorem 2 of [9] and Theorem 4 of [5], (i) and (ii) hold.

If we suppose that the elements of the signature \( \Gamma \) fulfill (i) and (ii), we define the homomorphism \( \theta : \Gamma \to Z_n \) in the following way:

if \( g \neq 0 \):

\[
\theta(a_i) = \bar{1}, \quad \theta(a_i) = \bar{n}, \quad i = 2, \ldots, g, \quad \theta(x_i) = \frac{\bar{n}}{m_i},
\]

\[
\theta(b_i) = \bar{1}, \quad \theta(b_i) = \bar{n}, \quad \theta(c_i) = \frac{\bar{n}}{2},
\]

\[
\theta(e_i) = -\sum_{i=1}^{\tau} \frac{n}{m_i}, \quad \theta(e_i) = \bar{n}, \quad i = 2, \ldots, k;
\]

if \( g = 0 \), \( k = 1 \):

\[
\theta(x_i) = \frac{\bar{n}}{m_i}, \quad \theta(c_i) = \frac{\bar{n}}{2}, \quad \theta(e_1) = -\sum_{i=1}^{\tau} \frac{n}{m_i};
\]

if \( g = 0 \), \( k > 1 \):

\[
\theta(x_i) = \frac{\bar{n}}{m_i}, \quad \theta(c_i) = \frac{\bar{n}}{2}, \quad \theta(e_1) = \bar{1},
\]

\[
\theta(c_2) = -1 - \sum_{i=1}^{\tau} \frac{n}{m_i}, \quad \theta(e_i) = \bar{n}, \quad i = 3, \ldots, k;
\]

in every case there is a \( \gamma \in \Gamma^+ \) such that \( \theta(\gamma) = \bar{1} \):

- if \( g \neq 0 \), \( \gamma = a_i \);
- if \( g = 0 \), \( k = 1 \), by (ii) \( \operatorname{lcm}(m_1, \ldots, m_\tau) = n \), for there exist integers \( \alpha_1, \ldots, \alpha_\tau \) such that \( \alpha_i n/m_1 + \cdots + \alpha_\tau n/m_\tau = 1 \), therefore \( \gamma = x_1^{\alpha_1} \cdots x_\tau^{\alpha_\tau} \);
  - if \( g = 0 \), \( k > 1 \), \( \gamma = e_1 \).

Therefore \( \theta(\Gamma^+) = Z_n \) and \( \theta \) is a non-orientable surface-kernel homomorphism.

(3.6) Theorem. Let \( \Gamma \) be a proper NEC group of signature

\[
\left\{ \begin{array}{c} g; -; [m_1, \ldots, m_\tau]; \\ \left( (-)^k, \ldots, (-) \right) \end{array} \right\}
\]

and let \( \theta \) be a non-orientable surface-kernel homomorphism \( \theta : \Gamma \to Z_n \) with \( n \) even. Then

(i) \( m_i \nmid n \forall i \in I, I = \{1, 2, \ldots, \tau\} \);

(ii) if \( g = 1, k = 0 \), then \( \operatorname{lcm}(m_1 \cdots m_\tau) = n \).
Proof. The Conditions (i) and (ii) hold by Theorem 2 of [9] and Theorem 4 of [5].

(3.7) Theorem. Let \( \Gamma \) be a proper NEC group of signature \((g; \tau; [m_1 \cdots m_r])\) and let \( n \) be odd. Then there exists a non-orientable surface-kernel homomorphism \( \theta: \Gamma \to \mathbb{Z}_n \) if and only if

(i) \( m_i \nmid n \ \forall \ i \in I, \ I = \{1, \ldots, \tau\}; \)
(ii) if \( g = 1 \), then \( \text{l.c.m.}(m_1 \cdots m_r) = n. \)

Proof. The necessity is similar to (3.6). Let us see the sufficiency. If we suppose that the elements of \( \Gamma \) fulfill (i) and (ii) we define the homomorphism \( \theta: \Gamma \to \mathbb{Z}_n \) in the following way: assume \( \Sigma_{i \in I} n/m_i = \bar{p}. \)

If \( g = 1 \) and \( p \) odd:

\[
\theta(x_i) = \frac{n}{m_i}, \quad \theta(a_1) = \frac{1}{2}(n - p).
\]

If \( g = 1 \) and \( p \) even:

\[
\theta(x_i) = \frac{n}{m_i}, \quad \theta(a_1) = -\frac{1}{2}p.
\]

If \( g > 1 \) and \( p \) odd:

\[
\theta(x_i) = \frac{n}{m_i}, \quad \theta(a_1) = \frac{n - 2p - 1}{2}, \quad \theta(a_i) = \bar{n}, \quad i > 2.
\]

If \( g > 1 \) and \( p \) even:

\[
\theta(x_i) = \frac{n}{m_i}, \quad \theta(a_1) = \frac{n + 1}{2}, \quad \theta(a_i) = \bar{n}, \quad i > 3.
\]

In every case there is \( \gamma \in \Gamma^+ \) such that \( \theta(\gamma) = 1 \): if \( g = 1 \), by (ii) \( \text{l.c.m.}(m_1 \cdots m_r) = n \), for there exist integers \( \alpha_1, \ldots, \alpha_r \) such that \( \alpha_1 n/m_1 + \cdots + \alpha_r n/m_r = 1 \), therefore \( \gamma = x_1^{\alpha_1} \cdots x_r^{\alpha_r}; \)

if \( g > 1 \) and \( p \) odd, \( \gamma = a_1^4 \cdot a_2^2; \)

if \( g > 1 \) and \( p \) even, \( \gamma = a_2^2. \)

Therefore \( \theta(\Gamma^+) = \mathbb{Z}_n \) and \( \theta \) is a non-orientable surface-kernel homomorphism.
4. Minimum genus. In this section we shall compute the minimum genus of a non-orientable Klein surface which has a cyclic group of automorphisms. We know by (2.4) that if $G$ is a group of automorphisms of a non-orientable Klein surface of genus $p \geq 3$, then $G \cong \Gamma / \Gamma_p$, where $\Gamma$ is a proper NEC group, and $\Gamma_p$ is a group of a non-orientable surface. Thus if $\text{order}(G) = n$, we have

$$n = 2\pi(p - 2)/|\Gamma|$$

and $p = 2 + (n/2\pi)|\Gamma|$, so we can reduce the problem to the search of a proper NEC group for which there exists a non-orientable surface-kernel homomorphism $\theta: \Gamma \to Z_n$ which minimizes $p$.

(4.1) Theorem. If $n = 1$, $q$ prime, then the minimum genus $p$ of a non-orientable Klein surface with a group of automorphisms isomorphic to $Z_n$ is:

- if $q = 2$, $p = 3$,
- if $q \neq 2$, $p = q$.

Proof. If $q = 2$, we consider an NEC group of signature

$$(0; +; [2, 2, 2]; \{(-)\}).$$

This group fulfills the conditions of Theorem (3.5), so

$$p - 2)/2 = 1/2, \quad \text{i.e.} \quad p = 3.$$ 

If $q \neq 2$, we have that an NEC group of signature $(1; -; [q, q])$ fulfills the conditions of Theorem (3.7), therefore it is the group of a surface and

$$p - 2)/q = 1 - 2/q, \quad \text{i.e.} \quad p = q.$$ 

Now let us see that $q$ is the minimum genus.

If we take any other NEC group $\Gamma$ with the conditions of Theorem (3.7), $\Gamma$ would have the signature $(g; -; [q, \ldots, q])$ and

$$p - 2)/q = g - 2 + \tau(1 - 1/q) = (\tau + g - 2) - \tau/q,$$

$$p = 2 + (\tau + g - 2)q - \tau,$$

since $\tau > 1$ if $g = 1$, and $g \geq 1$, then the following expression is always $\geq q$. 


(4.2) **Theorem.** If \( n = 2^\beta q_1^{\alpha_1} \cdots q_a^{\alpha_a} \), where \( 2 < q_1 < \cdots < q_a \) and \( q_1 \cdots q_a \) are prime, then the minimum genus \( p \) of a non-orientable Klein surface with group of automorphisms isomorphic to \( \mathbb{Z}_n \) is

\[
p = \begin{cases} 
  n/2 & \text{if } \beta = 1, \\
  n/2 + 1 & \text{if } \beta > 1.
\end{cases}
\]

**Proof.** If \( \beta = 1 \), we consider an NEC group \( \Gamma \) of signature

\[
(0; +; [2, n/2]; \{-\}).
\]

This group fulfills the conditions of Theorem (3.5), so

\[
\frac{p - 2}{n} = \frac{1}{2} - \frac{2}{n}, \quad \text{i.e.} \quad p = \frac{n}{2}.
\]

Now let us see that \( n/2 \) is the minimum genus. If we take any other group \( \Gamma \) in the conditions of (3.5), \( \Gamma \) would have the signature \((g; +; [m_1 \cdots m_k]; \{(-), \ldots, (-)})\), where \( m_i \nmid n \), so

\[
p = 2 + n(2g - 2 + k) + n \sum_{i=1}^{k} \left( 1 - \frac{1}{m_i} \right).
\]

If \( 2g - 2 + k > 0 \), then the genus would be greater than the one we had calculated before; if \( 2g - 2 + k \leq 0 \) as \( g \geq 0 \) and \( k \geq 1 \), we have that only the following cases can hold: \( g = 0, k = 1; g = 0, k = 2 \). If \( g = 0, k = 2 \), as \( |\Gamma| > 0 \) then \( \tau \geq 1 \).

\[
p = 2 + n \sum_{i=1}^{\tau} \left( 1 - \frac{1}{m_i} \right) > \frac{n}{2}.
\]

If \( g = 0, k = 1 \),

\[
p = 2 - n + n \sum_{i=1}^{\tau} \left( 1 - \frac{1}{m_i} \right),
\]

as \( p \geq 3, \tau \geq 2 \) necessarily. But \( \sum_{i=1}^{\tau} (1 - 1/m_i) < 2 \), since if it is greater or equal, the genus would be greater than the one calculated before. Thus \( \tau \) can only be 2 or 3. In both cases, keeping in mind that l.c.m. \( (m_1 \cdots m_k) = n \), one can check easily that the minimum genus one gets is \( \geq n/2 \).

If we take an NEC group \( \Gamma \) with signature

\[
\left\{ g; -; [m_1, \ldots, m_k]; \{(-), \ldots, (-) \} \right\}
\]
then

\[ p = 2 + n(g - 2 + k) + n \sum_{i \in I} \left(1 - \frac{1}{m_i}\right). \]

If \( g - 2 + k > 0 \), then the genus would be greater than the one we had calculated before. If \( g - 2 + k \leq 0 \), then, necessarily:

\[
\begin{align*}
g &= 1, \quad k = 1, \\
g &= 1, \quad k = 0, \\
g &= 2, \quad k = 0.
\end{align*}
\]

In the three cases, using Theorem (3.6), we have \( p \geq n/2 \).

If \( \beta \neq 1 \), we consider an NEC group \( \Gamma \) of signature

\[ (0; +; [n, 2]; \{(-)\}). \]

This group fulfills the conditions of Theorem (3.5), so

\[
\frac{p - 2}{n} = \frac{1}{2} - \frac{1}{n}, \quad \text{i.e.} \quad p = \frac{n}{2} + 1.
\]

If we take any other group \( \Gamma \), by (3.5) and (3.6) and operating in the same way as before, we get that \( n/2 + 1 \) is the minimum genus.

(4.3) \textbf{Theorem.} Let \( n = q_1^{r_1} \cdots q_a^{r_a} \), with \( q_1 < q_2 < \cdots < q_a \) being prime numbers and \( q_1 \neq 2 \). Then the minimum genus \( p \) of a non-orientable Klein surface with group of automorphisms isomorphic to \( Z_n \) is

\[
\begin{align*}
p &= 2 - q_1 + n - n/q_1 \quad \text{if } r_1 = 1, \\
p &= 1 + n - n/q_1 \quad \text{if } r_1 > 1.
\end{align*}
\]

\textit{Proof.} Similar to the proof of the above theorem, bearing in mind (3.7).

The following corollary has also been obtained by W. Hall in [4]. The corresponding result for orientable Klein surfaces without boundary is due to A. Wiman [10].
(4.4) Corollary. The maximum order for an automorphism of a non-orientable Klein surface of genus $p \geq 3$ is

$$
2p \quad \text{if } p \text{ is odd},
$$

$$
2(p - 1) \quad \text{if } p \text{ is even},
$$

and it is always reached.

Proof. Given a non-orientable Klein surface of genus $p \geq 3$, we have by Theorems (4.1), (4.2) and (4.3) that the genus $p$ satisfies $p \geq n/2$, i.e. $2p \geq n$. If $p = n/2$, then $n = 2$ and $n \neq 4$, so that bound is only reached when $p$ is odd: in fact, given an NEC group $\Gamma$ of signature $(0; +; [2, p]; \{(\cdot)\})$, by (3.5) there is a non-orientable Klein surface of genus $p$, with a group of automorphisms isomorphic to $Z_{2p}$.

If $p$ is even, the maximum order for an automorphism is $2(p - 1)$, since given an NEC group $\Gamma$ of signature $(0; +; [2(p - 1), 2]; \{(\cdot)\})$, by (3.5) there is a non-orientable Klein surface of genus $p$, with a group of automorphisms isomorphic to $Z_{2(p-1)}$.

If $p$ is the topological genus of a compact non-orientable Klein surface without boundary, the algebraic genus is $g = p - 1$.

If we express the above corollary in terms of algebraic genus, these bounds are the same as the ones obtained by C. L. May in [6] for the order of an automorphism of an orientable bordered Klein surface.

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References


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