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CERTAIN SUBVARIETIES OF A WEAKLY PSEUDOCONVEX
DOMAIN**

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EXTENDING BOUNDED HOLOMORPHIC FUNCTIONS FROM CERTAIN SUBVARIETIES OF A WEAKLY PSEUDOCONVEX DOMAIN

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Let D be a weakly pseudoconvex domain in C^n with C^∞ -boundary and Δ be a hypersurface in D which intersects ∂D transversally. If $\partial\Delta$ consists of strictly pseudoconvex boundary points of D , then any bounded holomorphic function in Δ can be extended to a bounded holomorphic function in D .

1. Introduction. G. M. Henkin [5] proved that any bounded holomorphic function defined on an analytic closed submanifold in general position in a strongly pseudoconvex domain can be continued to a bounded holomorphic function in the entire domain. The related results have been given by the author [1] and J. E. Fornæss [4]. In this paper, we extend this problem to the weakly pseudoconvex case. Our proof depends on the integral formula constructed by E. L. Stout [8], and the kernel function constructed by F. Beatrous, Jr. [3] which was used to obtain a Hölder estimate for solutions to $\bar{\partial}$ -problem in weakly pseudoconvex domains.

2. Let Ω be a bounded domain in C^{N+1} with C^∞ -boundary. We shall denote by $O(\Omega)$ the space of holomorphic functions in Ω . We shall also denote by $H^\infty(\Omega)$ the space of bounded holomorphic functions on Ω and by $A(\Omega)$ the subspace of $H^\infty(\Omega)$ of functions which extend continuously to $\bar{\Omega}$.

DEFINITION 1. (R. M. Range [7]) A point $\lambda \in \partial\Omega$ is a strictly pseudoconvex boundary point if there are a neighborhood U of λ and a C^∞ function $\phi: U \rightarrow \mathbb{R}$ such that:

- (a) $U \cap \Omega = \{z \in U: \phi(z) < 0\}$;
- (b) $\Sigma(\partial^2\phi(\lambda)/\partial z_i \partial \bar{z}_j)w_i \bar{w}_j > 0$ for all $w \in C^{N+1} - (0)$;
- (c) $d\phi(\lambda) \neq 0$.

The set of strictly pseudoconvex boundary points of Ω will be denoted by $S(\Omega)$. It follows from Definition 1 that $S(\Omega)$ is an open subset of the boundary $\partial\Omega$.

Let D be a pseudoconvex domain in C^{N+1} with C^∞ -boundary. We fix a function $F \in O(\bar{D})$, $F \not\equiv 0$. Then F is holomorphic in a domain \tilde{D} with $\bar{D} \subset \tilde{D}$. We set $\tilde{\Delta} = \{z \in \tilde{D}: F(z) = 0\}$ and $\Delta = \tilde{\Delta} \cap D$. We make the following assumptions:

- (a) Δ is a non-empty connected set;
- (b) $dF \neq 0$ on $\partial\Delta$;
- (c) $\tilde{\Delta}$ meets ∂D transversally;
- (d) $\partial\Delta \subset S(D)$.

In this setting, we have the following:

THEOREM. *Under hypotheses (a)–(d), there exists a continuous linear extension operator $L: H^\infty(\Delta) \rightarrow H^\infty(D)$. Moreover if Δ has no singular points then $L(A(\Delta)) \subset A(D)$.*

In order to prove this theorem, we use the function $\Phi(\zeta, z)$ in the following proposition which was constructed by F. Beatrous, Jr. ([3], Theorem 2.1).

PROPOSITION 1. *Let k be a positive integer ($k \geq 3$). There are a neighborhood U of $\partial\Delta$, a smooth positive function r on U , and a C^k function Φ on $U \times \bar{D}$ with the following properties:*

- (i) *For each $\zeta \in U$, $\Phi(\zeta, \cdot) \in C^k(\bar{D}) \cap O(D)$.*
- (ii) *$G(\zeta, z) = \Phi(\zeta, z)/T(\zeta, z)$ is a non-vanishing C^k function on $\{(\zeta, z) \in U \times \bar{D}: |\zeta - z| \leq r(z)\}$.*
- (iii) *$\Phi(\zeta, z) \neq 0$ if $|\zeta - z| \geq r(z)$.*
- (iv) *$\operatorname{Re} T(\zeta, z) > \rho(\zeta) - \rho(z) + r(z)|\zeta - z|^2$ if $|\zeta - z| \leq r(\zeta)$,*
where ρ is the defining function for the domain D constructed by F. Beatrous, Jr., and

$$T(\zeta, z) = -2 \sum_i \frac{\partial \rho}{\partial z_i}(\zeta)(z_i - \zeta_i) - \sum_{i,j} \frac{\partial^2 \rho}{\partial z_i \partial z_j}(\zeta)(z_i - \zeta_i)(z_j - \zeta_j).$$

Moreover we can extend $\Phi(\zeta, z)$ to a C^k function on a neighborhood of $\partial D \times \bar{D}$, holomorphic in z such that $\Phi(\zeta, z)$ satisfies $\Phi(\zeta, z) = \sum_{j=1}^{N+1} P_j(\zeta, z)(\zeta_j - z_j)$, and $\Phi(\zeta, z) \neq 0$ if $\rho(\zeta) > \rho(z)$, where $P_j(\zeta, z)$ is a C^k function on a neighborhood of $\partial D \times \bar{D}$, holomorphic in z .

Let $D_\nu = \{z \in D: \rho(z) < -\varepsilon_\nu\}$ and $\Delta_\nu = \Delta \cap D_\nu$, where $\{\varepsilon_\nu\}$ is a sequence of sufficiently small strictly decreasing positive numbers converging to 0. By E. L. Stout [8], we have the following:

PROPOSITION 2. *If $f \in H^\infty(\Delta)$, then the following formula holds for all $z \in \Delta_\nu$ and all sufficiently large ν :*

$$(1) \quad f(z) = \int_{\partial\Delta_\nu} f(\zeta) \frac{\tilde{\Psi}(\zeta, z) \tilde{\omega}_F}{\Phi(\zeta, z)^N \|\text{grad } F(\zeta)\|} = \int_{\partial\Delta_\nu} \frac{f(\zeta) K(\zeta, z)}{\Phi(\zeta, z)^N},$$

where $\tilde{\Psi}(\zeta, z)$ is a $C^{k-1}(0, N-1)$ form in a neighborhood of $\partial D \times \bar{D}$ and, for each ζ near ∂D , coefficients of $\tilde{\Psi}(\zeta, \cdot)$ are holomorphic in D . One could arrange for $\tilde{\Psi}(\zeta, \cdot)$ to be holomorphic on \bar{D} if \bar{D} were assumed to have a pseudoconvex neighborhood basis. $\tilde{\omega}_F$ is given by

$$\tilde{\omega}_F = \sum_{j=1}^{N+1} (-1)^{j-1} \bar{F}_j d\zeta_1 \wedge \cdots \wedge \widehat{d\zeta_j} \wedge \cdots \wedge d\zeta_{N+1},$$

where $F_j = \partial F / \partial z_j$, $j = 1, \dots, N+1$, and \wedge means the symbol is to be omitted. Therefore $K(\zeta, z)$ is a $C^{k-1}(N, N-1)$ form on a neighborhood of $\partial D \times \bar{D}$ and for each ζ near ∂D , coefficients of $K(\zeta, \cdot)$ are holomorphic in D .

We set

$$H_\nu(z) = \int_{\partial\Delta_\nu} \frac{f(\zeta) K(\zeta, z)}{\Phi(\zeta, z)^N} \quad \text{for } z \in \bar{D}_\nu | \partial\Delta_\nu,$$

and

$$L(f)(z) = H(z) = \lim_{\nu \rightarrow \infty} H_\nu(z) \quad \text{for } z \in \bar{D} | \partial\Delta.$$

LEMMA 1. $H(z)$ is holomorphic on D and $H(z) = f(z)$ for all $z \in \Delta$.

Proof. For $z \in W \subset D_\nu$, $\nu > \mu \geq \nu_0$, we have

$$H_\nu(z) - H_\mu(z) = \int_{\partial(\Delta_\nu - \Delta_\mu)} \frac{f(\zeta) K(\zeta, z)}{\Phi(\zeta, z)^N} = \int_{\Delta_\nu - \Delta_\mu} f(\zeta) \bar{\partial}_\zeta \left(\frac{K(\zeta, z)}{\Phi(\zeta, z)^N} \right).$$

Since the form $\bar{\partial}_\zeta(K(\zeta, z)/\Phi(\zeta, z)^N)$ is bounded for $\zeta \in \Delta_\nu - \Delta_\mu$ and $z \in W$, there exists a constant K such that

$$|H_\nu(z) - H_\mu(z)| \leq K \sup_{\zeta \in \Delta} |f(\zeta)| \text{Vol}(\Delta_\nu - \Delta_\mu).$$

Hence $H_\nu(z)$ converges locally uniformly on D . Therefore $H(z)$ is holomorphic in D . By Proposition 2, $H(z) = f(z)$ for all $z \in \Delta$. Therefore Lemma 1 is proved.

We want to show that $H(z) \in H^\infty(D)$. Let $S_{z^0, \sigma} = \{z: |z - z^0| < \sigma\}$. Let $z^0 \in \partial\Delta$. Then there exist a constant $\sigma_1 > 0$ and a biholomorphic

change of coordinates on a neighborhood of z^0 such that ρ is strictly convex in a neighborhood of $\bar{D} \cap \bar{S}_{z^0, \sigma_1}$, $\Delta \cap S_{z^0, \sigma_1} = \{z \in S_{z^0, \sigma_1} : z_{N+1} = 0\}$ and $\partial\rho(z^0)/\partial z_1 \neq 0$. Let $0 < \sigma_2 < \sigma_1$. Let $z \in S_{z^0, \sigma_2} \cap D_\nu$. We write

$$H_\nu(z) = \int_{\partial\Delta_\nu \cap S_{z^0, \sigma_1}} \frac{f(\zeta)K(\zeta, z)}{\Phi(\zeta, z)^N} + \int_{\partial\Delta_\nu |_{S_{z^0, \sigma_1}}} \frac{f(\zeta)K(\zeta, z)}{\Phi(\zeta, z)^N}.$$

Then

$$\left| \int_{\partial\Delta_\nu |_{S_{z^0, \sigma_1}}} \frac{f(\zeta)K(\zeta, z)}{\Phi(\zeta, z)^N} \right| \leq \gamma_1 \sup_{\zeta \in \Delta_\nu} |f(\zeta)|,$$

where γ_1 depends only on D and Δ . We set

$$\tilde{H}_\nu(z) = \int_{\partial\Delta_\nu \cap S_{z^0, \sigma_1}} \frac{f(\zeta)K(\zeta, z)}{\Phi(\zeta, z)^N}.$$

Then it is sufficient to show that $|\tilde{H}_\nu(z)| \leq \gamma_2 \sup_{\zeta \in \Delta} |f(\zeta)|$, where γ_2 depends only on D and Δ .

We consider the system of equations for $\zeta^0 = (\zeta_1^0, \dots, \zeta_{N+1}^0)$ of the following form for $z \in S_{z^0, \sigma_2}$:

$$(2) \quad \begin{aligned} \sum_{i=1}^{N+1} \frac{\partial\rho}{\partial\zeta_i}(\zeta^0)(\zeta_i^0 - z_i) &= 0, \\ \zeta_i^0 &= z_i \quad (i = 2, 3, \dots, N), \\ \zeta_{N+1}^0 &= 0 \end{aligned}$$

Then we have the following lemma which was proved by G. M. Henkin [5]. But we give the proof of E. Amar [2] which is simpler than Henkin's.

LEMMA 2. *There exist positive constants $\sigma_3 (< \sigma_2)$, γ_3 and γ_4 , depending only on D and Δ , such that for any $\sigma \leq \sigma_3$ and any $z \in S_{z^0, \sigma/2}$ there exists a unique solution $\zeta^0 = \zeta^0(z)$ of system (2) which belongs to the set $S_{z^0, \sigma} \cap \tilde{\Delta}$. Here the point $\zeta^0 = \zeta^0(z)$ has the following properties:*

$$(3) \quad |z - \zeta^0|^2 \leq \frac{1}{\gamma_3} [\rho(z) - \rho(\zeta^0)],$$

$$(4) \quad |z - \zeta^0|^2 \geq |z_{N+1}|^2 \geq \gamma_4 [\rho(z) - \rho(\zeta^0)],$$

$$\zeta^0 = z \quad \text{for any } z \in S_{z^0, \sigma/2} \cap \tilde{\Delta}.$$

Proof. From (2), we have

$$z_1 = \zeta_1 - \frac{(\partial\rho(\zeta)/\partial z_{N+1})z_{N+1}}{\partial\rho(\zeta)/\partial z_1} = \zeta_1 - a(\zeta)z_{N+1},$$

where $a(\zeta)$ is C^∞ in a neighborhood of z^0 . There exists $\sigma_3 > 0$ such that for any $\zeta \in B(z^0, \sigma_3)$, $z \in B(z^0, \sigma_3)$ we have $|\nabla a(\zeta)||z_{N+1}| \leq \frac{1}{2}$. We set by recurrence that

$$\begin{aligned}\zeta_1^{(1)} &= z_1, \\ \zeta^{(j)} &= (\zeta_1^{(j)}, z_2, \dots, z_N, z_{N+1}^0), \\ \zeta_1^{(j)} &= z_1 + a(\zeta^{(j-1)})z_{N+1}.\end{aligned}$$

If z and $\zeta^{(j)}$ are in $B(z^0, \sigma_3)$, then

$$|\zeta_i^{(j)} - \zeta_i^{(j-1)}| < |z_{N+1}| |\nabla a| |\zeta_i^{(j-1)} - \zeta_i^{(j-2)}| < \frac{1}{2} |\zeta_i^{(j-1)} - \zeta_i^{(j-2)}|.$$

Therefore $\zeta^{(j)}$ converges. Then $\lim_{j \rightarrow \infty} \zeta^{(j)} = \zeta^0$ is the solution of (2). The strict convexity of the function ρ and equations (2) imply the inequalities:

$$(5) \quad \rho(\zeta^0) - \rho(z) + \gamma_3 |z^0 - z|^2 \leq 2 \operatorname{Re} \sum_{i=1}^{N+1} \frac{\partial \rho}{\partial \zeta_i}(\zeta^0) (\zeta_i^0 - z_i) = 0,$$

$$(6) \quad \rho(\zeta^0) - \rho(z) + \gamma_4 |z^0 - z|^2 \geq 2 \operatorname{Re} \sum_{i=1}^{N+1} \frac{\partial \rho}{\partial \zeta_i}(\zeta^0) (\zeta_i^0 - z_i) = 0,$$

where $z \in S_{z^0, \sigma_3/2}$. From (5) we have (3). From (6) we have

$$|z^0 - z|^2 \geq \frac{1}{\gamma_4} [\rho(z) - \rho(\zeta^0)].$$

But

$$|z^0 - z|^2 \geq |z_{N+1}|^2 + |\zeta_1^0 - z_1|^2 \leq \gamma_4' |z_{N+1}|^2.$$

Therefore we have $|z_{N+1}|^2 \geq (1/\gamma_4)[\rho(z) - \rho(\zeta^0)]$. Therefore Lemma 2 is proved.

For any $z \in \bar{D}_v \cap S_{z^0, \sigma_2} | \partial \Delta_v$ and any vector $w = (w_1, \dots, w_{N+1}) \neq 0$, we have

$$(7) \quad \frac{d\tilde{H}_v(z + \lambda w)}{d\lambda} \Big|_{\lambda=0} = \int_{\partial \Delta_v \cap S_{z^0, \sigma_1}} \frac{f(\zeta) \sum_{j=1}^{N+1} (\partial/\partial z_j) K(\zeta, z) w_j}{\Phi(\zeta, z)^{N+1}} - \int_{\partial \Delta_v \cap S_{z^0, \sigma_1}} \frac{N \sum_{j=1}^{N+1} (\partial \Phi(\zeta, z)/\partial z_j) w_j K(\zeta, z)}{\Phi(\zeta, z)^{N+1}}.$$

LEMMA 3. Let $f(\zeta) \in H^\infty(\Delta)$. Then for any point $z^0 \in \partial\Delta$ and any point $z \in \partial(S_{z^0, \sigma} \cap D_\nu) \setminus \partial\Delta_\nu$ ($\sigma < (\sigma_3/2)$), we have

$$\left| \frac{d\tilde{H}_\nu(\zeta^0 + \lambda(z - \zeta^0))}{d\lambda} \Big|_{\lambda=1} \right| \leq \gamma_5 \sup_{\zeta \in \Delta} |f(\zeta)|,$$

where $\zeta^0 = \zeta^0(z)$ and γ_5 depends only on D and Δ .

Proof. We set $\varepsilon = |z_{N+1}|$, where

$$z = (z_1, \dots, z_{N+1}) \in \partial(S_{z^0, \sigma} \cap D_\nu) \setminus \partial\Delta_\nu.$$

Then Lemma 2 implies the inequalities

$$\varepsilon \leq |\zeta^0 - z| \leq \left\{ \frac{\rho(z) - \rho(\zeta^0)}{\gamma_3} \right\}^{1/2} \leq \frac{\varepsilon}{(\gamma_3 \gamma_4) 1/2}.$$

Since $\sum_{i=1}^{N+1} (\partial\rho/\partial\zeta_i)(\zeta^0)(\zeta_i^0 - z_i) = 0$, it follows that

$$\begin{aligned} \left| \sum_{i=1}^{N+1} \frac{\partial\Phi}{\partial z_i}(\zeta, z)(\zeta_i^0 - z_i) \right| &= \left| \sum_{i=1}^{N+1} \left(\frac{\partial\Phi(\zeta, z)}{\partial z_i} + 2 \frac{\partial\rho}{\partial\zeta_i}(\zeta^0) \right) (\zeta_i^0 - z_i) \right| \\ &\leq \left| \sum_{i=1}^{N+1} \left(\frac{\partial\Phi}{\partial z_i}(\zeta, z) - \frac{\partial\Phi}{\partial z_i}(\zeta^0, z) + O(|\zeta^0 - z|) \right) (\zeta_i^0 - z_i) \right| \\ &\leq \gamma_6 \varepsilon (|\zeta - z| + \varepsilon). \end{aligned}$$

Here we have used the equation

$$\frac{\partial\Phi}{\partial z_i}(\zeta^0, z) = -2 \frac{\partial\rho}{\partial\zeta_i}(\zeta^0) + O(|\zeta^0 - z|).$$

By (7), we have

$$\begin{aligned} \left| \frac{d\tilde{H}_\nu(\zeta^0 + \lambda(z - \zeta^0))}{d\lambda} \Big|_{\lambda=1} \right| &= \left| \frac{d\tilde{H}_\nu(z + \lambda(z - \zeta^0))}{d\lambda} \Big|_{\lambda=0} \right| \\ &\leq \gamma_7 \int_{\partial\Delta_\nu \cap S_{z^0, \sigma_1}} \frac{|f(\zeta)| |z - \zeta^0|}{|\Phi(\zeta, z)|^{N+1}} d\lambda + \gamma_8 \int_{\partial\Delta_\nu \cap S_{z^0, \sigma_1}} \frac{|f(\zeta)| \varepsilon (|\zeta - z| + \varepsilon)}{|\Phi(\zeta, z)|^{N+1}} d\lambda. \end{aligned}$$

We can choose coordinates $(\eta_1(\zeta), \dots, \eta_{N+1}(\zeta))$ in S_{z^0, σ_3} such that $\eta_1(\zeta) = \rho(\zeta) - \rho(z) + i \operatorname{Im} \Phi(\zeta, z)$. Then

$$|\Phi(\zeta, z)| \geq \gamma_9 \left[(t_1 + |\zeta - z|^2)^2 + t_2^2 \right]^{1/2}$$

and

$$|\zeta - z| \geq \gamma_{10}(t_1^2 + \cdots + t_{2N}^2 + \varepsilon^2)^{1/2} \geq \gamma_{11}|\zeta - z|,$$

where we have written $\eta_i(\zeta) = t_{2i-1} + \sqrt{-1}t_{2i}$ ($i = 1, 2, \dots, N+1$). Then we have

$$\begin{aligned} & \left| \frac{dF(\zeta^0 + \lambda(z - \zeta^0))}{d\lambda} \right|_{\lambda=1} \leq \gamma_{12} \sup_{\zeta \in \Delta} |f(\zeta)| \\ & \times \left\{ \varepsilon \int_{\substack{t_2^2 + \cdots + t_{2N}^2 \leq 1 \\ t_1 \geq 0}} \frac{dt_2 dt_3 \cdots dt_{2N}}{\left[(t_1 + t_2^2 + \cdots + t_{2N}^2 + \varepsilon^2)^2 + t_2^2 \right]^{N/2}} \right. \\ & + \varepsilon \int_{\substack{t_2^2 + \cdots + t_{2N}^2 \leq 1 \\ t_1 \geq 0}} \frac{(t_1^2 + t_2^2 + \cdots + t_{2N}^2 + \varepsilon^2)^{1/2} dt_2 \cdots dt_{2N}}{\left[(t_1 + t_2^2 + \cdots + t_{2N}^2 + \varepsilon^2)^2 + t_2^2 \right]^{N+1/2}} \\ & \left. + \varepsilon^2 \int_{\substack{t_2^2 + \cdots + t_{2N}^2 \leq 1 \\ t_1 \geq 0}} \frac{dt_2 \cdots dt_{2N}}{\left[(t_1 + t_2^2 + \cdots + t_{2N}^2 + \varepsilon^2)^2 + t_2^2 \right]^{N+1/2}} \right\} \end{aligned}$$

(by G. M. Henkin [5])

$$\leq \gamma_{13} \sup_{\zeta \in \Delta} |f(\zeta)|.$$

We want to have

$$\sup_{z \in D_\nu} |H_\nu(z)| \leq \gamma_{14} \sup_{\zeta \in \Delta} |f(\zeta)|,$$

where γ_{14} depends only on D and Δ . We shall denote by $(\partial\Delta_\nu)_\sigma$ the σ -neighborhood of $\partial\Delta_\nu$. Since the function $H_\nu(z)$ is holomorphic at all points $z \in \bar{D}_\nu \setminus \partial\Delta_\nu$, we have

$$\sup_{z \in D_\nu} |H_\nu(z)| \leq \sup_{z \in \partial D_\nu \setminus (\partial\Delta_\nu)_\sigma} |H_\nu(z)| + \sup_{z \in [(\partial\Delta_\nu)_\sigma \setminus \partial\Delta_\nu] \cap \partial D_\nu} |H_\nu(z)|.$$

We obtain

$$\begin{aligned} & \sup_{z \in \partial D_\nu \setminus (\partial\Delta_\nu)_\sigma} |H_\nu(z)| \\ & \leq \gamma_{15} \left[\int_{t_2^2 + \cdots + t_{2N}^2 \leq 1} \frac{dt_2 \cdots dt_{2N}}{\left[(t_2^2 + \cdots + t_{2N}^2 + \sigma^2)^2 + t_2^2 \right]^{N/2}} \right] \sup_{\zeta \in \Delta} |f(\zeta)| \\ & \leq \gamma_{16} \sup_{\zeta \in \Delta} |f(\zeta)|. \end{aligned}$$

Let $\sigma < 16\sigma_3$. We now fix $z \in [(\partial\Delta_\nu)_\sigma - \partial\Delta_\nu] \cap \partial D_\nu$. We take ν so large that one can find $z^0 \in \partial\Delta$ such that $z \in S_{z^0, 2\sigma}$. Then by Lemma 2, there exists a solution $\zeta^0 = \zeta^0(z)$ of system (2) belonging to the set $S_{z^0, 0.4\sigma} \cap \tilde{\Delta}$ and satisfying the inequalities

$$(8) \quad \gamma_3 |z_{N+1}|^2 \leq \rho(z) - \rho(\zeta^0) \leq |z_{N+1}|^2 / \gamma_4.$$

Let $T_\nu = \{\lambda \in C: z(\lambda) = \zeta^0 + \lambda(z - \zeta^0) \in D_\nu \cap S_{z^0, 0.4\sigma}\}$. T_ν is a convex domain containing $\lambda = 0$. For any λ we have

$$\sum_{i=1}^{N+1} \frac{\partial \rho}{\partial \zeta_i}(\zeta^0)(\zeta_i^0 - z_i(\lambda)) = 0.$$

From this we have

$$|z(\lambda) - \zeta^0|^2 \leq \frac{1}{\gamma_{17}} \{\rho(z(\lambda)) - \rho(\zeta^0)\}.$$

Hence for $\lambda \in \partial T_\nu$ with $z(\lambda) \in \partial D_\nu$, we obtain

$$\begin{aligned} |z(\lambda) - z^0| &\leq |z(\lambda) - \zeta^0| + |\zeta^0 - z^0| \\ &\leq \frac{1}{\sqrt{\gamma_{17}}} (\rho(z(\lambda)) - \rho(\zeta^0))^{1/2} + \frac{\sigma_3}{4} \\ &= \frac{1}{\sqrt{\gamma_{17}}} (\rho(z) - \rho(\zeta^0))^{1/2} + \frac{\sigma_3}{4} \\ &\leq \frac{\varepsilon}{\sqrt{\gamma_4 \gamma_{17}}} + \frac{\sigma_3}{4} \leq \frac{\sigma}{\sqrt{\gamma_4 \gamma_{17}}} + \frac{\sigma_3}{4}. \end{aligned}$$

We impose the further restriction that the constant $\sigma < \sigma_3 \sqrt{\gamma_4 \gamma_{17}} / 4$. Then $|z(\lambda) - z^0| < \sigma_3 / 2$. Therefore $z(\lambda) \in S_{z^0, 0.5\sigma_3/2}$. Since the point $\zeta^0(z)$ satisfies system (2) with any $z(\lambda)$ satisfying $\lambda \in \partial T_\nu$ and $z(\lambda) \in \partial D_\nu$, it follows that $\zeta^0(z(\lambda)) = \zeta^0(z)$ for any $\lambda \in \partial T_\nu$ with $z(\lambda) \in \partial D_\nu$. Moreover

$$\begin{aligned} \frac{|\lambda| \varepsilon}{\gamma_3 \gamma_4} &\geq \frac{|\lambda|}{\gamma_3} (\rho(z) - \rho(\zeta^0)) \geq |\lambda| |z - \zeta^0| = |z(\lambda) - \zeta^0| \\ &\geq (\gamma_4 (\rho(z(\lambda)) - \rho(\zeta^0)))^{1/2} \\ &= [\gamma_4 (\rho(z) - \rho(\zeta^0))]^{1/2} \geq (\gamma_3 \gamma_4)^{1/2} \varepsilon. \end{aligned}$$

Therefore $|\lambda| \geq \gamma_3 \gamma_4$ for any $\lambda \in \partial T_\nu$ with $z(\lambda) \in \partial D_\nu$. If $\lambda \in \partial T_\nu$ and $z(\lambda) \in S_{z^0, 0.4\sigma}$, there exists $\gamma_{18} > 0$ such that $|\lambda| \geq \gamma_{18}$. Let $\gamma_{19} = \min(\gamma_3 \gamma_4, \gamma_{18})$. Then

$$(9) \quad |\lambda| \geq \gamma_{19} \quad \text{for any } \lambda \in \partial T_\nu.$$

By Lemma 3, we have

$$(10) \quad \left| \frac{dH_\nu(\zeta^0 + t(z(\lambda) - \zeta^0))}{dt} \right|_{t=1} \leq \gamma_5 \sup_{\zeta \in \Delta} |f(\zeta)|$$

for any $\lambda \in \partial T_\nu$. We note that

$$\frac{dH_\nu(\zeta^0 + t(z(\lambda) - \zeta^0))}{dt} \Big|_{t=1} = \frac{dH_\nu(\zeta^0 + \lambda(z - \zeta^0))}{d\lambda}.$$

From (8), (9) and (10), we have

$$\left| \frac{dH_\nu(\zeta^0 + \lambda(z - \zeta^0))}{d\lambda} \right| \leq \frac{\gamma_5}{|\lambda|} \sup_{\zeta \in \Delta} |f(\zeta)| \leq \frac{\gamma_5}{\gamma_{19}} \sup_{\zeta \in \Delta} |f(\zeta)|$$

for any $\lambda \in \partial T_\nu$. Since the function $dH_\nu(\zeta^0 + \lambda(z - \zeta^0))/d\lambda$ is holomorphic in λ for all $\lambda \in \bar{T}_\nu$, it follows that

$$\sup_{\lambda \in T_\nu} \left| \frac{dH_\nu(\zeta^0 + \lambda(z - \zeta^0))}{d\lambda} \right| \leq \frac{\gamma_5}{\gamma_{19}} \sup_{\zeta \in \Delta} |f(\zeta)|.$$

Consequently

$$|H_\nu(z) - H_\nu(\zeta^0)| = \left| \int_0^1 \frac{d}{d\lambda} H_\nu(\zeta^0 + \lambda(z - \zeta^0)) d\lambda \right| \leq \frac{\gamma_5}{\gamma_{19}} \sup_{\zeta \in \Delta} |f(\zeta)|.$$

From (8), $\zeta^0 \in \Delta_\nu$. Since $H_\nu(\zeta^0) = f(\zeta^0)$, we have

$$|H_\nu(z)| \leq \left(\frac{\gamma_5}{\gamma_{19}} + 1 \right) \sup_{\zeta \in \Delta} |f(\zeta)|.$$

Therefore

$$\sup_{z \in D_\nu} |H_\nu(z)| \leq \gamma_{20} \sup_{\zeta \in \Delta} |f(\zeta)|.$$

Hence

$$\sup_{z \in D} |H(z)| \leq \gamma_{20} \sup_{\zeta \in \Delta} |f(\zeta)|.$$

The next step is to show that if $f \in A(\Delta)$, then also $H(z) = L(f)(z) \in A(D)$. In this case we have assumed that Δ has no singular points. Therefore by N. Kerzman [6], there exists a sequence $\{f_k\}_{k=1}^\infty$ of functions holomorphic in a neighborhood of $\bar{\Delta}$ in $\tilde{\Delta}$ such that $\|f_k - f\|_\Delta \rightarrow 0$ when $k \rightarrow \infty$. By the continuity of L it suffices to prove that each Lf_k is in $A(D)$. Hence we can suppose f is holomorphic in $\bar{\Delta}'$ ($\bar{\Delta} \subset \Delta' \subset \bar{\Delta} \subset \tilde{\Delta}$).

Let $z^0 \in \partial\Delta$ and let $z \in S_{z^0, \sigma/2} \cap (\bar{D}_\nu | \partial\Delta_\nu)$. By Stokes' formula, we have

$$\begin{aligned} H_\nu(z) &= \int_{\partial\Delta_\nu} \frac{f(\zeta)K(\zeta, z)}{\Phi(\zeta, z)^N} \\ &= \int_{\partial\Delta'} \frac{f(\zeta)K(\zeta, z)}{\Phi(\zeta, z)^N} - \int_{\Delta' - \Delta_\nu} f(\zeta) \bar{\partial}_\zeta \left(\frac{K(\zeta, z)}{\Phi(\zeta, z)^N} \right) \\ &= \int_{\partial\Delta'} \frac{f(\zeta)K(\zeta, z)}{\Phi(\zeta, z)^N} - \int_{(\Delta' - \Delta_\nu) \cap S_{z^0, 2\sigma}} f(\zeta) \bar{\partial}_\zeta \left(\frac{K(\zeta, z)}{\Phi(\zeta, z)^N} \right) \\ &\quad - \int_{(\Delta' - \Delta_\nu) \setminus S_{z^0, 2\sigma}} f(\zeta) \bar{\partial}_\zeta \left(\frac{K(\zeta, z)}{\Phi(\zeta, z)^N} \right). \end{aligned}$$

The first and the third term on the left are continuous in z^0 . Therefore it is sufficient to show that, if we set

$$F_\nu(z) = \int_{(\Delta' - \Delta_\nu) \cap S_{z^0, 2\sigma}} f(\zeta) \bar{\partial}_\zeta \left(\frac{K(\zeta, z)}{\Phi(\zeta, z)^N} \right),$$

then $F_\nu(z)$ is continuous at z^0 .

LEMMA 4. *Let $z \in S_{z^0, \sigma/2} \cap (\bar{D}_\nu | \partial\Delta)$. Then*

$$\left| \frac{dF_\nu(\zeta^0 + \lambda(z - \zeta^0))}{d\lambda} \right|_{\lambda=1} \leq \gamma_{21} \varepsilon |\log \varepsilon| \sup_{\zeta \in \bar{\Delta}} |f(\zeta)|.$$

Proof. We can write

$$\begin{aligned} F_\nu(z) &= \int_{(\Delta' - \Delta_\nu) \cap S_{z^0, 2\sigma}} f(\zeta) \frac{A(\zeta, z)}{\Phi(\zeta, z)^N} \\ &\quad + \int_{(\Delta' - \Delta_\nu) \cap S_{z^0, 2\sigma}} \frac{f(\zeta) \sum_{j=1}^{N+1} (\zeta_j - z_j) B_j(\zeta, z)}{\Phi(\zeta, z)^{N+1}} \end{aligned}$$

where $A(\zeta, z)$ and $B_j(\zeta, z)$ are (N, N) forms which are continuous in ζ and holomorphic in z . Therefore

$$\begin{aligned} \left| \frac{dF_\nu(\zeta^0 + \lambda(z - \zeta^0))}{d\lambda} \right|_{\lambda=1} &\leq \gamma_{22} \int_{(\Delta' - \Delta_\nu) \cap S_{z^0, 2\sigma}} \frac{\varepsilon}{\Phi(\zeta, z)^{N+1}} d\lambda \\ &\quad + \gamma_{23} \int_{(\Delta' - \Delta_\nu) \cap S_{z^0, 2\sigma}} \frac{|\zeta - z| \varepsilon (|\zeta - z| + \varepsilon)}{|\Phi(\zeta, z)|^{N+2}} d\lambda \end{aligned}$$

(by the estimates of G. M. Henkin [5])

$$\leq \gamma_{24} \varepsilon |\log \varepsilon| \sup_{\zeta \in \bar{\Delta}} |f(\zeta)|.$$

Therefore Lemma 4 is proved.

Using the method of Henkin [5], we have

$$|F_\nu(z) - F_\nu(z^0)| \leq \gamma_{25} \sigma |\log \sigma| \sup_{\zeta \in \Delta'} |f(\zeta)| + \gamma_{26} \sigma \sup_{\zeta \in \Delta'} |\text{grad } f(\zeta)|.$$

Therefore $F_\nu(z)$ is continuous at z^0 . Therefore the theorem is proved.

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