

Pacific Journal of Mathematics

THE p -EQUIVALENCE OF $SO(2n + 1)$ AND $Sp(n)$

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Elementary homotopy methods are used to construct homotopy equivalences of the localized spaces $SO(2n + 1)_{\mathfrak{P}}$ and $Sp(n)_{\mathfrak{P}}$, where \mathfrak{P} is the set of odd primes. The equivalences are H -maps.

Serre [1] conjectured a \mathcal{C} -isomorphism $\pi_k(Sp(n)) \approx \pi_k(SO(2n + 1))$ where \mathcal{C} is the class of 2-primary abelian groups. This was proved by Harris [3]. Since the development of localization techniques for spaces [4, 8], other proofs of equivalence via decomposition as products have been given [6]. Friedlander [2] has proved the p -equivalence of $BSO(2n + 1)$ and $BSp(n)$, for odd primes p , by the use of etale homotopy theory. None of these methods prove the equivalence by actually giving a map.

The purpose of this note is to use the results of Harris [3], a map described in [5], and elementary homotopy theory to construct homotopy equivalences of the localized spaces $SO(2n + 1)_{\mathfrak{P}}$ and $Sp(n)_{\mathfrak{P}}$, where \mathfrak{P} is the set of odd primes. These equivalences are H -maps, but the author does not know if they can be delooped to obtain Friedlander's result.

The author wishes to thank the referee, whose comments helped to improve the exposition of this result.

1. Notation. The unitary group $U(n)$ is the group of non-singular complex $n \times n$ matrices with inverse the conjugate transpose. The orthogonal group $O(n)$ is the subgroup of $U(n)$ left pointwise fixed under complex conjugation, i.e. the subgroup of real matrices. We denote by $SO(n) \subset O(n)$ and $SU(n) \subset U(n)$ the subgroups of elements of determinant 1, and by $\alpha: O(n) \rightarrow U(n)$ (or $\alpha: SO(n) \rightarrow SU(n)$) the inclusion monomorphism.

If $J \in SU(n)$ is the matrix with 2×2 blocks $(\begin{smallmatrix} - & 1 \\ & 0 \end{smallmatrix})$ down the diagonal, then $Sp(n)$ is the subgroup of $SU(2n)$ left pointwise fixed by the automorphism $g \rightarrow J\bar{g}J^{-1}$, where \bar{g} is the complex conjugate matrix of g (i.e. $(\bar{g}_{ij}) = (\bar{g}_{ji})$). We denote the inclusion monomorphism by $\beta: Sp(n) \rightarrow SU(2n)$.

The monomorphisms α, β are natural with respect to inclusions $U(n - k) \rightarrow U(n)$ described in matrix notation by $A \rightarrow \begin{pmatrix} A & 0 \\ 0 & I_k \end{pmatrix}$ where I is the $k \times k$ identity.

If \mathfrak{S} is a set of prime numbers and X is a space which admits localizations, then $X_{\mathfrak{S}}$ will denote a localization of X at \mathfrak{S} and $e_{\mathfrak{S}}: X \rightarrow X$ a localization map.

2. The map ϕ . In [5] the author defined a map $\phi: O(n) \rightarrow U(n - 1)$ so that the diagram

$$\begin{array}{ccc} O(n) & \xrightarrow{\phi} & U(n - 1) \\ & \searrow \alpha & \downarrow j \\ & & U(n) \end{array}$$

homotopy commutes. For the reader's convenience we repeat the definition here.

Let u be a complex number with $|u| = 1$ and define a cross-section $\sigma_u: S^{2n-1} - \{ue_n\} \rightarrow U(n)$ by the formula

$$\sigma_u(x_1, x_2, \dots, x_n) = \begin{bmatrix} & x_2 & & \\ \left[\frac{\delta_{pq} - x_p Q^{-1} \bar{x}_q}{P \bar{x}_1 P \bar{x}_2 \cdots P \bar{x}_{n-1}} \right] & \vdots & & \\ & x_{n-1} & & \\ & & & x_n \end{bmatrix},$$

where $Q = 1 - \bar{x}u$ and $P = u\bar{Q}Q^{-1}$. Taking $u = i$ in this formula $j\phi(x) = [\sigma_i p \alpha(x)]^{-1} \alpha(x)$, where $p: U(n) \rightarrow S^{2n-1}$ is the bundle projection which picks out the last column of a matrix in $U(n)$. A proof of the homotopy commutativity of the diagram above as well as other properties of ϕ can be found in [5].

We first remark that $\det \sigma_u(x) = -P$, so if we multiply $\sigma_u(x)$ on the right by the matrix

$$\begin{pmatrix} I_{n-2} & & \\ & -\bar{P} & \\ & & 1 \end{pmatrix}$$

we obtain a cross-section $\sigma'_u: S^{2n-1} - \{ue_n\} \rightarrow \text{SU}(n)$. For $x \in \text{SO}(n)$, the map $\phi': \text{SO}(n) \rightarrow \text{SU}(n - 1)$ such that $j\phi'(x) = [\sigma'_i p \alpha(x)]^{-1} \alpha(x)$ factors $\alpha: \text{SO}(n) \rightarrow \text{SU}(n)$ through $\text{SU}(n - 1)$ up to homotopy and has properties analogous to ϕ . From now on we will suppress primes, writing $\sigma_u = \sigma'_u$ and $\phi = \phi'$.

PROPOSITION 2.1. *The map ϕ and its complex conjugate $\bar{\phi}$ are homotopic maps $\text{SO}(n) \rightarrow \text{SU}(n - 1)$.*

Proof. One easily sees that for the complex conjugate, $\overline{\sigma_i p\alpha(x)} = \sigma_{-i} p\alpha(x)$, and that

$$\bar{\phi}(x) = [\sigma_{-i} p\alpha(x)]^{-1} [\sigma_i p\alpha(x)] \phi(x).$$

For $y \in S^{2n-1} - \{\pm ie_n\}$, we have $[\sigma_{-i}(y)]^{-1} [\sigma_i(y)] \in \text{SU}(n - 1)$, and if we set

$$h(x, t) = \left(\cos \frac{\pi t}{2} \right) p\alpha(x) + i \left(\sin \frac{\pi t}{2} \right) e_{n-1},$$

and $H(x, t) = [\sigma_{-i} h(x, t)]^{-1} [\sigma_i h(x, t)] \phi(x)$, we have $H: \text{SO}(n) \times I \rightarrow \text{SU}(n - 1)$ with $H(x, 0) = \bar{\phi}(x)$, $H(x, 1) = \phi(x)$. \square

3. Construction of the map. We will be concerned with the fibre bundles

$$(*) \quad \text{SO}(2n + 1) \xrightarrow{\alpha} \text{SU}(2n + 1) \xrightarrow{p_1} \text{SU}(2n + 1)/\text{SO}(2n + 1)$$

and

$$(**) \quad \text{Sp}(n) \xrightarrow{\beta} \text{SU}(2n) \xrightarrow{p_2} \text{SU}(2n)/\text{Sp}(n).$$

Harris [3] showed that the maps

$$q_1: \text{SU}(2n + 1)/\text{SO}(2n + 1) \rightarrow \text{SU}(2n + 1)$$

and

$$q_2: \text{SU}(2n)/\text{Sp}(n) \rightarrow \text{SU}(2n)$$

defined by $q_1 p_1(x) \rightarrow x \cdot x^t$ and $q_2 p_2(x) = x \cdot J \cdot x^t \cdot J^{-1}$ have the property that $p_1 q_1$ and $p_2 q_2$ induce \mathcal{C} isomorphisms in homotopy, where \mathcal{C} is the Serre class of 2-primary abelian groups. If we let \mathcal{P} be the set of odd prime integers, the result of Harris implies that after \mathcal{P} -localization of spaces and maps, $p_1 q_1$ and $p_2 q_2$ induce isomorphisms of homotopy groups and are therefore homotopy equivalences [7, p. 405]. Let h_i be a (\mathcal{P} -local) homotopy inverse of the \mathcal{P} -localization of $p_i q_i$. Of course the localized maps $q_i h_i$ can be deformed to cross-sections of the \mathcal{P} -local versions of (*) and (**).

LEMMA 3.1. *If W is a connected CW-complex, the maps of based homotopy sets*

$$\alpha_{\mathcal{P}*}: [W, \text{SO}(2n + 1)_{\mathcal{P}}] \rightarrow [W, \text{SU}(2n + 1)_{\mathcal{P}}]$$

$$\beta_{\mathcal{P}*}: [W, \text{Sp}(n)_{\mathcal{P}}] \rightarrow [W, \text{SU}(2n)_{\mathcal{P}}]$$

are monomorphisms of groups.

Proof. We give the proof for $\beta_{\mathfrak{P}^*}$; the proof for $\alpha_{\mathfrak{P}^*}$ is similar. We consider a portion of the long exact homotopy sequence of (**):

$$\begin{aligned} \cdots \rightarrow \left[\sum W, \text{SU}(2n)_{\mathfrak{P}} \right] &\xrightleftharpoons[q_{2, \mathfrak{P}^*}]{p_{2, \mathfrak{P}^*}} \left[\sum W, (\text{SU}(2n)/\text{Sp}(n))_{\mathfrak{P}} \right], \\ &\xrightarrow{d_*} \left[W, \text{Sp}(n)_{\mathfrak{P}} \right] \xrightarrow{\beta_{\mathfrak{P}^*}} \left[W, \text{SU}(2n)_{\mathfrak{P}} \right]. \end{aligned}$$

Since $p_{2, \mathfrak{P}}q_{2, \mathfrak{P}}$ is a homotopy equivalence, d_* is the trivial map and $\beta_{\mathfrak{P}^*}$ is injective. □

Let ψ be the composite monomorphism $\psi: \text{Sp}(n) \xrightarrow{\beta} \text{SU}(2n) \xrightarrow{j} \text{SU}(2n + 1)$, and $J' = j(J)$ so that $\overline{\psi(x)} = J' \cdot \psi(x) \cdot J'^{-1} = \psi(\bar{x})$.

PROPOSITION 3.2. *Let \mathfrak{P} be the set of odd primes.*

(i) *There is a map $\Phi: \text{SO}(2n + 1) \rightarrow \text{Sp}(n)_{\mathfrak{P}}$ such that $\beta_{\mathfrak{P}}\Phi$ is homotopic to $\text{SO}(2n + 1) \xrightarrow{\phi} \text{SU}(2n) \xrightarrow{e_{\mathfrak{P}}} \text{SU}(2n)_{\mathfrak{P}}$.*

(ii) *There is a map $\Psi: \text{Sp}(n) \rightarrow \text{SO}(2n + 1)_{\mathfrak{P}}$ such that $\alpha_{\mathfrak{P}}\Psi$ is homotopic to $\text{Sp}(n) \xrightarrow{\psi} \text{SU}(2n + 1) \xrightarrow{e_{\mathfrak{P}}} \text{SU}(2n + 1)_{\mathfrak{P}}$.*

Proof. Using a path in $\text{SU}(2n)$ from J to the identity and the homotopy of ϕ with $\bar{\phi}$ of Proposition 2.1, we have

$$q_2 p_2 \phi = \phi \cdot J \cdot \phi' \cdot J^{-1} \simeq \phi \cdot \phi' \simeq \bar{\phi} \cdot \phi' = I_{2n} \quad (\text{constant}).$$

Thus (partially suppressing the subscript \mathfrak{P}),

$$p_2 e_{\mathfrak{P}} \phi \simeq h_2 p_2 q_2 p_2 e_{\mathfrak{P}} \phi \simeq h_2 p_2 e_{\mathfrak{P}} (q_2 p_2 \phi) \simeq \text{constant}.$$

By the covering homotopy property, there is a map $\Phi: \text{SO}(2n + 1) \rightarrow \text{Sp}(n)_{\mathfrak{P}}$ so that $\beta_{\mathfrak{P}}\Phi$ is homotopic to $e_{\mathfrak{P}}\phi$.

Similarly,

$$q_1 p_1 \psi = \psi \cdot \psi' \simeq \psi \cdot J' \cdot \psi' \cdot J'^{-1} = I_{2n+1} \quad (\text{constant}).$$

An analogous argument completes the proof of (ii). □

Note that this proposition implies that $\alpha_{\mathfrak{P}}\Psi_{\mathfrak{P}} \simeq \psi_{\mathfrak{P}}$ and $\beta_{\mathfrak{P}}\Phi_{\mathfrak{P}} = \phi_{\mathfrak{P}}$.

Since $\psi: \text{Sp}(n) \rightarrow \text{SU}(2n + 1)$ is a homomorphism and the localization of an H -space is an H -space, we obtain

PROPOSITION 3.3. *The map $\Psi: \text{Sp}(n) \rightarrow \text{SO}(2n + 1)_{\mathfrak{P}}$ is an H -map of H -spaces.*

Proof. Let $\mu_G: G \times G \rightarrow G$ be multiplication. Then since $\alpha_{\mathfrak{p}}$, $e_{\mathfrak{p}}$ and ψ are H -maps,

$$\begin{aligned} \alpha_{\mathfrak{p}}\Psi\mu_{\mathrm{Sp}} &\simeq e_{\mathfrak{p}}\psi\mu_{\mathrm{Sp}} \simeq \mu_{\mathrm{SU},\mathfrak{p}}(e_{\mathfrak{p}}\psi \times e_{\mathfrak{p}}\psi) \simeq \mu_{\mathrm{SU},\mathfrak{p}}(\alpha_{\mathfrak{p}}\Psi \times \alpha_{\mathfrak{p}}\Psi) \\ &\simeq \alpha_{\mathfrak{p}}\mu_{\mathrm{SO},\mathfrak{p}}(\Psi \times \Psi). \end{aligned}$$

Since $\alpha_{\mathfrak{p}^*}$ is injective, taking $W = \mathrm{Sp}(n) \times \mathrm{Sp}(n)$ in 3.1, we have $\Psi\mu_{\mathrm{Sp}} \simeq \mu_{\mathrm{SO},\mathfrak{p}}(\Psi \times \Psi)$. \square

COROLLARY 3.4. *The localized map $\Psi_{\mathfrak{p}}: \mathrm{Sp}(n)_{\mathfrak{p}} \rightarrow \mathrm{SO}(2n + 1)_{\mathfrak{p}}$ is an H -map of H -spaces.*

Proof. This follows by localizing the homotopy of 3.3. \square

A proof for Φ analogous to the one above fails because ϕ is not a group homomorphism.

We are now ready to state the main result.

THEOREM 3.5. *If \mathfrak{P} is the set of odd primes there exist maps $\Phi: \mathrm{SO}(2n + 1) \rightarrow \mathrm{Sp}(n)_{\mathfrak{p}}$ and $\Psi: \mathrm{Sp}(n) \rightarrow \mathrm{SO}(2n + 1)_{\mathfrak{p}}$ whose \mathfrak{P} -localizations*

$$\begin{aligned} \Phi_{\mathfrak{p}}: \mathrm{SO}(2n + 1)_{\mathfrak{p}} &\rightarrow \mathrm{Sp}(n)_{\mathfrak{p}}, \\ \Psi_{\mathfrak{p}}: \mathrm{Sp}(n)_{\mathfrak{p}} &\rightarrow \mathrm{SO}(2n + 1)_{\mathfrak{p}} \end{aligned}$$

are inverse homotopy equivalences and H -maps.

Proof. By Proposition 3.2 we have a commutative diagram of homotopy sets

$$\begin{array}{ccc} [W, \mathrm{Sp}(n)_{\mathfrak{p}}] & \xrightarrow{\beta_{\mathfrak{p}^*}} & [W, \mathrm{SU}(2n)_{\mathfrak{p}}] \\ \Psi_{\mathfrak{p}^*} \downarrow \uparrow \Phi_{\mathfrak{p}^*} & & \downarrow j_{\mathfrak{p}^*} \\ [W, \mathrm{SO}(2n + 1)_{\mathfrak{p}}] & \xrightarrow{\alpha_{\mathfrak{p}^*}} & [W, \mathrm{SU}(2n + 1)_{\mathfrak{p}}]. \end{array}$$

We have

$$\alpha_{\mathfrak{p}} \simeq j_{\mathfrak{p}}\phi_{\mathfrak{p}} \simeq j_{\mathfrak{p}}\beta_{\mathfrak{p}}\Phi_{\mathfrak{p}} \simeq \psi_{\mathfrak{p}}\Phi_{\mathfrak{p}} \simeq \alpha_{\mathfrak{p}}\Psi_{\mathfrak{p}}\Phi_{\mathfrak{p}}.$$

Taking $W = \mathrm{SO}(2n + 1)_{\mathfrak{p}}$ and using brackets to denote homotopy class, we have $\alpha_{\mathfrak{p}^*}[1_{\mathrm{SO}(2n+1),\mathfrak{p}}] = \alpha_{\mathfrak{p}^*}[\Psi_{\mathfrak{p}}\Phi_{\mathfrak{p}}]$, or $\Psi_{\mathfrak{p}}\Phi_{\mathfrak{p}} \simeq 1_{\mathrm{SO}(2n+1),\mathfrak{p}}$, since $\alpha_{\mathfrak{p}^*}$ is injective by 3.1. Thus $\Psi_{\mathfrak{p}^*}\Phi_{\mathfrak{p}^*}$ is the identity, $\Psi_{\mathfrak{p}^*}$ is surjective and $\Phi_{\mathfrak{p}^*}$ is injective.

Now

$$j_{\mathfrak{q}}\beta_{\mathfrak{q}} \simeq \psi_{\mathfrak{q}} \simeq \alpha_{\mathfrak{q}}\Psi_{\mathfrak{q}} \simeq j_{\mathfrak{q}}\phi_{\mathfrak{q}}\Psi_{\mathfrak{q}} \simeq j_{\mathfrak{q}}\beta_{\mathfrak{q}}\Phi_{\mathfrak{q}}\Psi_{\mathfrak{q}}.$$

Take $W = S^k$ so the sets are homotopy groups, and

$$j_{\mathfrak{q}*}\beta_{\mathfrak{q}*} = j_{\mathfrak{q}*}\beta_{\mathfrak{q}*}(\Phi\Psi)_{\mathfrak{q}*} : \pi_k(\mathrm{Sp}(n)_{\mathfrak{q}}) \rightarrow \pi_k(\mathrm{SU}(2n + 1)_{\mathfrak{q}}).$$

Since $\beta_{\mathfrak{q}*}$ is a monomorphism and $j_{\mathfrak{q}*}$ is an isomorphism for $k < 4n$, $(\Phi_{\mathfrak{q}}\Psi_{\mathfrak{q}})_{\mathfrak{q}*}$ is the identity on homotopy groups in dimensions $k < 4n$. By the results of Harris [3], $\pi_k(\mathrm{SO}(2n + 1)_{\mathfrak{q}})$ and $\pi_k(\mathrm{Sp}(n)_{\mathfrak{q}})$ are finite groups of the same order in dimension $k \geq 4n$. Since $\Psi_{\mathfrak{q}*}$ is an epimorphism, it is an isomorphism (as is $\Phi_{\mathfrak{q}*}$). Thus $\Phi_{\mathfrak{q}}$ and $\Psi_{\mathfrak{q}}$ induce isomorphisms on homotopy groups, and are therefore homotopy equivalences. But $\Phi_{\mathfrak{q}}$ is a right homotopy inverse for the homotopy equivalence $\Psi_{\mathfrak{q}}$, hence is a left homotopy inverse for $\Psi_{\mathfrak{q}}$ and $\Phi_{\mathfrak{q}}\Psi_{\mathfrak{q}} \simeq 1_{\mathrm{Sp}(n)_{\mathfrak{q}}}$.

Finally, since $\Phi_{\mathfrak{q}}$ is a homotopy inverse for $\Psi_{\mathfrak{q}}$ and $\Psi_{\mathfrak{q}}$ is an H -map, $\Phi_{\mathfrak{q}}$ is an H -map. □

REMARKS. Since $\Phi = \Phi_{\mathfrak{q}}e_{\mathfrak{q}}$ and both $\Phi_{\mathfrak{q}}$ and $e_{\mathfrak{q}}$ are H -maps, Φ is an H -map.

By Theorem 6.6 of [4], there exist maps $\Phi' : \mathrm{SO}(2n + 1) \rightarrow \mathrm{Sp}(n)$ and $\Psi' : \mathrm{Sp}(n) \rightarrow \mathrm{SO}(2n + 1)$ so that $\Phi'_{\mathfrak{q}}$ and $\Psi'_{\mathfrak{q}}$ are homotopy equivalences. We do not know if Φ' and Ψ' can be chosen to be H -maps or if they can be delooped to maps on the classifying spaces.

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Received October 8, 1981 and in revised form May 10, 1982.

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