A UNIFIED APPROACH TO CARLESON MEASURES AND $A_p$ WEIGHTS. II

Francisco José Ruiz and José Luis Torrea
A UNIFIED APPROACH TO CARLESON MEASURES
AND $A_p$ WEIGHTS. II

FRANCISCO J. RUIZ AND JOSÉ L. TORREA

In this note we find for each $p$, $1 < p < \infty$, a necessary and sufficient condition on the pair $(\mu, \nu)$ (where $\mu$ is a measure on $\mathbb{R}^{n+1}_+ = \mathbb{R}^n \times [0, \infty)$, and $\nu$ a weight on $\mathbb{R}^n$) for the Poisson integral to be a bounded operator from $L^p(\mathbb{R}^n, \nu(x) \, dx)$ into $L^p(\mathbb{R}^{n+1}_+, \mu)$.

1. Introduction. In this note we find for each $p$, $1 < p < \infty$, a necessary and sufficient condition on the pair $(\mu, \nu)$ (where $\mu$ is a measure on $\mathbb{R}^{n+1}_+ = \mathbb{R} \times [0, \infty)$ and $\nu$ a weight on $\mathbb{R}^n$) for the Poisson integral to be a bounded operator from $L^p(\mathbb{R}^n, \nu(x) \, dx)$ into $L^p(\mathbb{R}^{n+1}_+, \mu)$.

Our proof follows the ideas of Sawyer [7] and the condition we find is

\[
(F_p) \quad \int_{\tilde{Q}} [\mathcal{M}(v^{1-p'}X_Q)(x, t)]^p \, d\mu(x, t) \leq C \int_{Q} v^{1-p'}(x) \, dx < +\infty
\]

for all cubes in $\mathbb{R}^n$ (cube will always means a compact cube with sides parallel to the coordinate axes).

For $\mathcal{M}$ we denote the maximal operator

\[
(*) \quad \mathcal{M}f(x, t) = \sup_{Q} \frac{1}{|Q|} \int_{Q} |f(x)| \, dx, \quad x \in \mathbb{R}^n, \, t \geq 0,
\]

where the supremum is taken over the cubes $Q$ in $\mathbb{R}^n$, containing $x$ and having side length at least $t$.

As usual $\tilde{Q}$ denotes the cube in $\mathbb{R}^{n+1}_+$, with the cube $Q$ as its basis.

Carleson [1] showed that $\mathcal{M}$ is bounded from $L^p(\mathbb{R}^n, dx)$ into $L^p(\mathbb{R}^{n+1}_+, \mu)$ if and only if $\mu$ satisfies the so-called “Carleson condition”

\[
(1) \quad \mu(\tilde{Q}) \leq C|Q| \quad \text{for each cube in } \mathbb{R}^n.
\]

Afterwards, Fefferman and Stein [2] found that

\[
(2) \quad \sup_{x \in Q} \frac{\mu(\tilde{Q})}{Q} \leq Cv(x) \quad \text{a.e. } x
\]

is sufficient for $\mathcal{M}$ to be bounded from $L^p(\mathbb{R}^n, \nu(x) \, dx)$ into $L^p(\mathbb{R}^{n+1}_+, \mu)$.
Recently F. Ruiz [6] found the condition
\[
\frac{\mu(Q)}{|Q|} \left( \frac{1}{|Q|} \int_Q v^{1-p'}(x) \, dx \right)^{p-1} \leq C
\]
to be necessary and sufficient for the boundedness of the operator \( M \) from \( L^p(\mathbb{R}^n, v(x) \, dx) \) into weak-\( L^p(\mathbb{R}^n_{+1}, \mu) \). The condition (3) will be denoted by \((C_p)\) as in [6].

The paper is set out as follows: in §2 we give results and some consequences, whilst §3 contains detailed proofs.

2. Results. Throughout this paper, \( Q \) will denote a cube in \( \mathbb{R}^n \) with sides parallel to the coordinate planes. For \( r > 0 \), \( rQ \) will denote the cube with the same centre as \( Q \) diameter \( r \) times that of \( Q \). \(|Q|_v\) will denote \( \int_Q v(x) \, dx \).

We shall say that \( Q \) is a dyadic cube and we shall write \( Q \in \mathcal{D} \), if \( Q \) is a subset of \( \mathbb{R}^n \) of the form \( \prod_{i=1}^n [x_i, x_i + 2^k) \), where \( x \in 2^k \mathbb{Z}^n \), with \( k \) in \( \mathbb{Z} \). We define the dyadic maximal operator \( N \) associated with the Poisson integral by
\[
(*) \quad Nf(x, t) = \sup_Q \frac{1}{|Q|} \int_Q |f(x)| \, dx, \quad x \in \mathbb{R}^n, \ t \geq 0,
\]
where the supremum is taken over the dyadic cubes in \( \mathbb{R}^n \) containing \( x \) and having side length at least \( t \).

The main results in this paper are the following:

**THEOREM A.** Given a weight \( v \) in \( \mathbb{R}^n \), a positive measure \( \mu \) in \( \mathbb{R}^n_{+1} \), and \( p, 1 < p < \infty \), the following conditions are equivalent.

(i) The operator \( M \) defined in (*) is bounded from \( L^p(\mathbb{R}^n, v(x) \, dx) \) into \( L^p(\mathbb{R}^n_{+1}, \mu) \); i.e.
\[
\int_{\mathbb{R}^n_{+1}} \left( Mf(x, t) \right)^p \, d\mu(x, t) \leq C \int_{\mathbb{R}^n} |f(x)|^p v(x) \, dx.
\]

(ii) The pair \( (\mu, r) \) verifies \((F_p)\).

**THEOREM B.** Given a weight \( v \) in \( \mathbb{R}^n \), a positive measure \( \mu \) in \( \mathbb{R}^n_{+1} \), and \( p, 1 < p < \infty \), the following conditions are equivalent.

(i) The operator \( N \) defined in (**) is bounded from \( L^p(\mathbb{R}^n, vx \, dx) \) into \( L^p(\mathbb{R}^n_{+1}, \mu) \).
(ii) The pair \((\mu, v)\) verifies
\[
\int_{Q} \left[ \mathcal{N}(v^{1-p'}x_{Q})(x, t) \right]^{p} \, d\mu(x, t) \leq C \int_{Q} v^{1-p'}(x) \, dx < +\infty
\]
for all dyadic cubes \(Q\) in \(\mathbb{R}^{n}\).

The above results have certain consequences. (I) In the particular case in Theorem A where \(v(x) \equiv 1\), the condition \((F_p)\) reduces to
\[
\int_{Q} \left[ \mathcal{M}(x_Q)(x, t) \right]^{p} \, d\mu(x, t) \leq C \int_{Q} dx = C|Q|
\]
and since \(\mathcal{M}(x_Q)(x, t) = 1\) for \((x, t) \in \tilde{Q}\), we see that Theorem A gives Carleson's result mentioned in the introduction.

(II) If the measure \(\mu\) in \(\mathbb{R}^{n+1} = \mathbb{R}^{n} \times [0, \infty)\) is of the form \(d(x) = u(x) \, dx\) concentrated in \(\mathbb{R}^{n} \times \{0\}\), then \((F_p)\) is equivalent to Sawyer's condition
\[
(S_p) \quad \int_{Q} \left[ M(v^{1-p'}x_{Q})(x) \right]^{p} u(x) \, dx \leq C \int_{Q} v^{1-p'}(x) \, dx < +\infty
\]
where \(Mf\) denotes the Hardy Littlewood maximal operator.

Since \(\mathcal{M}f(x, 0) = Mf(x), x \in \mathbb{R}^{n}\). Then from Theorem A we obtain

**Theorem (Sawyer [7]).** Let \(1 < p < \infty\). Given weights \(u\) and \(v\) in \(\mathbb{R}^{n}\) the following statements are equivalent:

(i) \((u, v)\) satisfies the \((S_p)\) condition

(ii) \(\int_{\mathbb{R}^{n}} (Mf(x))^{p} u(x) \, dx \leq C_{p} \int_{\mathbb{R}^{n}} |f(x)|^{p} v(x) \, dx\).

(III) Hunt, Kurtz and Neugebauer [3] have shown by a direct proof that if a weight \(v\) belongs to the \(A_p\) class, \(1 < p < \infty\), of Muckenhoupt, i.e.
\[
\sup_{Q} \left( \frac{1}{|Q|} \int_{Q} v(x) \, dx \right) \left( \frac{1}{|Q|} \int_{Q} v^{1-p'}(x) \right)^{p-1} \leq C
\]
then \(v\) satisfies the \((S_p)\) condition in (II) with \(u = v\).

In our case it can be shown, see [6], that if the pair \((\mu, v)\) satisfies the \((C_p)\) condition, \(1 < p < \infty\), and \(v\) belongs to the class \(A_p\) of Muckenhoupt, then the operator \(\mathcal{M}\) is bounded from \(L^p(\mathbb{R}^{n}, v(x) \, dx)\) into \(L^p(\mathbb{R}^{n+1}, \mu)\) and this tells us that in particular \((\mu, v)\) will satisfy the \((F_p)\) condition.
In the particular case considered in (II), this suggests that for a pair of weights \((u, v)\) satisfying the \(A_p\) condition, \(1 < p < \infty\), of Muckenhoupt, i.e.

\[
\sup_Q \left( \frac{1}{|Q|} \int_Q u(x) \, dx \right) \left( \frac{1}{|Q|} \int_Q v^{1-p'}(x) \, dx \right)^{p+1} \leq C
\]

the fact that \(v \in A_p\) is sufficient for \((u, v)\) to satisfy the \(S_p\) condition.

(IV) If a weight \(v\) is given and we call \(F_p(v)\) (respectively \(C_p(v)\)) the set of measures \(\mu\) on \(\mathbb{R}^{n+1}_+\) such that \((\mu, v)\) satisfies the \(F_p\) condition (respectively the \(C_p\) condition) we can state that for \(1 < p \leq q\)

\[
C_1(v) \subset F_p(v) \subset C_p(v) \subset \cdots \subset F_q(v) \subset C_q(v) \subset \cdots.
\]

The inclusion \(C_p(v) \subset C_q(v)\) is proved in [6]. To see that \(F_p(v) \subset C_p(v)\) let us observe that for \((x, t) \in \bar{Q}\)

\[
\mathcal{M}(v^{1-p'}x_Q)(x, t) \geq \frac{1}{|Q|} \int_Q v^{1-p'}(y) \, dy
\]

and this implies for \(\mu \in F_p(v)\)

\[
\left( \frac{1}{|Q|} \int_Q v^{1-p'}(x) \, dx \right)^p \mu(\bar{Q}) \leq C \int_Q v^{1-p'}(x) \, dx.
\]

So \(\mu \in C_p(v)\).

Now, given \(p < q\), and \(\mu \in C_p(v)\), using the Marcinkiewicz interpolation theorem between the boundedness of \(\mathcal{M}\) from \(L^p(\mathbb{R}^n, v(x) \, dx)\) into weak-\(L^p(\mathbb{R}^{n+1}_+, \mu)\) and the trivial \(L^\infty\) boundedness, we obtain \(\mu \in F_q(v)\).

**REMARK.** If, for a given \(p\), \(v\) belongs to the \(A_p\) class of Muckenhoupt, then it can be shown that \(C_p(v) = C_q(v)\), \(p \leq q \leq \infty\), see [6]. This fact and the fact that for \(v \in A_p\) there exists \(\varepsilon > 0\) such that \(v \in A_{p-\varepsilon}\) allows us to obtain that

\[
F_p(v) = C_p(v) = C_q(v) = F_q(v), \quad p \leq q \leq \infty.
\]

3. **Detailed proofs.** The proof of the implication \((i) \Rightarrow (ii)\) is the same in both Theorems A and B, the only difference being the use of non dyadic or dyadic cubes.

Firstly, let us see that \(\int_Q v^{1-p'}(x) \, dx < +\infty\) for all cubes. If

\[
\int_Q v^{1-p'}(x) \, dx = \int_Q (v^{-1}(x))^p v(x) \, dx = \infty
\]

this would imply the existence of a function \(f \in L^p(v)\) such that

\[
\int_Q f(x) \, dx = \int_Q f(x)v^{-1}(x)v(x) \, dx = +\infty,
\]
and in particular $Mf(x, t) = + \infty$ for $(x, t) \in \mathbb{R}_+^{n+1}$ which contradicts the hypothesis:

$$\int_{\mathbb{R}_+^{n+1}} [Mf(x, t)]^p \, d\mu(x, t) \leq C \int_{\mathbb{R}^n} f^p(x) v(x) \, dx < +\infty.$$ 

To show the inequality in (ii) it is sufficient to choose $f(x) = \chi_Q(x) v^{1-p'}(x)$ in the hypothesis.

Proof of (ii) $\Rightarrow$ (i) in Theorem B. In order to handle a Calderón-Zygmund decomposition we introduce the operators

$$\mathcal{N}^Rf(x, t) = \sup_Q \frac{1}{Q} \int_Q |f(x)| \, dx, \quad x \in \mathbb{R}^n, t > 0,$$

the supremum being taken over all dyadic cubes in $\mathbb{R}^n$ containing $x$ and having side length at least $t$ and at most $R$.

Observe that $\mathcal{N}^Rf(x, t) = 0$ for $t > R$ and that

$$\lim_{R \to \infty} \mathcal{N}^Rf(x, t) = Nf(x, t)$$

with increasing limit.

Let $\Omega_k$ be the set

$$\Omega_k = \{(x, t): \mathcal{N}^Rf(x, t) > 2^k\}, \quad k \in \mathbb{Z}.$$

**Lemma.** For each $k \in \mathbb{Z}$ there exists a family $\{Q^k_j\}, j \in J_k$, of dyadic cubes in $\mathbb{R}^n$ such that

(i) $1/|Q^k_j| \int_{Q^k_j} |f(x)| \, dx > 2^k$.

(ii) The interiors of $\tilde{Q}^k_j$ are disjoints

(iii) $\Omega_k = \bigcup_{j \in J_k} \tilde{Q}^k_j$.

Proof of the lemma. If $(x, t) \in \Omega_k$ it means that there exists a dyadic cube with $x \in Q$, $l(Q) \geq t$, $l(Q) \leq R$ and $1/|Q| \int_Q |f(x)| \, dx > 2^k$. This implies the existence of a dyadic maximal $Q^k_j$ such that $Q \subset Q^k_j$, $l(Q^k_j) \leq R$, $l(Q^k_j) \geq t$ and

$$\frac{1}{|Q^k_j|} \int_{Q^k_j} |f(x)| \, dx > 2^k.$$

In particular, $(x, t) \in \tilde{Q}^k_j$. The fact that the interiors of $\tilde{Q}^k_j$ are disjoint is an obvious consequence of the same property for the $\tilde{Q}^k_j$'s.

Now let us consider the sets

$$E^k_j = \tilde{Q}^k_j \setminus \{(x, t): \mathcal{N}^Rf(x, t) \geq 2^{k+1}\}.$$
Then we have a family of sets \( \{ E_{jk}^k \} \) with disjoint interiors and

\[
\int_{\mathbb{R}^{n+1}} \left[ \mathcal{N}^R f(x, t) \right]^p \, d\mu(x, t) \leq \sum_{j,k} \int_{E_{jk}^k} \left[ \mathcal{N}^R f(x, t) \right]^p \, d\mu(x, t)
\]

\[
\leq \sum_{j,k} 2^{(k+1)p} \mu(E_{jk}^k) \leq 2^p \sum_{j,k} \mu(E_{jk}^k) \left( \int_{Q_j^k} |f(x)| \, dx \right)^p.
\]

Following the ideas of Sawyer [7] and Jawerth [4], we introduce the following notations:

\[
\sigma(x) = v^{1-p'}(x), \quad \sigma(Q) = \int_Q \sigma(x) \, dx
\]

\[
\gamma_{jk} = \mu(E_{jk}^k) \left( \frac{\sigma(Q_j^k)}{|Q_j^k|} \right)^p,
\]

\[
g_{jk} = \left( \frac{1}{\sigma(Q_j^k)} \int_{Q_j^k} |f(x)| \, \sigma(x) \, dx \right)^p,
\]

\[
X = \{(k, j): k \in \mathbb{Z}, j \in J_k\} \quad \text{with atomic measure } \gamma_{jk}.
\]

\[
\Gamma(\lambda) = \{(k, j) \in X: g_{jk} > \lambda \}.
\]

Then we can write

\[
\int_{\mathbb{R}^{n+1}} \left[ \mathcal{N}^R f(x, t) \right]^p \, d\mu(x, t) \leq 2^p \sum_{j,k} \gamma_{jk} g_{jk}
\]

\[
= 2^p \int_0^\infty \gamma\{(k, j): g_{jk} > \lambda\} \, d\lambda = 2^p \int_0^\infty \left\{ \sum_{(k, j) \in \Gamma(\lambda)} \gamma_{jk} \right\} \, d\lambda
\]

\[
= 2^p \int_0^\infty \sum_{(k, j) \in \Gamma(\lambda)} \mu(E_{jk}^k) \left( \frac{\sigma(Q_j^k)}{|Q_j^k|} \right)^p \, d\lambda
\]

\[
= 2^p \int_0^\infty \sum_{(k, j) \in \Gamma(\lambda)} \int_{E_{jk}^k} \left( \frac{\sigma(Q_j^k)}{|Q_j^k|} \right)^p \, d\mu(x, t) \, d\lambda,
\]

calling \( Q_i \) the maximal cubes of the family \( \{ Q_j^k: (k, j) \in \Gamma(\lambda) \} \). This is equal to

\[
2^p \int_0^\infty \sum_{j} \sum_{Q_j^k \subset Q} \int_{E_{jk}^k} \left( \frac{\sigma(Q_j^k)}{|Q_j^k|} \right)^p \, d\mu(x, t) \, d\lambda
\]

\[
\leq 2^p \int_0^\infty \sum_{i} \sum_{Q_i^k \subset Q} \int_{E_{jk}^k} \left( \mathcal{N}^R (\sigma \chi_{Q_i})(x, t) \right)^p \, d\mu(x, t) \, d\lambda
\]
by the disjointness of the $E_j$'s. This is less than

$$2^p \int_0^\infty \sum_i \int_{Q_i} \left( \mathcal{N}^R(\sigma \chi_Q) \right)^p d\mu(x, t) d\lambda.$$ 

Following hypothesis (i) this is less than

$$2^p \int_0^\infty \sum_i \left( \int_{Q_i} \sigma(x) dx \right) d\lambda = 2^p \int_0^\infty \sigma(\cup Q_i) d\lambda = 2^p \int \sigma \left( \bigcup_{(k, j) \in \Gamma(\lambda)} Q_j^k \right) d\lambda.$$

The definition of $\Gamma(\lambda)$ states that

$$\bigcup_{(k, j) \in \Gamma(\lambda)} Q_j^k \subset \left\{ x: N_\sigma \left( \frac{|f|}{\sigma} \right)(x) > \lambda^{1/p} \right\}$$

where

$$N_\sigma(x) = \sup \frac{1}{\sigma(Q)} \int_Q g(x) \sigma(x) dx,$$

the supremum being taken over all dyadic cubes in $\mathbb{R}^n$ containing $x$.

Then we have

$$\int_{\mathbb{R}^{n+1}} \left[ \mathcal{N}^R f(x, t) \right]^p d\mu(x, t) \leq 2^p \int_0^\infty \sigma \left\{ x: N_\sigma \left( \frac{|f|}{\sigma} \right)(x) > \lambda \right\} d\lambda \leq 2^p \int_{\mathbb{R}^n} \left( N_\sigma \left( \frac{|f|}{\sigma} \right)(x) \right)^p \sigma(x) dx \leq 2^p \int_{\mathbb{R}^n} \frac{|f(x)|^p}{\sigma(x)^p} \sigma(x) dx$$

since the dyadic maximal operator with respect to any positive measure $\nu$ maps $L^p(d\nu)$, $1 < p < \infty$, into itself.

The proof ends by applying Fatou's lemma and observing that $\sigma^{1-p} = \nu$.

Proof of (ii) $\Rightarrow$ (i) in Theorem A. The proof of this part follows easily from the ensuing lemma due to Sawyer [7].

**Lemma 2.** Define for each $y \in \mathbb{R}^n$

$$\mathcal{N}^y f(x, t) = \sup \frac{1}{|Q|} \int_Q |f(u)| du,$$
the supremum being taken in all cubes $Q$ with $x \in Q$, side length less than $t$ and such that the set $Q - y = \{ u - y : u \in Q \}$ is a dyadic cube. Then,

$$\mathcal{M}^{2^k} f(x, t) \leq C \int_{[-2^{k+2}, 2^{k+2}]}^{\mathcal{N} f(x, t) \frac{dy}{2^n(k+3)}}$$

where the constant $C$ depends only on the dimension.

By $\mathcal{M}^R$ we mean the maximal operator obtained considering cubes with side length less than $R$.

Observe that the proof of Theorem B can be repeated for the operator $\lambda \mathcal{N}$ where the dyadic cubes are now of the type $\prod_{i=1}^n [x_i, x_i + 2^k)$ with $x - y \in 2^k \mathbb{Z}^n$.

Then, by Lemma 2 we have

$$\int_{\mathbb{R}_+^{n+1}} [\mathcal{M}^{2^k} f(x, t)]^p d\mu(x, t)$$

$$\leq C \int_{\mathbb{R}_+^{n+1}} d\mu(x, t) \left[ \int_{[-2^{k+2}, 2^{k+2}]}^{\mathcal{N} f(x, t) \frac{dy}{2^n(k+3)}} \right]^p$$

$$\leq C \left[ \int_{[-2^{k+2}, 2^{k+2}]} \frac{dy}{2^n(k+3)} \left( \int |f(x)|^p v(x) \, dx \right)^{1/p} \right]^p$$

$$= C \int |f(x)|^p v(x) \, dx.$$

By letting $k \to \infty$ we conclude (i) in Theorem A.

The proof of Lemma 2 follows along the lines of the corresponding result in [7] and is therefore omitted.

Acknowledgments. The authors are grateful to José L. Rubio for his helpful suggestions.

REFERENCES


Received November 21, 1983 and in revised form April 4, 1984.

UNIVERSIDAD DE ZARAGOZA
ZARAGOZA, SPAIN

AND

UNIVERSIDAD AUTONOMA DE MADRID
MADRID-34, SPAIN
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ulrich F. Albrecht</td>
<td>A note on locally $A$-projective groups</td>
<td>1</td>
</tr>
<tr>
<td>Marilyn Breen</td>
<td>A Krasnosel’skiǐ-type theorem for unions of two starshaped sets in the plane</td>
<td>19</td>
</tr>
<tr>
<td>Anthony Carbery, Sun-Yung Alice Chang and John Brady Garnett</td>
<td>Weights and $L \log L$</td>
<td>33</td>
</tr>
<tr>
<td>Joanne Marie Dombrowski</td>
<td>Tridiagonal matrix representations of cyclic self-adjoint operators. II</td>
<td>47</td>
</tr>
<tr>
<td>Heinz W. Engl and Werner Römisch</td>
<td>Approximate solutions of nonlinear random operator equations: convergence in distribution</td>
<td>55</td>
</tr>
<tr>
<td>P. Ghez, R. Lima and J. E. Roberts</td>
<td>$W^*$-categories</td>
<td>79</td>
</tr>
<tr>
<td>Barry E. Johnson</td>
<td>Continuity of homomorphisms of Banach $G$-modules</td>
<td>111</td>
</tr>
<tr>
<td>Elyahu Katz and Sidney Allen Morris</td>
<td>Free products of topological groups with amalgamation. II</td>
<td>123</td>
</tr>
<tr>
<td>Neal I. Koblitz</td>
<td>$p$-adic integral transforms on compact subgroups of $C_p$</td>
<td>131</td>
</tr>
<tr>
<td>Albert Edward Livingston</td>
<td>A coefficient inequality for functions of positive real part with an application to multivalent functions</td>
<td>139</td>
</tr>
<tr>
<td>Scott Carroll Metcalf</td>
<td>Finding a boundary for a Hilbert cube manifold bundle</td>
<td>153</td>
</tr>
<tr>
<td>Jack Ray Porter and R. Grant Woods</td>
<td>When all semiregular $H$-closed extensions are compact</td>
<td>179</td>
</tr>
<tr>
<td>Francisco José Ruiz and José Luis Torrea</td>
<td>A unified approach to Carleson measures and $A_p$ weights. II</td>
<td>189</td>
</tr>
<tr>
<td>Timothy DuWayne Sauer</td>
<td>The number of equations defining points in general position</td>
<td>199</td>
</tr>
<tr>
<td>John Brendan Sullivan</td>
<td>Universal observability and codimension one subgroups of Borel subgroups</td>
<td>215</td>
</tr>
<tr>
<td>Akihito Uchiyama</td>
<td>Extension of the Hardy-Littlewood-Fefferman-Stein inequality</td>
<td>229</td>
</tr>
</tbody>
</table>