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**SOME REMARKS ON ACTIONS OF COMPACT MATRIX
QUANTUM GROUPS ON C^* -ALGEBRAS**

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SOME REMARKS ON ACTIONS OF COMPACT MATRIX QUANTUM GROUPS ON C^* -ALGEBRAS

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In this paper we construct an action of a compact matrix quantum group on a Cuntz algebra or a UHF-algebra, and investigate the fixed point subalgebra of the algebra under the action. Especially we consider the action of ${}_{\mu}U(2)$ on the Cuntz algebra \mathcal{O}_2 and the action of $S_{\mu}U(2)$ on the UHF-algebra of type 2^{∞} . We show that these fixed point subalgebras are generated by a sequence of Jones' projections.

1. Compact matrix quantum groups and their actions. We use the terminology introduced by Woronowicz([6]).

DEFINITION. Let A be a unital C^* -algebra and $u = (u_{kl})_{kl} \in M_n(A)$, and \mathcal{A} be the $*$ -subalgebra of A generated by the entries of u . Then $G = (A, u)$ is called a compact matrix quantum group (a compact matrix pseudogroup) if it satisfies the following three conditions:

- (1) \mathcal{A} is dense in A .
- (2) There exists a $*$ -homomorphism Φ (comultiplication) from A to $A \otimes_{\alpha} A$ such that

$$\Phi(u_{kl}) = \sum_{r=1}^n u_{kr} \otimes u_{rl} \quad (1 \leq k, l \leq n),$$

where the symbol \otimes_{α} means the spatial C^* -tensor product.

- (3) There exists a linear, antimultiplicative mapping κ from \mathcal{A} to \mathcal{A} such that

$$\kappa(\kappa(a^*)^*) = a \quad (a \in \mathcal{A})$$

and

$$\kappa(u_{kl}) = (u^{-1})_{kl} \quad (1 \leq k, l \leq n).$$

We call $w \in B(C^N) \otimes A \cong M_N \otimes A$ a representation of a compact matrix quantum group $G = (A, u)$ on C^N if $w \oplus w = (\text{id} \otimes \Phi)w$, where \oplus is a bilinear map of $(M_N \otimes A) \times (M_N \otimes A)$ to $M_N \otimes A \otimes A$ as follows:

$$(l \otimes a) \oplus (m \otimes b) = lm \otimes a \otimes b$$

for any $l, m \in M_N$ and $a, b \in A$.

It is known that a compact matrix quantum group $G = (A, u)$ has the Haar measure h , that is, h is a state on A satisfying

$$(h \otimes \text{id})\Phi(a) = (\text{id} \otimes h)\Phi(a) = h(a)1 \quad \text{for any } a \in A.$$

So any finite dimensional representation is equivalent to a unitary representation. In this paper we only treat a unitary representation of a compact matrix quantum group.

DEFINITION. Let B be a C^* -algebra and π be a $*$ -homomorphism from B to $B \otimes_\alpha A$. Then we call π an action of a compact matrix quantum group $G = (A, u)$ on B if $(\pi \otimes \text{id}_A)\pi = (\text{id}_B \otimes \Phi)\pi$.

Let w be a unitary representation of a compact matrix quantum group $G = (A, u)$ and belong to $M_N(A)$. We denote by \mathcal{O}_N the Cuntz algebra which is generated by isometries S_1, \dots, S_N satisfying $\sum_{i=1}^N S_i S_i^* = 1$ ([1]). Then we can construct an action of $G = (A, u)$ on \mathcal{O}_N simultaneously to [2], [3].

THEOREM 1. *For a unitary representation $w \in M_N(A)$ of a compact matrix quantum group $G = (A, u)$, there exists an action φ of the compact matrix quantum group $G = (A, u)$ on the Cuntz algebra \mathcal{O}_N such that*

$$\varphi(S_i) = \sum_{j=1}^N S_j \otimes w_{ji} \quad \text{for any } 1 \leq i \leq N.$$

Proof. We set $T_i = \varphi(S_i) = \sum_{j=1}^N S_j \otimes w_{ji}$ for any $i = 1, 2, \dots, N$. By the relation $S_i^* S_j = \delta_{ij}$ and the unitarity of w , T_i 's are isometries and $\sum_{i=1}^N T_i T_i^* = 1$. So φ can be extended to the $*$ -homomorphism from \mathcal{O}_N to $\mathcal{O}_N \otimes_\alpha A$. Then we have

$$(\varphi \otimes \text{id})\varphi(S_i) = \sum_{j,k=1}^N S_k \otimes w_{kj} \otimes w_{ji} = (\text{id} \otimes \Phi)\varphi(S_i)$$

for any $1 \leq i \leq N$. This implies that $(\varphi \otimes \text{id})\varphi = (\text{id} \otimes \Phi)\varphi$ on \mathcal{O}_N . \square

REMARK 2. Let ε be a $*$ -character from \mathcal{A} to the algebra C of all the complex numbers such that

$$\varepsilon(u_{ij}) = \delta_{ij}$$

for any $1 \leq i, j \leq n$ ([6]). If the above unitary representation w belongs to $M_N(\mathcal{A})$, then the relation,

$$(id \otimes \varepsilon)\varphi = id_{\mathcal{O}_N},$$

holds on the dense $*$ -subalgebra of \mathcal{O}_N generated by S_1, S_2, \dots, S_N . □

We denote by M_N^K the K -times tensor product of the $N \times N$ -matrix algebra M_N , and define a canonical embedding ι from M_N^K to \mathcal{O}_N by

$$\iota(e_{i_1 j_1} \otimes \cdots \otimes e_{i_k j_k}) = S_{i_1} \cdots S_{i_k} S_{j_k}^* \cdots S_{j_1}^*,$$

where $\{e_{ij}\}_{i,j=1}^N$ is a system of matrix units of M_N . This embedding ι is compatible with the canonical inclusion of M_N^K into M_N^{K+1} . We denote by M_N^∞ the UHF-algebra of type N^∞ , which is obtained as the inductive limit C^* -algebra of $\{M_N^K\}_{K=1}^\infty$. We may consider the UHF-algebra M_N^∞ as a C^* -subalgebra of \mathcal{O}_N through the embedding.

COROLLARY 3. *Let φ be the action of a compact matrix quantum group $G = (A, u)$ on the Cuntz algebra \mathcal{O}_N defined by the unitary representation $w \in M_N(A)$ as in Theorem 1. Then the restriction ψ of φ on the UHF-algebra M_N^∞ is also an action of $G = (A, u)$ on M_N^∞ satisfying*

$$\begin{aligned} \psi(e_{i_1 j_1} \otimes \cdots \otimes e_{i_k j_k}) &= \sum_{\substack{a_1, \dots, a_k \\ b_1, \dots, b_k}} e_{a_1 b_1} \otimes \cdots \otimes e_{a_k b_k} \\ &\otimes w_{a_1 i_1} \cdots w_{a_k i_k} w_{b_k j_k}^* \cdots w_{b_1 j_1}^* \end{aligned}$$

for any positive integer K .

REMARK 4. We define a bilinear map \oplus of $(M_N \otimes A) \times (M_N \otimes A)$ to $M_N \otimes M_N \otimes A$ as follows:

$$(l \otimes a) \oplus (m \otimes b) = l \otimes m \otimes ab$$

for any $l, m \in M_N$ and $a, b \in A$. We denote $\overbrace{w \oplus \cdots \oplus w}^{K \text{ times}}$ by w^K . Then w^K is a unitary representation of a compact matrix quantum group $G = (A, u)$ if w is a unitary representation of $G = (A, u)$. The above action ψ is represented by the following form

$$\psi(x) = w^K(x \otimes 1_A)(w^K)^* \quad \text{for any } x \in M_N^K.$$

So we call the action ψ the product type action of $G = (A, u)$ on the UHF-algebra M_N^∞ . \square

DEFINITION. Let B be a C^* -algebra and π be an action of a compact matrix quantum group $G = (A, u)$ on B . We define the fixed point subalgebra B^π of B by π as follows :

$$B^\pi = \{x \in B \mid \pi(x) = x \otimes 1_A\}.$$

Let \mathcal{P}_N be the dense $*$ -subalgebra of \mathcal{O}_N generated by S_1, S_2, \dots, S_N and \mathcal{M}_N be the dense $*$ -subalgebra $\bigcup_{K=1}^\infty M_N^K$ of M_N^∞ .

LEMMA 5. *Let h be the Haar measure on a compact matrix quantum group $G = (A, u)$, and we define $E_\varphi = (\text{id} \otimes h)\varphi$ and $E_\psi = (\text{id} \otimes h)\psi$. Then E_φ (resp. E_ψ) is a projection of norm one from \mathcal{O}_N onto $(\mathcal{O}_N)^\varphi$ (resp. from M_N^∞ onto $(M_N^\infty)^\psi$) such that*

$$E_\varphi(\mathcal{P}_N) \subset \mathcal{P}_N, \quad E_\psi(\mathcal{M}_N) \subset \mathcal{M}_N.$$

Proof. Clearly E_φ is a unital, completely positive map, $E_\varphi(x) = x$ for any $x \in (\mathcal{O}_N)^\varphi$, and $E_\varphi(\mathcal{P}_N) \subset \mathcal{P}_N$. By the property of the Haar measure, for any $x \in \mathcal{O}_N$, we have

$$\begin{aligned} E_\varphi(E_\varphi(x)) &= (\text{id} \otimes h \otimes \text{id})(\varphi \otimes \text{id})(\text{id} \otimes h)\varphi(x) \\ &= (\text{id} \otimes h \otimes h)(\varphi \otimes \text{id})\varphi(x) \\ &= (\text{id} \otimes h \otimes h)(\text{id} \otimes \Phi)\varphi(x) = (\text{id} \otimes (h \otimes h)\Phi)\varphi(x) \\ &= (\text{id} \otimes h)\varphi(x) = E_\varphi(x). \end{aligned}$$

So the assertion holds for E_φ .

Similarly the assertion also holds for E_ψ . \square

We can easily get the following lemma.

LEMMA 6. *Let π be an action of a compact matrix quantum group $G = (A, u)$ on a C^* -algebra B and B_0 be a dense $*$ -subalgebra of B . If E is a projection of norm one from B onto the fixed point subalgebra B^π of B by the action π such that $E(B_0) \subset B_0$, then $B_0 \cap B^\pi$ is dense in B^π .*

We define a $*$ -endomorphism σ of \mathcal{O}_N by $\sigma(X) = \sum_{i=1}^N S_i X S_i^*$ for any $X \in \mathcal{O}_N$. Then the restriction of σ to the UHF-algebra M_N^∞ of type N^∞ satisfies that $\sigma(X) = 1_{M_N} \otimes X$ for any $X \in M_N^\infty$.

LEMMA 7. (1) If $X \in (\mathcal{O}_N)^\varphi$, then $\sigma(X) \in (\mathcal{O}_N)^\varphi$.
 (2) If $X \in (M_N^\infty)^\psi$, then $\sigma(X) \in (M_N^\infty)^\psi$.

Proof. (1) For $X \in (\mathcal{O}_N)^\varphi$, we have

$$\begin{aligned} \varphi(\sigma(X)) &= \sum_{i=1}^N \varphi(S_i X S_i^*) = \sum_{i=1}^N \varphi(S_i)(X \otimes 1_A) \varphi(S_i)^* \\ &= \sum_{i,j,k=1}^N S_j X S_k^* \otimes u_{ij} u_{ik}^* = \sum_{i=1}^N S_i X S_i^* \otimes 1_A = \sigma(X) \otimes 1_A. \end{aligned}$$

(2) The assertion follows that ψ is the restriction of φ . \square

2. Jones' projections and compact matrix quantum groups $S_\mu U(2)$ and ${}_\mu U(2)$. We shall consider the actions of $S_\mu U(2)$ and ${}_\mu U(2)$ coming from their fundamental representations.

DEFINITION ([7]). A compact matrix quantum group $G = (A, u)$ is called $S_\mu U(2)$ if A is the universal C^* -algebra generated by α, γ satisfying

$$\alpha^* \alpha + \gamma^* \gamma = 1, \quad \alpha \alpha^* + \mu^2 \gamma \gamma^* = 1, \quad \gamma^* \gamma = \gamma \gamma^*,$$

$$\mu \gamma \alpha = \alpha \gamma, \quad \mu \gamma^* \alpha = \alpha \gamma^*, \quad \mu \alpha^* \gamma = \gamma \alpha^*, \quad \mu \alpha^* \gamma^* = \gamma^* \alpha^*,$$

where $-1 \leq \mu \leq 1$. Its fundamental representation u is as follows:

$$u = \begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix} \in M_2(A).$$

The comultiplication Φ associated with $S_\mu U(2)$ is defined by

$$\Phi(\alpha) = \alpha \otimes \alpha - \mu \gamma^* \otimes \gamma, \quad \Phi(\gamma) = \gamma \otimes \alpha + \alpha^* \otimes \gamma.$$

We shall introduce the quantum $U(2)$ group ${}_\mu U(2)$, which is obtained by the unitarization of the quantum $GL(2)$ group.

DEFINITION. A compact matrix quantum group $H = (B, v)$ is called ${}_\mu U(2)$ if B is the universal C^* -algebra generated by α, γ, D satisfying

$$D^* D = D D^* = 1, \quad \alpha D = D \alpha, \quad \gamma D = D \gamma, \quad \alpha^* D = D \alpha^*,$$

$$\gamma^* D = D \gamma^*, \quad \alpha^* \alpha + \gamma^* \gamma = 1, \quad \alpha \alpha^* + \mu^2 \gamma \gamma^* = 1, \quad \gamma^* \gamma = \gamma \gamma^*,$$

$$\mu \gamma \alpha = \alpha \gamma, \quad \mu \gamma^* \alpha = \alpha \gamma^*, \quad \mu \alpha^* \gamma = \gamma \alpha^*, \quad \mu \alpha^* \gamma^* = \gamma^* \alpha^*,$$

where $-1 \leq \mu \leq 1$. Its fundamental representation v is as follows:

$$v = \begin{pmatrix} \alpha & -\mu D\gamma^* \\ \gamma & D\alpha^* \end{pmatrix} \in M_2(B).$$

The comultiplication Ψ associated with ${}_\mu U(2)$ is defined by

$$\Psi(\alpha) = \alpha \otimes \alpha - \mu D\gamma^* \otimes \gamma, \quad \Psi(\gamma) = \gamma \otimes \alpha + D\alpha^* \otimes \gamma,$$

$$\Psi(D) = D \otimes D.$$

REMARK 8. The above C^* -algebra B associated with the compact matrix quantum group ${}_\mu U(2) = H = (B, v)$ is isomorphic to $A \otimes_\alpha C(T)$ as a C^* -algebra, where A is the C^* -algebra associated with the compact matrix quantum group $S_\mu U(2) = G = (A, u)$ and $C(T)$ is the algebra of all the continuous functions on the one dimensional torus T . The elements α and γ in H satisfy the same relation of α and γ in G . But the values of the comultiplication Ψ at α, γ differ from ones of the comultiplication Φ at α, γ . \square

In the rest of the paper, we fix a number $\mu \in [-1, 1] \setminus \{0\}$.

We denote by φ_1 (resp. by φ_2) the action of the compact matrix quantum group ${}_\mu U(2) = (B, v)$ (resp. $S_\mu U(2) = (A, u)$) on the Cuntz algebra \mathcal{O}_2 coming from the fundamental representation v (resp. u) as in Theorem 1. We also denote ψ_1 (resp. ψ_2) the product type action of the compact matrix quantum group ${}_\mu U(2) = (B, v)$ (resp. $S_\mu U(2) = (A, u)$) on the UHF-algebra M_2^∞ of type 2^∞ coming from v (resp. u) as in Corollary 3.

From now on, we shall determine the fixed point subalgebras of the above actions.

In [8] Woronowicz defines the 4×4 -matrix

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 \\ 0 & \mu & 1 - \mu^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in M_2 \otimes M_2 \subset M_2^\infty$$

and shows that the algebra $\{x \in M_2^K | u^K(x \otimes 1_A) = (x \otimes 1_A)u^K\}$ is generated by g_1, g_2, \dots, g_{K-1} , where $g_{i+1} = \sigma^i(g)$ ($i = 0, 1, \dots, K-2$).

We set

$$e_i = \frac{1}{1 + \mu^2}(1 - g_i) \quad \text{for any } i = 1, 2, \dots, K-1,$$

then the sequence $\{e_n\}_{n=1}^\infty$ of projections satisfies the Jones' relation

$$e_i e_{i\pm 1} e_i = \frac{\mu^2}{(1 + \mu^2)^2} e_i, \quad e_i e_j = e_j e_i \quad (\text{if } |i - j| > 1).$$

We denote by $C^*(\{e_n\}_{n=1}^\infty)$ the unital C^* -algebra generated by the projections $\{e_n\}_{n=1}^\infty$.

PROPOSITION 9. *The fixed point subalgebra $(M_2^\infty)^{S_\mu U(2)}$ of the UHF-algebra M_2^∞ by the action ψ_2 of $S_\mu U(2)$ is generated by the above Jones' projections $\{e_n\}_{n=1}^\infty$.*

Proof. By Remark 4, $M_2^K \cap (M_2^\infty)^{\psi_2} = \{x \in M_2^K \mid u^K(x \otimes 1_A) = (x \otimes 1_A)u^K\}$. So $M_2^K \cap (M_2^\infty)^{\psi_2}$ is generated by e_1, e_2, \dots, e_{K-1} . The assertion follows from Lemma 5 and Lemma 6. \square

THEOREM 10. *The fixed point subalgebra $(\mathcal{O}_2)^\mu U(2)$ of the Cuntz algebra \mathcal{O}_2 by the action φ_1 of ${}_\mu U(2) = (B, v)$ coincides with the fixed point subalgebra $(M_2^\infty)^{S_\mu U(2)}$ of the UHF-algebra M_2^∞ by the action ψ_2 of $S_\mu U(2) = (A, u)$.*

In particular,

$$(\mathcal{O}_2)^\mu U(2) = (M_2^\infty)^\mu U(2) = (M_2^\infty)^{S_\mu U(2)} = C^*(\{e_n\}_{n=1}^\infty).$$

Proof. It is clear that $(\mathcal{O}_2)^\mu U(2) \supset (M_2^\infty)^\mu U(2)$. In order to show that $(\mathcal{O}_2)^\mu U(2) \subset (M_2^\infty)^\mu U(2)$, it is sufficient to show that $\mathcal{P}_2 \cap (\mathcal{O}_2)^{\varphi_1} \subset (\mathcal{M}_2 \cap (M_2^\infty)^{\psi_1})$ by Lemma 5 and Lemma 6. Let $x \in \mathcal{P}_2 \cap (\mathcal{O}_2)^{\varphi_1}$ and θ be a $*$ -homomorphism of B onto $C^*(D)$ such that $\theta(\alpha) = D$, $\theta(\gamma) = 0$ and $\theta(D) = D^2$. The element x has the unique representation

$$x = \sum_{i>0} (S_1^*)^i A_{-i} + A_0 + \sum_{i>0} A_i (S_1)^i,$$

where each A_i ($i = 0, \pm 1, \pm 2, \dots$) belongs to \mathcal{M}_2 ([1]). Since $(\text{id}_{\mathcal{O}_2} \otimes \theta)\varphi_1(S_i) = S_i \otimes D$ for any $i = 1, 2$,

$$\begin{aligned} x \otimes 1_B &= (\text{id}_{\mathcal{O}_2} \otimes \theta)\varphi_1(x) \\ &= \sum_{i>0} (S_1^*)^i A_{-i} \otimes (D^*)^i + A_0 \otimes 1_B + \sum_{i>0} A_i (S_1)^i \otimes D^i. \end{aligned}$$

Hence $x = A_0 \in \mathcal{M}_2 \cap (M_2^\infty)^{\psi_1}$. Therefore $(\mathcal{O}_2)^\mu U(2) = (M_2^\infty)^\mu U(2)$.

We define a *-homomorphism η of B onto A such that $\eta(\alpha) = \alpha$, $\eta(\gamma) = \gamma$ and $\eta(D) = 1$. Then the following diagram commutes

$$\begin{array}{ccc} M_2^\infty & \xrightarrow{\psi_1} & M_2^\infty \otimes_\alpha B \\ \parallel & & \downarrow \text{id} \otimes \eta \\ M_2^\infty & \xrightarrow{\psi_2} & M_2^\infty \otimes_\alpha A. \end{array}$$

So $(M_2^\infty)_\mu^{U(2)} \subset (M_2^\infty)^{S_\mu U(2)}$.

We shall show that $(M_2^\infty)_\mu^{U(2)} \supset (M_2^\infty)^{S_\mu U(2)}$. It is sufficient to show that $(M_2^\infty)_\mu^{U(2)}$ contains $\{e_n\}_{n=1}^\infty$ by Proposition 9. We set

$$\tau = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & D & 0 \\ 0 & 0 & 0 & D^2 \end{pmatrix} \in M_4(B) \cong M_2 \otimes M_2 \otimes B,$$

then

$$v \oplus v = \left(\begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \right) \tau$$

and

$$\tau(e_1 \otimes 1_B) = (e_1 \otimes 1_B)\tau.$$

Then we have

$$\begin{aligned} \psi_1(e_1) &= (v \oplus v)(e_1 \otimes 1_B)(v \oplus v)^* \\ &= \left(\begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \right) \tau(e_1 \otimes 1_B) \tau^* \left(\begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \right)^* \\ &= \left(\begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \right) (e_1 \otimes 1_B) \left(\begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \right)^* \\ &= e_1 \otimes 1_B. \end{aligned}$$

By this fact and Lemma 7, $e_n \in (M_2^\infty)_\mu^{U(2)}$ for any positive integer n .

So the theorem holds. \square

REMARK 11. In the case $\mu = 1$,

$$e_i e_{i\pm 1} e_i = \frac{\mu^2}{(1 + \mu^2)^2} e_i = \frac{1}{4} e_i,$$

and the projection e_1 is represented as follows:

$$e_1 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Therefore the above theorem is a C^* -version of a deformation of Jones' result ([2], [4], [5]). \square

REFERENCES

- [1] J. Cuntz, *Simple C^* -algebras generated by isometries*, *Comm. Math. Phys.*, **57** (1977), 173–185.
- [2] M. Enomoto and Y. Watatani, *Young diagrams and C^* -algebras*, *Math. Japon.*, **25** (1980), 395–404.
- [3] D. E. Evans, *On \mathcal{O}_n* , *Publ. Res. Inst. Math. Sci.*, **16** (1980), 915–927.
- [4] F. M. Goodman, P. de la Harpe and V. F. R. Jones, *Coxeter Graphs and Towers of Algebras*, *Mathematical Sciences Research Institute Publications 14*, Springer-Verlag.
- [5] V. F. R. Jones, *Index for subfactors*, *Invent. Math.*, **72** (1983), 1–25.
- [6] S. L. Woronowicz, *Compact matrix pseudogroups*, *Comm. Math. Phys.*, **111** (1987), 613–665.
- [7] ———, *Twisted $SU(2)$ group. An example of a non-commutative differential calculus*, *Publ. Res. Inst. Math. Sci.*, **23** (1987), 117–181.
- [8] ———, *Tannaka-Krein duality for compact matrix pseudogroups. Twisted $SU(N)$ groups*, *Invent. Math.*, **93** (1988), 35–76.

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