SOME REMARKS ON ACTIONS OF COMPACT MATRIX QUANTUM GROUPS ON $C^*$-ALGEBRAS

Yuji Konishi, Masaru Nagisa and Yasuo Watatani
In this paper we construct an action of a compact matrix quantum group on a Cuntz algebra or a UHF-algebra, and investigate the fixed point subalgebra of the algebra under the action. Especially we consider the action of $\mu U(2)$ on the Cuntz algebra $\mathcal{O}_2$ and the action of $S_{\mu}U(2)$ on the UHF-algebra of type $2^\infty$. We show that these fixed point subalgebras are generated by a sequence of Jones’ projections.

1. Compact matrix quantum groups and their actions. We use the terminology introduced by Woronowicz([6]).

**Definition.** Let $A$ be a unital $C^*$-algebra and $u = (u_{kl})_{kl} \in M_n(A)$, and $\mathcal{A}$ be the $*$-subalgebra of $A$ generated by the entries of $u$. Then $G = (A, u)$ is called a compact matrix quantum group (a compact matrix pseudogroup) if it satisfies the following three conditions:

1. $\mathcal{A}$ is dense in $A$.
2. There exists a $*$-homomorphism $\Phi$ (comultiplication) from $A$ to $A \otimes _\alpha A$ such that
$$\Phi(u_{kl}) = \sum_{r=1}^{n} u_{kr} \otimes u_{rl} \quad (1 \leq k, l \leq n),$$
where the symbol $\otimes _\alpha$ means the spatial $C^*$-tensor product.
3. There exists a linear, antimultiplicative mapping $\kappa$ from $\mathcal{A}$ to $\mathcal{A}$ such that
$$\kappa(\kappa(a^*)^*) = a \quad (a \in \mathcal{A})$$
and
$$\kappa(u_{kl}) = (u^{-1})_{kl} \quad (1 \leq k, l \leq n).$$

We call $w \in B(C^N) \otimes A \cong M_N \otimes A$ a representation of a compact matrix quantum group $G = (A, u)$ on $C^N$ if $w \circ w = (\text{id} \otimes \Phi)w$, where $\circ$ is a bilinear map of $(M_N \otimes A) \times (M_N \otimes A)$ to $M_N \otimes A \otimes A$ as follows:
$$(l \otimes a) \circ (m \otimes b) = lm \otimes a \otimes b$$
for any $l, m \in M_N$ and $a, b \in A$. 

---

Yuji Konishi, Masaru Nagisa, and Yasuo Watatani
It is known that a compact matrix quantum group $G = (A, u)$ has the Haar measure $h$, that is, $h$ is a state on $A$ satisfying 

$$(h \otimes \text{id})\Phi(a) = (\text{id} \otimes h)\Phi(a) = h(a)1 \quad \text{for any } a \in A.$$ 

So any finite dimensional representation is equivalent to a unitary representation. In this paper we only treat a unitary representation of a compact matrix quantum group.

**Definition.** Let $B$ be a $C^*$-algebra and $\pi$ be a $^*$-homomorphism from $B$ to $B \otimes_\alpha A$. Then we call $\pi$ an action of a compact matrix quantum group $G = (A, u)$ on $B$ if $(\pi \otimes \text{id}_A)\pi = (\text{id} \otimes \Phi)\pi$.

Let $w$ be a unitary representation of a compact matrix quantum group $G = (A, u)$ and belong to $M_N(A)$. We denote by $\mathcal{O}_N$ the Cuntz algebra which is generated by isometries $S_1, \ldots, S_N$ satisfying $\sum_{i=1}^N S_iS_i^* = 1$ ([1]). Then we can construct an action of $G = (A, u)$ on $\mathcal{O}_N$ simultaneously to [2], [3].

**Theorem 1.** For a unitary representation $w \in M_N(A)$ of a compact matrix quantum group $G = (A, u)$, there exists an action $\phi$ of the compact matrix quantum group $G = (A, u)$ on the Cuntz algebra $\mathcal{O}_N$ such that

$$\phi(S_i) = \sum_{j=1}^N S_j \otimes w_{ji} \quad \text{for any } 1 \leq i \leq N.$$ 

**Proof.** We set $T_i = \phi(S_i) = \sum_{j=1}^N S_j \otimes w_{ji}$ for any $i = 1, 2, \ldots, N$. By the relation $S_i^*S_j = \delta_{ij}$ and the unitarity of $w$, $T_i$'s are isometries and $\sum_{i=1}^N T_iT_i^* = 1$. So $\phi$ can be extended to the $^*$-homomorphism from $\mathcal{O}_N$ to $\mathcal{O}_N \otimes_\alpha A$. Then we have

$$(\phi \otimes \text{id})\phi(S_i) = \sum_{j,k=1}^N S_k \otimes w_{kj} \otimes w_{ji} = (\text{id} \otimes \Phi)\phi(S_i)$$

for any $1 \leq i \leq N$. This implies that $(\phi \otimes \text{id})\phi = (\text{id} \otimes \Phi)\phi$ on $\mathcal{O}_N$. $\square$

**Remark 2.** Let $\varepsilon$ be a $^*$-character from $\mathcal{A}$ to the algebra $C$ of all the complex numbers such that

$$\varepsilon(u_{ij}) = \delta_{ij}$$
for any $1 \leq i, j \leq n$ ([6]). If the above unitary representation $w$ belongs to $M_N(\mathcal{A})$, then the relation,

$$(id \otimes \varepsilon)\varphi = id_{\mathcal{A}_N},$$

holds on the dense $*$-subalgebra of $\mathcal{A}_N$ generated by $S_1, S_2, \ldots, S_N$. □

We denote by $M_N^K$ the $K$-times tensor product of the $N \times N$-matrix algebra $M_N$, and define a canonical embedding $\iota$ from $M_N^K$ to $\mathcal{A}_N$ by

$$\iota(e_{i_1,j_1} \otimes \cdots \otimes e_{i_k,j_k}) = S_{i_1} \cdots S_{i_k} S_{j_k}^* \cdots S_{j_1}^*,$$

where $\{e_{ij}\}_{i,j=1}^N$ is a system of matrix units of $M_N$. This embedding $\iota$ is compatible with the canonical inclusion of $M_N^K$ into $M_N^{K+1}$. We denote by $M_N^{\infty}$ the UHF-algebra of type $N^\infty$, which is obtained as the inductive limit $C^*$-algebra of $\{M_N^K\}_{K=1}^\infty$. We may consider the UHF-algebra $M_N^{\infty}$ as a $C^*$-subalgebra of $\mathcal{A}_N$ through the embedding.

**Corollary 3.** Let $\varphi$ be the action of a compact matrix quantum group $G = (A, u)$ on the Cuntz algebra $\mathcal{A}_N$ defined by the unitary representation $w \in M_N(A)$ as in Theorem 1. Then the restriction $\psi$ of $\varphi$ on the UHF-algebra $M_N^{\infty}$ is also an action of $G = (A, u)$ on $M_N^{\infty}$ satisfying

$$\psi(e_{i_1,j_1} \otimes \cdots \otimes e_{i_k,j_k}) = \sum_{a_1,\ldots,a_k} e_{a_1,b_1} \otimes \cdots \otimes e_{a_k,b_k} \otimes w_{a_1,i_1} \cdots w_{a_k,i_k} w_{b_1,j_1}^* \cdots w_{b_k,j_k}^*$$

for any positive integer $K$.

**Remark 4.** We define a bilinear map $\oplus$ of $(M_N \otimes A) \times (M_N \otimes A)$ to $M_N \otimes M_N \otimes A$ as follows:

$$(l \otimes a) \oplus (m \otimes b) = l \otimes m \otimes ab$$

for any $l, m \in M_N$ and $a, b \in A$. We denote $\overbrace{w \oplus \cdots \oplus w}^{K \text{ times}}$ by $w^K$. Then $w^K$ is a unitary representation of a compact matrix quantum group $G = (A, u)$ if $w$ is a unitary representation of $G = (A, u)$. The above action $\psi$ is represented by the following form

$$\psi(x) = w^K(x \otimes 1_A)(w^K)^*$$

for any $x \in M_N^K$. 
So we call the action $\psi$ the product type action of $G = (A, u)$ on the UHF-algebra $M_N^\infty$.

**DEFINITION.** Let $B$ be a $C^*$-algebra and $\pi$ be an action of a compact matrix quantum group $G = (A, u)$ on $B$. We define the fixed point subalgebra $B^\pi$ of $B$ by $\pi$ as follows:

$$B^\pi = \{ x \in B | \pi(x) = x \otimes 1_A \}.$$  

Let $\mathcal{P}_N$ be the dense *-subalgebra of $\mathcal{O}_N$ generated by $S_1, S_2, \ldots, S_N$ and $\mathcal{M}_N$ be the dense *-subalgebra $\bigcup_{K=1}^\infty M_N^K$ of $M_N^\infty$.

**LEMMA 5.** Let $h$ be the Haar measure on a compact matrix quantum group $G = (A, u)$, and we define $E_\varphi = (id \otimes h)\varphi$ and $E_\psi = (id \otimes h)\psi$. Then $E_\varphi$ (resp. $E_\psi$) is a projection of norm one from $\mathcal{O}_N$ onto $(\mathcal{O}_N)^\varphi$ (resp. from $M_N^\infty$ onto $(M_N^\infty)^\psi$) such that

$$E_\varphi(\mathcal{P}_N) \subset \mathcal{P}_N, \quad E_\psi(\mathcal{M}_N) \subset \mathcal{M}_N.$$  

**Proof.** Clearly $E_\varphi$ is a unital, completely positive map, $E_\varphi(x) = x$ for any $x \in (\mathcal{O}_N)^\varphi$, and $E_\varphi(\mathcal{P}_N) \subset \mathcal{P}_N$. By the property of the Haar measure, for any $x \in \mathcal{O}_N$, we have

$$E_\varphi(E_\varphi(x)) = (id \otimes h \otimes id)(\varphi \otimes id)(id \otimes h)\varphi(x)$$

$$= (id \otimes h \otimes h)(\varphi \otimes id)\varphi(x)$$

$$= (id \otimes h)(id \otimes \Phi)\varphi(x) = (id \otimes (h \otimes h)\Phi)\varphi(x)$$

$$= (id \otimes h)\varphi(x) = E_\varphi(x).$$

So the assertion holds for $E_\varphi$.

Similarly the assertion also holds for $E_\psi$.  

We can easily get the following lemma.

**LEMMA 6.** Let $\pi$ be an action of a compact matrix quantum group $G = (A, u)$ on a $C^*$-algebra $B$ and $B_0$ be a dense *-subalgebra of $B$. If $E$ is a projection of norm one from $B$ onto the fixed point subalgebra $B^\pi$ of $B$ by the action $\pi$ such that $E(B_0) \subset B_0$, then $B_0 \cap B^\pi$ is dense in $B^\pi$.

We define a *-endomorphism $\sigma$ of $\mathcal{O}_N$ by $\sigma(X) = \sum_{i=1}^N S_iXS_i^*$ for any $X \in \mathcal{O}_N$. Then the restriction of $\sigma$ to the UHF-algebra $M_N^\infty$ of type $N^\infty$ satisfies that $\sigma(X) = 1_{M_N} \otimes X$ for any $X \in M_N^\infty$.  


LEMMA 7. (1) If $X \in (\Omega_N)^\phi$, then $\sigma(X) \in (\Omega_N)^\phi$.
(2) If $X \in (M_N^\infty)^\psi$, then $\sigma(X) \in (M_N^\infty)^\psi$.

Proof. (1) For $X \in (\Omega_N)^\phi$, we have

$$\phi(\sigma(X)) = \sum_{i=1}^{N} \phi(S_iXS^*_i) = \sum_{i=1}^{N} \phi(S_i)(X \otimes 1_A)\phi(S_i)^* = \sum_{i,j,k=1}^{N} S_jXS^*_k \otimes u_{ij}u^*_{ik} = \sum_{i=1}^{N} S_iXS^*_i \otimes 1_A = \sigma(X) \otimes 1_A.$$ 

(2) The assertion follows that $\psi$ is the restriction of $\phi$. \hfill \Box

2. Jones’ projections and compact matrix quantum groups $S_\mu U(2)$ and $\mu U(2)$. We shall consider the actions of $S_\mu U(2)$ and $\mu U(2)$ coming from their fundamental representations.

DEFINITION ([7]). A compact matrix quantum group $G = (A, \upsilon)$ is called $S_\mu U(2)$ if $A$ is the universal $C^*$-algebra generated by $\alpha, \gamma$ satisfying

$$\alpha^*\alpha + \gamma^*\gamma = 1, \quad \alpha\alpha^* + \mu^2\gamma\gamma^* = 1, \quad \gamma^*\gamma = \gamma\gamma^*,$$

$$\mu\gamma\alpha = \alpha\gamma, \quad \mu\gamma^*\alpha = \alpha\gamma^*, \quad \mu\alpha^*\gamma = \gamma\alpha^*, \quad \mu\alpha^*\gamma^* = \gamma^*\alpha^*,$$

where $-1 < \mu < 1$. Its fundamental representation $\upsilon$ is as follows:

$$\upsilon = \begin{pmatrix} \alpha & -\mu\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \in M_2(A).$$

The comultiplication $\Phi$ associated with $S_\mu U(2)$ is defined by

$$\Phi(\alpha) = \alpha \otimes \alpha - \mu\gamma^* \otimes \gamma, \quad \Phi(\gamma) = \gamma \otimes \alpha + \alpha^* \otimes \gamma.$$

We shall introduce the quantum $U(2)$ group $\mu U(2)$, which is obtained by the unitarization of the quantum GL(2) group.

DEFINITION. A compact matrix quantum group $H = (B, \upsilon)$ is called $\mu U(2)$ if $B$ is the universal $C^*$-algebra generated by $\alpha, \gamma, D$ satisfying

$$D^*D = DD^* = 1, \quad \alpha D = D\alpha, \quad \gamma D = D\gamma, \quad \alpha^*D = D\alpha^*,$$

$$\gamma^*D = D\gamma^*, \quad \alpha^*\alpha + \gamma^*\gamma = 1, \quad \alpha\alpha^* + \mu^2\gamma\gamma^* = 1, \quad \gamma^*\gamma = \gamma\gamma^*,$$

$$\mu\gamma\alpha = \alpha\gamma, \quad \mu\gamma^*\alpha = \alpha\gamma^*, \quad \mu\alpha^*\gamma = \gamma\alpha^*, \quad \mu\alpha^*\gamma^* = \gamma^*\alpha^*,$$
where \(-1 < \mu < 1\). Its fundamental representation \(\upsilon\) is as follows:

\[
\upsilon = \begin{pmatrix} \alpha & -\mu D\gamma^* \\ \gamma & D\alpha^* \end{pmatrix} \in M_2(B).
\]

The comultiplication \(\Psi\) associated with \(\mu U(2)\) is defined by

\[
\Psi(\alpha) = \alpha \otimes \alpha - \mu D\gamma^* \otimes \gamma, \quad \Psi(\gamma) = \gamma \otimes \alpha + D\alpha^* \otimes \gamma,
\]

\[
\Psi(D) = D \otimes D.
\]

**Remark 8.** The above \(C^*\)-algebra \(B\) associated with the compact matrix quantum group \(\mu U(2) = H = (B, \upsilon)\) is isomorphic to \(A \otimes_\alpha C(T)\) as a \(C^*\)-algebra, where \(A\) is the \(C^*\)-algebra associated with the compact matrix quantum group \(S_\mu U(2) = G = (A, u)\) and \(C(T)\) is the algebra of all the continuous functions on the one dimensional torus \(T\). The elements \(\alpha\) and \(\gamma\) in \(H\) satisfy the same relation of \(\alpha\) and \(\gamma\) in \(G\). But the values of the comultiplication \(\Psi\) at \(\alpha, \gamma\) differ from ones of the comultiplication \(\Phi\) at \(\alpha, \gamma\).


In the rest of the paper, we fix a number \(\mu \in [-1, 1]\setminus\{0\}\).

We denote by \(\varphi_1\) (resp. by \(\varphi_2\)) the action of the compact matrix quantum group \(\mu U(2) = (B, \upsilon)\) (resp. \(S_\mu U(2) = (A, u)\)) on the Cuntz algebra \(\mathcal{O}_2\) coming from the fundamental representation \(\upsilon\) (resp. \(u\)) as in Theorem 1. We also denote \(\psi_1\) (resp. \(\psi_2\)) the product type action of the compact matrix quantum group \(\mu U(2) = (B, \upsilon)\) (resp. \(S_\mu U(2) = (A, u)\)) on the UHF-algebra \(M_2^\infty\) of type \(2^\infty\) coming from \(\upsilon\) (resp. \(u\)) as in Corollary 3.

From now on, we shall determine the fixed point subalgebras of the above actions.

In [8] Woronowicz defines the \(4 \times 4\)-matrix

\[
g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 \\ 0 & \mu & 1 - \mu^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in M_2 \otimes M_2 \subset M_2^\infty
\]

and shows that the algebra \(\{x \in M_2^K| u^K(x \otimes 1_A) = (x \otimes 1_A)u^K\}\) is generated by \(g_1, g_2, \ldots, g_{K-1}\), where \(g_{i+1} = \sigma^i(g)\) \((i = 0, 1, \ldots, K-2)\).

We set

\[
e_i = \frac{1}{1 + \mu^2}(1 - g_i) \quad \text{for any } i = 1, 2, \ldots, K - 1,
\]
then the sequence \( \{e_n\}_{n=1}^{\infty} \) of projections satisfies the Jones’ relation

\[
e_i e_{i \pm 1} e_i = \frac{\mu^2}{(1 + \mu^2)^2} e_i, \quad e_i e_j = e_j e_i \quad (\text{if } |i - j| > 1).
\]

We denote by \( C^* \{\{e_n\}_{n=1}^{\infty}\} \) the unital \( C^* \)-algebra generated by the projections \( \{e_n\}_{n=1}^{\infty} \).

**Proposition 9.** The fixed point subalgebra \( (M_2^{\infty})^{\psi_2} \) of the UHF-algebra \( M_2^{\infty} \) by the action \( \psi_2 \) of \( S_\mu U(2) \) is generated by the above Jones’ projections \( \{e_n\}_{n=1}^{\infty} \).

**Proof.** By Remark 4, \( M_2^K \cap (M_2^{\infty})^{\psi_2} = \{x \in M_2^K|u^K(x \otimes 1_A) = (x \otimes 1_A)u^K\} \). So \( M_2^K \cap (M_2^{\infty})^{\psi_2} \) is generated by \( e_1, e_2, \ldots, e_{K-1} \). The assertion follows from Lemma 5 and Lemma 6.

**Theorem 10.** The fixed point subalgebra \( (\mathcal{O}_2)^{\mu U(2)} \) of the Cuntz algebra \( \mathcal{O}_2 \) by the action \( \varphi_1 \) of \( \mu U(2) = (B, v) \) coincides with the fixed point subalgebra \( (M_2^{\infty})^{\psi_2} \) of the UHF-algebra \( M_2^{\infty} \) by the action \( \psi_2 \) of \( S_\mu U(2) = (A, u) \).

In particular,

\[
(\mathcal{O}_2)^{\mu U(2)} = (M_2^{\infty})^{\mu U(2)} = (M_2^{\infty})^{\psi_2} = C^* \{\{e_n\}_{n=1}^{\infty}\}.
\]

**Proof.** It is clear that \( (\mathcal{O}_2)^{\mu U(2)} \supset (M_2^{\infty})^{\mu U(2)} \). In order to show that \( (\mathcal{O}_2)^{\mu U(2)} \subset (M_2^{\infty})^{\mu U(2)} \), it is sufficient to show that \( \mathcal{O}_2 \cap (\mathcal{O}_2)^{\varphi_1} \subset (\mathcal{M}_2 \cap (M_2^{\infty})^{\psi_1}) \) by Lemma 5 and Lemma 6. Let \( x \in \mathcal{O}_2 \cap (\mathcal{O}_2)^{\varphi_1} \) and \( \theta \) be a \(*\)-homomorphism of \( B \) onto \( C^*(D) \) such that \( \theta(\alpha) = D, \theta(\gamma) = 0 \) and \( \theta(D) = D^2 \). The element \( x \) has the unique representation

\[
x = \sum_{i>0} (S_1^*)^i A_{-i} + A_0 + \sum_{i>0} A_i (S_1)^i,
\]

where each \( A_i \) (\( i = 0, \pm 1, \pm 2, \ldots \)) belongs to \( \mathcal{M}_2 \) ([1]). Since \( (id_{\mathcal{O}_2} \otimes \theta)\varphi_1(S_i) = S_i \otimes D \) for any \( i = 1, 2 \),

\[
x \otimes 1_B = (id_{\mathcal{O}_2} \otimes \theta)\varphi_1(x)
= \sum_{i>0} (S_1^*)^i A_{-i} \otimes (D^*)^i + A_0 \otimes 1_B + \sum_{i>0} A_i (S_1)^i \otimes D^i.
\]

Hence \( x = A_0 \in \mathcal{M}_2 \cap (M_2^{\infty})^{\psi_1} \). Therefore \( (\mathcal{O}_2)^{\mu U(2)} = (M_2^{\infty})^{\mu U(2)} \).
We define a \( \ast \)-homomorphism \( \eta \) of \( B \) onto \( A \) such that \( \eta(\alpha) = \alpha \), \( \eta(\gamma) = \gamma \) and \( \eta(D) = 1 \). Then the following diagram commutes:

\[
\begin{array}{cc}
M_2^\infty & \overset{\psi_1}{\longrightarrow} & M_2^\infty \otimes_\alpha B \\
\uparrow & & \downarrow \text{id} \otimes \eta \\
M_2^\infty & \overset{\psi_2}{\longrightarrow} & M_2^\infty \otimes_\alpha A.
\end{array}
\]

So \( (M_2^\infty)^{uU(2)} \subset (M_2^\infty)^{S_uU(2)} \).

We shall show that \( (M_2^\infty)^{uU(2)} \supset (M_2^\infty)^{S_uU(2)} \). It is sufficient to show that \( (M_2^\infty)^{uU(2)} \) contains \( \{e_n\}_{n=1}^\infty \) by Proposition 9. We set

\[\tau = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & D & 0 & 0 \\
0 & 0 & D & 0 \\
0 & 0 & 0 & D^2
\end{pmatrix} \in M_4(B) \cong M_2 \otimes M_2 \otimes B,
\]

then

\[v \oplus v = \left(\begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix}\right) \tau\]

and

\[\tau(e_1 \otimes 1_B) = (e_1 \otimes 1_B)\tau.
\]

Then we have

\[
\psi_1(e_1) = (v \oplus v)(e_1 \otimes 1_B)(v \oplus v)^* \\
= \left(\begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix}\right) \tau(e_1 \otimes 1_B)\tau^* \left(\begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix}\right)^* \\
= \left(\begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix}\right)(e_1 \otimes 1_B) \left(\begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix} \oplus \begin{pmatrix} \alpha & -\mu \gamma^* \\ \gamma & \alpha^* \end{pmatrix}\right)^* \\
= e_1 \otimes 1_B.
\]

By this fact and Lemma 7, \( e_n \in (M_2^\infty)^{uU(2)} \) for any positive integer \( n \).

So the theorem holds.

\[\square\]

**Remark 11.** In the case \( \mu = 1 \),

\[e_i e_{i \pm 1} e_i = \frac{\mu^2}{(1 + \mu^2)^2} e_i = \frac{1}{4} e_i,
\]

and the projection \( e_1 \) is represented as follows:

\[e_1 = \frac{1}{2} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.
\]
Therefore the above theorem is a $C^*$-version of a deformation of Jones' result ([2], [4], [5]).

References


Received April 4, 1990.

Niigata University
Niigata 950-21, Japan

Chiba University
Chiba 260, Japan

AND

Hokkaido University
Sapporo 060, Japan
<table>
<thead>
<tr>
<th>Authors</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patrick Robert Ahern and Carmen Cascante</td>
<td>Exceptional sets for Poisson integrals of potentials on the unit sphere in $\mathbb{C}^n$, $p \leq 1$</td>
<td>1</td>
</tr>
<tr>
<td>David Peter Blecher</td>
<td>The standard dual of an operator space</td>
<td>15</td>
</tr>
<tr>
<td>Patrick Gilmer</td>
<td>Real algebraic curves and link cobordism</td>
<td>31</td>
</tr>
<tr>
<td>Simon M. Goberstein</td>
<td>On orthodox semigroups determined by their bundles of correspondences</td>
<td>71</td>
</tr>
<tr>
<td>John Kalliongis and Darryl John McCullough</td>
<td>Homeotopy groups of irreducible 3-manifolds which may contain two-sided projective planes</td>
<td>85</td>
</tr>
<tr>
<td>Yuji Konishi, Masaru Nagisa and Yasuo Watatani</td>
<td>Some remarks on actions of compact matrix quantum groups on $C^*$-algebras</td>
<td>119</td>
</tr>
<tr>
<td>Guojun Liao and Luen-Fai Tam</td>
<td>On the heat equation for harmonic maps from noncompact manifolds</td>
<td>129</td>
</tr>
<tr>
<td>John Marafino</td>
<td>Boundary behavior of a conformal mapping</td>
<td>147</td>
</tr>
<tr>
<td>Ji Min</td>
<td>A remark on the symmetry of solutions to nonlinear elliptic equations</td>
<td>157</td>
</tr>
<tr>
<td>Paul Nevai and Walter Van Assche</td>
<td>Compact perturbations of orthogonal polynomials</td>
<td>163</td>
</tr>
<tr>
<td>Kyril Tintarev</td>
<td>Level set maxima and quasilinear elliptic problems</td>
<td>185</td>
</tr>
</tbody>
</table>