

*Pacific
Journal of
Mathematics*

**A SPLITTING CRITERION FOR RANK 2 VECTOR BUNDLES
ON P^n**

EDOARDO BALLICO

Volume 169 No. 1

May 1995

A SPLITTING CRITERION FOR RANK 2 VECTOR BUNDLES ON \mathbf{P}^n

E. BALLICO

This is an addendum to a recent paper of V. Ancona, T. Peternell and J. Wisniewski. Here we prove (using heavily their paper) two criteria for the splitting of rank 2 algebraic vector bundles (one on \mathbf{P}^n and one on certain algebraic complete manifolds).

More precisely, the aim here is to show why the proofs of [1, Th. 10.5], and [1, Th. 10.13], give the following two theorems.

THEOREM 1. *Let E be a rank 2 algebraic vector bundle on \mathbf{P}^n which satisfies the assumptions of [1, Th. 10.5]. Then E splits.*

THEOREM 2. *Let E be a rank 2 algebraic vector bundle on a projective manifold X with (X, E) satisfying the assumption of [1, Th. 10.13]. Then E splits.*

The assumptions on X in Theorem 2 are very restrictive (e.g. X is a Fano manifold with $\text{Pic}(X) \cong \mathbf{Z}$). We only remark that the assumptions of Theorem 1 are satisfied if there is a two dimensional projective family, S , of lines in \mathbf{P}^n such that the splitting type of $E|L$ is the same for all $L \in S$.

Proof of Theorem 1. By the statement of [1, Th. 10.13], E numerically splits, i.e. it has the same Chern classes of a direct sum of 2 line bundles, i.e. there are integers a_1, a_2 with $a_1 \leq a_2$ such that $c_1(E) = a_1 + a_2$ and $c_2(E) = a_1 a_2$. The key remark is that the proof of [1, Th. 10.5], shows the existence of a line L such that $E|L \cong \mathbf{O}_L(a_1) \oplus \mathbf{O}_L(a_2)$. Since $4c_2(E) - c_1(E)^2 \leq 0$, E is not stable. Hence there is an integer $t \geq (a_1 + a_2)/2$ such that $H^0(\mathbf{P}^n, E(-t)) \neq 0$; take as t the minimal one; the corresponding section s of $E(-t)$ will vanish on a codimension 2 subscheme, Z , with $\deg(Z) = c_2(E(-t))$. Since $c_2(E(-x)) < 0$ if $a_1 < x < a_2$, we have $t \geq a_2$. If $t = a_2$ we obtain $Z = \emptyset$; hence E splits. Hence we

may assume $t > a_2$. This implies that $s|D = 0$ for every line D such that $E|D \cong \mathbf{O}_D(b_1) \oplus \mathbf{O}_D(b_2)$ with $a_1 \leq b_1 \leq b_2 \leq a_2$; in particular by semicontinuity this is true for a general line of \mathbf{P}^n . Hence $s = 0$, contradiction. \square

The proof of Theorem 2 is simply the remark (following [1], Remark 10.12) that, having Theorem 1 instead of the statement of [1, Th. 10.5], we obtain the stronger assertion of Theorem 2 instead of the numerical splitting asserted by [1, Th. 10.13].

The proof of Theorem 1 (i.e. of the small part of [1] needed) works in positive characteristic. The same remark applies to Theorem 2 if we assume $\text{Pic}(X) \cong \mathbf{Z}$ instead of making the assumptions on X which by [2] imply in characteristic 0 that $\text{Pic}(X) \cong \mathbf{Z}$.

We think that [1, Remark 10.12], (on the extension of [1, §10], to other manifolds) is potentially very interesting and we hope that some reader will be able to use it.

REFERENCES

- [1] V. Ancona, T. Peternell and J. Wisniewski, *Fano bundles and splitting theorems on projective spaces and quadrics*, Pacific J. Math., (to appear).
- [2] J. Wisniewski, *On a conjecture of Mukai*, Manuscripta Math., **68** (1990), 135-141.

Received August 12, 1992.

UNIVERSITY OF TRENTO

38050 PROVO (TN)

ITALY

E-mail address: ballico@itncisca.bitnet

ballico@science.unitn.it

PACIFIC JOURNAL OF MATHEMATICS

Founded by E. F. Beckenbach (1906-1982) and F. Wolf (1904-1989)

EDITORS

Sun-Yung Alice Chang (Managing Editor)
University of California
Los Angeles, CA 90095-1555
pacific@math.ucla.edu

F. Michael Christ
University of California
Los Angeles, CA 90095-1555
christ@math.ucla.edu

Robert Finn
Stanford University
Stanford, CA 94305
finn@gauss.stanford.edu

Martin Scharlemann
University of California
Santa Barbara, CA 93106
mgscharl@math.ucsb.edu

Thomas Enright
University of California
San Diego, La Jolla, CA 92093
tenright@ucsd.edu

Vaughan F. R. Jones
University of California
Berkeley, CA 94720
vfr@math.berkeley.edu

Gang Tian
Courant Institute
New York University
New York, NY 10012-1100
tiang@taotao.cims.nyu.edu

Nicholas Ercolani
University of Arizona
Tucson, AZ 85721
ercolani@math.arizona.edu

Steven Kerckhoff
Stanford University
Stanford, CA 94305
spk@gauss.stanford.edu

V. S. Varadarajan
University of California
Los Angeles, CA 90095-1555
vsv@math.ucla.edu

SUPPORTING INSTITUTIONS

CALIFORNIA INSTITUTE OF TECHNOLOGY
NEW MEXICO STATE UNIVERSITY
OREGON STATE UNIVERSITY
STANFORD UNIVERSITY
UNIVERSITY OF ARIZONA
UNIVERSITY OF BRITISH COLUMBIA
UNIVERSITY OF CALIFORNIA
UNIVERSITY OF HAWAII

UNIVERSITY OF MONTANA
UNIVERSITY OF NEVADA, RENO
UNIVERSITY OF OREGON
UNIVERSITY OF SOUTHERN CALIFORNIA
UNIVERSITY OF UTAH
UNIVERSITY OF WASHINGTON
WASHINGTON STATE UNIVERSITY

The supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its contents or policies.

Manuscripts must be prepared in accordance with the instructions provided on the inside back cover.

The *Pacific Journal of Mathematics* (ISSN 0030-8730) is published monthly except for July and August. Regular subscription rate: \$215.00 a year (10 issues). Special rate: \$108.00 a year to individual members of supporting institutions.

Subscriptions, orders for back issues published within the last three years, and changes of subscribers address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. Prior back issues are obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

The Pacific Journal of Mathematics at the University of California, c/o Department of Mathematics, 981 Evans Hall, Berkeley, CA 94720 (ISSN 0030-8730) is published monthly except for July and August. Second-class postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 6143, Berkeley, CA 94704-0163.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS at University of California,
Berkeley, CA 94720, A NON-PROFIT CORPORATION

This publication was typeset using AMS-LATEX,
the American Mathematical Society's TEX macro system.

Copyright © 1995 by Pacific Journal of Mathematics

PACIFIC JOURNAL OF MATHEMATICS

Volume 169 No. 1 May 1995

Minimal sets of periods for torus maps via Nielsen numbers	1
LLUÍS ALSÈDÀ, STEWART BALDWIN, JAUME LLIBRE, RICHARD SWANSON and WIESLAW SZLENK	
Diagonalizing Hilbert cusp forms	33
TIMOTHY ATWILL	
A splitting criterion for rank 2 vector bundles on \mathbf{P}^n	51
EDOARDO BALLICO	
Controlling Tietze-Urysohn extensions	53
MARC FRANTZ	
Length of Julia curves	75
DAVID H. HAMILTON	
On the uniqueness of capillary surfaces over an infinite strip	95
JENN-FANG HWANG	
Volume estimates for log-concave densities with application to iterated convolutions	107
MARIUS JUNGE	
A reflection principle in complex space for a class of hypersurfaces and mappings	135
FRANCINE ANTOINETTE MEYLAN	
Jean Bourgain's analytic partition of unity via holomorphic martingales	161
PAUL F.X. MÜLLER	
Characters of Brauer's centralizer algebras	173
ARUN RAM	



0030-8730(1995)169:1;1-C