Pacific Journal of Mathematics

A SPLITTING CRITERION FOR RANK 2 VECTOR BUNDLES ON P^n

EDOARDO BALLICO

Volume 169 No. 1 May 1995

PACIFIC JOURNAL OF MATHEMATICS
Vol. 169, No. 1, 1995

A SPLITTING CRITERION FOR RANK 2 VECTOR BUNDLES ON $\mathbf{P}^n$

E. BALLICO

This is an addendum to a recent paper of V. Ancona, T. Peternell and J. Wisniewski. Here we prove (using heavily their paper) two criteria for the splitting of rank 2 algebraic vector bundles (one on $\mathbf{P}^n$ and one on certain algebraic complete manifolds).

More precisely, the aim here is to show why the proofs of [**1**, Th. 10.5], and [**1**, Th. 10.13], give the following two theorems.

THEOREM 1. *Let E be a rank 2 algebraic vector bundle on $\mathbf{P}^n$ which satisfies the assumptions of* [**1**, Th. 10.5]. *Then E splits.*

THEOREM 2. *Let E be a rank 2 algebraic vector bundle on a projective manifold X with (X, E) satisfying the assumption of* [**1**, Th. 10.13]. *Then E splits.*

The assumptions on X in Theorem 2 are very restrictive (e.g. X is a Fano manifold with $\mathrm{Pic}(X) \cong \mathbf{Z}$). We only remark that the assumptions of Theorem 1 are satisfied if there is a two dimentional projective family, S, of lines in $\mathbf{P}^n$ such that the splitting type of $E|L$ is the same for all $L \in S$.

Proof of Theorem 1. By the statement of [**1**, Th. 10.13], E numerically splits, i.e. it has the same Chern classes of a direct sum of 2 line bundles, i.e. there are integers a_1, a_2 with $a_1 \leq a_2$ such that $c_1(E) = a_1 + a_2$ and $c_2(E) = a_1 a_2$. The key remark is that the proof of [**1**, Th. 10.5], shows the existence of a line L such that $E|L \cong \mathbf{O}_L(a_1) \oplus \mathbf{O}_L(a_2)$. Since $4c_2(E) - c_1(E)^2 \leq 0$, E is not stable. Hence there is an integer $t \geq (a_1 + a_2)/2$ such that $H^0(\mathbf{P}^n, E(-t)) \neq 0$; take as t the minimal one; the corresponding section $\mathbf{s}$ of $E(-t)$ will vanish on a codimension 2 subscheme, Z, with $\deg(Z) = c_2(E(-t))$. Since $c_2(E(-x)) < 0$ if $a_1 < x < a_2$, we have $t \geq a_2$. If $t = a_2$ we obtain $Z = \varnothing$; hence E splits. Hence we

may assume $t > a_2$. This implies that $\mathbf{s}|D = 0$ for every line D such that $E|D \cong \mathbf{O}_D(b_1) \oplus \mathbf{O}_D(b_2)$ with $a_1 \leq b_1 \leq b_2 \leq a_2$; in particular by semicontinuity this is true for a general line of $\mathbf{P}^n$. Hence $\mathbf{s} = 0$, contradiction. □

The proof of Theorem 2 is simply the remark (following [**1**], Remark 10.12) that, having Theorem 1 instead of the statement of [**1**, Th. 10.5], we obtain the stronger assertion of Theorem 2 instead of the numerical splitting asserted by [**1**, Th. 10.13].

The proof of Theorem 1 (i.e. of the small part of [**1**] needed) works in positive characteristic. The same remark applies to Theorem 2 if we assume $\mathrm{Pic}(X) \cong \mathbf{Z}$ instead of making the assumptions on X which by [**2**] imply in characteristic 0 that $\mathrm{Pic}(X) \cong \mathbf{Z}$.

We think that [**1**, Remark 10.12], (on the extension of [**1**, §10], to other manifolds) is potentially very interesting and we hope that some reader will be able to use it.

References

[1] V. Ancona, T. Peternell and J. Wisniewski, *Fano bundles and splitting theorems on projective spaces and quadrics*, Pacific J. Math., (to appear).

[2] J. Wisniewski, *On a conjecture of Mukai*, Manuscripta Math., **68** (1990), 135-141.

Received August 12, 1992.

University of Trento
38050 Provo (TN)
Italy
E-mail address: ballico@itncisca.bitnet
ballico@science.unitn.it

PACIFIC JOURNAL OF MATHEMATICS

Founded by E. F. Beckenbach (1906-1982) and F. Wolf (1904-1989)

Manuscripts must be prepared in accordance with the instructions provided on the inside back cover.

The *Pacific Journal of Mathematics* (ISSN 0030-8730) is published monthly except for July and August. Regular subscription rate: $215.00 a year (10 issues). Special rate: $108.00 a year to individual members of supporting institutions.

Subscriptions, orders for back issues published within the last three years, and changes of subscribers address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. Prior back issues are obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

The Pacific Journal of Mathematics at the University of California, c/o Department of Mathematics, 981 Evans Hall, Berkeley, CA 94720 (ISSN 0030-8730) is published monthly except for July and August. Second-class postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 6143, Berkeley, CA 94704-0163.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS at University of California,
Berkeley, CA 94720, A NON-PROFIT CORPORATION
This publication was typeset using AMS-LATEX,
the American Mathematical Society's TEX macro system.

PACIFIC JOURNAL OF MATHEMATICS

Volume 169 No. 1 May 1995

Minimal sets of periods for torus maps via Nielsen numbers 1
LLUÍS ALSEDÀ, STEWART BALDWIN, JAUME LLIBRE, RICHARD SWANSON and WIESLAW SZLENK

Diagonalizing Hilbert cusp forms 33
TIMOTHY ATWILL

A splitting criterion for rank 2 vector bundles on $\mathbf{P}^n$ 51
EDOARDO BALLICO

Controlling Tietze-Urysohn extensions 53
MARC FRANTZ

Length of Julia curves 75
DAVID H. HAMILTON

On the uniqueness of capillary surfaces over an infinite strip 95
JENN-FANG HWANG

Volume estimates for log-concave densities with application to iterated convolutions 107
MARIUS JUNGE

A reflection principle in complex space for a class of hypersurfaces and mappings 135
FRANCINE ANTOINETTE MEYLAN

Jean Bourgain's analytic partition of unity via holomorphic martingales 161
PAUL F.X. MÜLLER

Characters of Brauer's centralizer algebras 173
ARUN RAM