CORRECTION TO: “ASYMPTOTIC RADIAL SYMMETRY FOR SOLUTIONS OF $\Delta u + e^u = 0$ IN A PUNCTURED DISC”

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The negative case ($K < 0$) in the Theorem 3 of the above mentioned paper is incomplete. In this case, the authors considered three separate cases (page 273 Pacific J. Math., 163, No. 2, 1994). After handling the first two, the authors thought a similar argument would work for the third which turns out to be incorrect. Therefore, we need to reconsider the third case, namely, the function $f$ may take the form

$$f(z) = e^{i\alpha} \frac{(1 + g(z) + \alpha \log z)}{1 - g(z) - \alpha \log z}$$

for some $\alpha \in \mathbb{R}$ and some single-valued analytic function $g$ on the punctured disc $D^* = \{ z \in \mathbb{C} | 0 < |z| < 1 \}$. As in this paper, we may assume that $K = -4$ and $|f| < 1$. Then we conclude that

$$\text{Re} \ g(z) + \alpha \log r < 0, \quad \text{where } r = |z|,$$

and hence

$$r^\alpha |e^{g(z)}| < 1.$$

Therefore, $0$ is not an essential singularity of $e^{g(z)}$. It implies that $g(z)$ analytically extends across $0$. So, in the negative case, we have

**Theorem 1.** Real smooth solutions of $\Delta u - 8e^u = 0$ in $D^*$ are of the form

$$u = \log \frac{|f'|^2}{(1 - |f|^2)^2}$$

with $f$ a multi-valued locally univalent meromorphic function of the form

$$f(z) = h(z)z^\beta, \quad \beta \geq 0$$

or

(1) $$f(z) = \frac{1 + h(z) + \alpha \log z}{1 - h(z) - \alpha \log z}, \quad \alpha \in \mathbb{R}$$
for some single-valued analytic function \( h(z) \) on the whole disc \( D = \{ z \in \mathbb{C} | |z| < 1 \} \).

To find the asymptotic formula, observe that (1) gives

\[
  u = \log \frac{|zh'(z) + \alpha|^2}{4r^2(\text{Re} h(z) + \alpha \log r)^2},
\]

which implies that

\[
  u = -2 \log \left( r \log \frac{1}{r} \right) + O(1) \quad \text{as} \quad r \to 0.
\]

Therefore, we have

**Theorem 2.** Let \( u \) be a smooth real solution of \( \Delta u + 2Ke^u = 0 \) for \( K < 0 \), then

\[
  u(z) = \alpha \log |z| + O(1), \quad \alpha > -2,
\]

or

\[
  u(z) = -2 \log \left( |z| \log \frac{1}{|z|} \right) + O(1)
\]

as \( |z| \to 0 \).

Finally, it is well-known that all such solution \( u \) are bounded by the Poincaré metric (the unique complete constant curvature \( K \) conformal metric) on \( D^* \) which has finite area near the origin. Therefore, all solution \( u \) satisfies \( \int e^u < +\infty \) in any small region containing the origin.

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