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THE COVERS OF A NOETHERIAN MODULE

JIAN-JUN CHUAI

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In this paper we define the covers of a module and describe some of their applications.

1. Introduction.

Let R be a commutative ring and A an R -module. A cover of A is defined to be a subset T of $\text{Max}(R)$ satisfying that for any $x \in A$, $x \neq 0$, there is $M \in T$ such that $0 :_R x \subseteq M$. If we denote by J the intersection of all the maximal ideals belonging to T and suppose that $A \neq 0$ is finitely generated, then we have $JA \neq A$. This generalises the Nakayama's lemma; if, in addition, R is Noetherian, then $\bigcap_{n=1}^{\infty} J^n A = 0$. This is a generalization of a well-known result. A key observation for the covers is that, in the case that R is Noetherian and A is finitely generated, there is a cover T of A which is itself a finite set. From this we have the following result: Let R be a Noetherian ring. Then there is a finite number of maximal ideals M_1, \dots, M_m of R such that $\bigcap_{n=1}^{\infty} J^n = 0$, where $J = \bigcap_{i=1}^m M_i$. This generalises the Krull's theorem for Jacobson radicals. Using this result we can embed the Noetherian ring R in the J -adic completion \hat{R} of R , which is a complete semi-local Noetherian ring; besides, if R is a Cohen-Macaulay (C-M for short) ring, then \hat{R} is a C-M ring. We also use the covers to deal with the maximal component of a finitely generated module over a Noetherian ring, which was introduced by Matlis in [3].

Throughout the paper, R will denote a (non-trivial) commutative ring with identity. Also, if T is a subset of $\text{Max}(R)$ we denote by $\cap T$ (resp. $\cup T$) the intersection (resp. union) of all the maximal ideals belonging to T .

2. The covers.

In this section we define the covers of a module and generalise some known results.

Definition. Let A be an R -module. A subset T of $\text{Max}(R)$ is called a cover of A if for any $x \in A$, $x \neq 0$, there is $M \in T$ such that $0 :_R x \subseteq M$.

Clearly, if T is a cover of A and B is a submodule of A , then T is a cover of B . If T is a cover of A and $T \subseteq T' \subseteq \text{Max}(R)$, then T' is a cover of A . We

say that T is a finite cover of A , or A has a finite cover T , if T is a cover of A and T is itself a finite set. If T is a cover of A , we also say that T covers A .

Lemma 2.1. *Let T be a cover of A . Then each $r \in R - \cup T$ is A -regular. Indeed if $a \in A - \{0\}$ and $ra = 0$, then $r \in (0 :_R a) \subseteq M$ for some $M \in T$, a contradiction.*

Proposition 2.2. *Let $A \neq 0$ be a finitely generated R -module and T a cover of A . Then $JA \neq A$, where $J = \cap T$.*

Proof. Suppose that $JA = A$, then there is $r \in J$ such that $(1 + r)A = 0$, which contradicts Lemma 2.1. \square

Proposition 2.3. *Let A be an R -module, T a cover of A , and $I \not\subseteq 0 :_R A$ an ideal of R . Set $J = \cap T$. If $A/0 :_A I$ is finitely generated, then $JA + (0 :_A I) \neq A$.*

Proof. Since $I \not\subseteq 0 :_R A$, $A/0 :_A I \neq 0$. Let $\bar{x} \in A/0 :_A I$ and $\bar{x} \neq 0$. Then $0 :_R \bar{x} = (0 :_A I) :_R x \subseteq 0 :_R Ix$. Since $x \notin 0 :_A I$, $Ix \neq 0$. Take $r \in I$ such that $rx \neq 0$, then $0 :_R \bar{x} \subseteq 0 :_R rx$. It follows that T is a cover of $A/0 :_A I$. By Proposition 2.2, $J(A/0 :_A I) \neq A/0 :_A I$, hence $JA + (0 :_A I) \neq A$. \square

Proposition 2.4. *Let R be a Noetherian ring, A a finitely generated R -module, T a cover of A , and $I \subseteq \cap T$ an ideal of R . Then $\bigcap_{n=1}^{\infty} I^n A = 0$.*

Proof. Set $\bigcap_{n=1}^{\infty} I^n A = B$. By Krull's theorem, there is $r \in I$ such that $(1 + r)B = 0$. From Lemma 2.1, $B = 0$. \square

Proposition 2.5. *Let T be a finite subset of $\text{Max}(R)$ and A an R -module. Set $J = \cap T$. If $\bigcap_{n=1}^{\infty} J^n A = 0$, then T is a cover of A .*

Proof. If it were not true, there would be a non-zero element x of A such that for any $M \in T$, $0 :_R x \not\subseteq M$. Thus for any integer $n > 0$ we have $(0 :_R x) + M^n = R$, so $M^n x = Rx$. It then follows that $J^n x = Rx$, and thus $\bigcap_{n=1}^{\infty} J^n A \neq 0$, a contradiction. \square

Let R be a Noetherian ring and A a finitely generated R -module. We know that $\text{Ass}(A)$ is a finite set. Let $\text{Ass}(A) = \{P_1, \dots, P_n\}$. Choose a finite subset T of $\text{Max}(R)$ in such a way that for any P_i , there is $M_i \in T$ such that $P_i \subseteq M_i$. Since for any $x \in A$, $x \neq 0$, there is P_i such that $0 :_R x \subseteq P_i$, it follows that T is a finite cover of A . Hence finite covers exist for any finitely

generated module over a Noetherian ring. In particular, any Noetherian ring (as a module over itself) has finite covers.

As a consequence of the above remarks and Proposition 2.4 we have the following theorem.

Theorem 2.6. *Let R be a Noetherian ring and A a finitely generated R -module. Then there is a finite subset T of $\text{Max}(R)$ such that $\bigcap_{n=1}^{\infty} J^n A = 0$, where $J = \cap T$. In particular, if $A = R$, $\bigcap_{n=1}^{\infty} J^n = 0$.*

It is clear that if R is a Noetherian ring and A is a finitely generated R -module, then for any cover T of A we have $T \supseteq \text{Ass}(A) \cap \text{Max}(R)$.

In general, if T is a cover of the module A and B is a submodule of A , T is not a cover of A/B . For example, if T is a cover of the ring R and $T \neq \text{Max}(R)$, then for any $M \in \text{Max}(R) - T$, T is not a cover of R/M .

Proposition 2.7. *Let R be a Noetherian ring, A a finitely generated R -module, B a submodule of A , and T a finite cover of A . Then T is a cover of A/B if and only if B is a closed submodule of A in the J -adic topology, where $J = \cap T$.*

Proof. Suppose first that B is closed, then we have $\bigcap_{n=1}^{\infty} (J^n A + B) = B$, so $\bigcap_{n=1}^{\infty} J^n (A/B) = 0$. By Proposition 2.5, T is a cover of A/B . Conversely, if T is a cover of A/B , then $\bigcap_{n=1}^{\infty} J^n (A/B) = 0$, by Proposition 2.4. So $\bigcap_{n=1}^{\infty} (J^n A + B) = B$, and hence B is closed. \square

Proposition 2.8. *Let R be a Noetherian ring, A a finitely generated R -module, B a submodule of A , and I an ideal of R . Then it is possible to choose a finite subset T of $\text{Max}(R)$ such that $\bigcap_{n=1}^{\infty} (J^n A + I^s B) = I^s B$, for all $s \geq 0$, where $J = \cap T$.*

Proof. By [5, Theorem 5.5(1)], the sequence $\text{Ass}(A/I^s B)$ is constant for large s , thus the set $\bigcup_{s=0}^{\infty} \text{Ass}(A/I^s B)$ is finite. Hence it is possible to choose a finite subset T of $\text{Max}(R)$ in such a way that T covers all $A/I^s B$. By Proposition 2.4, the Proposition follows. \square

3. The maximal component of a Noetherian module.

Throughout this section and the next section the ring R will be Noetherian and the modules will be finitely generated.

Let A be an R -module and define $X(A) = \{x \in A \mid \text{every prime ideal containing } 0 :_R x \text{ is maximal}\}$. Then $X(A)$ is a submodule of A . Matlis [3] called $X(A)$ the maximal component of A . By [3, Corollary (3)], $X(A)$ is the sum of all Artinian submodules of A , and hence is the largest Artinian submodule of A , since A is Noetherian. Further, $X(A/X(A)) = 0$.

Chatters [4] gave a similar discussion for Noetherian rings (not necessary to be commutative).

From [3, Corollary (1)] and the fact that $X(A)$ has finite length we have the following result.

Theorem 3.1. *Let T be a finite cover of A . Set $J = \cap T$. Then $X(A) = \bigcup_{n=1}^{\infty} (0 :_A J^n)$.*

The following result is standard.

Lemma 3.2. *Let I be an ideal of R and $A \neq 0$ an R -module. Then $\text{dep}_I(A) > 0$ if and only if $0 :_A I = 0$.*

Theorem 3.3. *Let A be an R -module, not Artinian. Let T be a finite cover of A and set $J = \cap T$. Then $X(A)$ is the least element of the set*

$$S = \{B \mid B \text{ is a proper submodule of } A \text{ and } \text{dep}_J(A/B) > 0\}.$$

Proof. Since A is not Artinian, $X(A)$ is a proper submodule of A . By Theorem 3.1, we may assume that $X(A) = 0 :_A J^N$. Now

$$0 :_{A/X(A)} J = (X(A) :_A J) / X(A) = 0 :_A J^{N+1} / 0 :_A J^N = 0.$$

From Lemma 3.2, $\text{dep}_J(A/X(A)) > 0$. Hence we have $X(A) \in S$. If B is a proper submodule of A satisfying that $\text{dep}_J(A/B) > 0$, again by Lemma 3.2, $0 :_{A/B} J = (B :_A J) / B = 0$, i.e., $B :_A J = B$. Hence for any integer $n > 0$, $B :_A J^n = B$. Thus we get that $B = B :_A J^N \supseteq 0 :_A J^N = X(A)$, i.e., $X(A)$ is the least element of S . \square

Corollary 3.4. *Let A be a non-zero R -module and T a finite cover of A . Set $J = \cap T$. Then $\text{dep}_J(A) > 0$ if and only if $X(A) = 0$.*

Let $T = \{M_1, \dots, M_n\}$ be a finite cover of the R -module A . We want to find the relations between $X(A)$ and $X(A_{M_i})$, $1 \leq i \leq n$. For any $P \in \text{Spec}(R)$, if K is an R_P -submodule of A_P , denote by K^c the contradiction of K to A . We have $(K^c)_P = K$. If B is a submodule of A , then $(B_P)^c = \bigcup_{r \in R-P} (B :_A r)$. It is also easily checked that if B is a submodule of A and

K is an R_P -submodule of B_P , then $(K^c \cap B)_P = K$. It follows that if B is an Artinian submodule of A , then B_P is an Artinian submodule of A_P . In particular, we have $X(A)_P \subseteq X(A_P)$.

Theorem 3.5. *Let A be an R -module and $T = \{M_1, \dots, M_n\}$ be a finite cover of A . Set $J = \cap T$. Then*

$$X(A) = \bigcap_{i=1}^n X(A_{M_i})^c.$$

Proof. Since $X(A) \subseteq (X(A)_{M_i})^c \subseteq X(A_{M_i})^c$ for all i , we have $X(A) \subseteq \bigcap_{i=1}^n X(A_{M_i})^c$. On the other hand, from Theorem 3.1 we can take a fixed integer $s > 0$ such that $X(A_{M_i}) = 0 :_{A_{M_i}} M_i^s R_{M_i}$, for all i . Hence

$$X(A_{M_i})^c = \left(0 :_{A_{M_i}} M_i^s R_{M_i}\right)^c = \left((0 :_A M_i^s)_{M_i}\right)^c = \bigcup_{r \in R - M_i} ((0 :_A M_i^s) :_A r).$$

If $x \in \bigcap_{i=1}^n X(A_{M_i})^c$, then for each i there is $r_i \in R - M_i$ such that $r_i M_i^s x = 0$. Since $r_i R + M_i = R$, we have $M_i^{s+1} x = M_i^s x$. Thus

$$M_1^{s+1} M_2^{s+1} x = M_1^{s+1} M_2^s x = M_2^s M_1^{s+1} x = M_2^s M_1^s x = M_1^s M_2^s x.$$

Similarly we have $M_1^{s+1} \dots M_n^{s+1} x = M_1^s \dots M_n^s x$. So $J^{s+1} x = J^s x$, and hence $J^s x = 0$ by Proposition 2.2. Thus $x \in 0 :_A J^s \subseteq X(A)$, and the proof is complete. \square

In the remainder of this section we consider modules over local rings.

Lemma 3.6. *Let (R, M) be a local ring (M is the unique maximal ideal of R) and A an R -module. If A is not Artinian, then $\dim(A) = \dim(A/X(A))$.*

Proof. By the definitions of $\dim(A)$ and $\dim(A/X(A))$ we need to show that $\text{rad}(0 :_R A) = \text{rad}(0 :_R (A/X(A)))$. Clearly, we need only to show that $0 :_R (A/X(A)) \subseteq \text{rad}(0 :_R A)$. This follows from the fact that if $r \in R$ such that $rA \subseteq X(A)$, then $rM^s A \subseteq M^s X(A) = 0$ for some integer $s > 0$, hence $r^{s+1} \in 0 :_R A$. \square

Lemma 3.7. [6, p. 105]. *Let R be a local ring and A an R -module. If r_1, \dots, r_n is an A -sequence, then*

$$\dim(A/(r_1, \dots, r_n)A) = \dim(A) - n.$$

Theorem 3.8. *Let (R, M) be a local ring and $A \neq 0$ an R -module. Then there is a strictly ascending chain $A_1 \subset \dots \subset A_s$ of submodules of A such that*

$$\sum_{i=1}^s \text{dep}(A/A_i) = \dim(A).$$

Proof. We use induction on $d = \dim(A)$. If $d = 0$, then $R/(0 :_R A)$ is Artinian. It follows that $0 :_R A$ is M -primary, and hence $M^r \subseteq 0 :_R A$ for some integer $r > 0$. It is clear that $\text{dep}(A) = 0$, and we can take $s = 1$ and $A_1 = 0$ in this case. If $d > 0$, then $0 :_R A$ is not M -primary, and thus $M^n \not\subseteq 0 :_R A$ for any integer $n > 0$. It then follows that $A \neq X(A)$, by Theorem 3.1. Since $X(A/X(A)) = 0$, $\text{dep}(A/X(A)) > 0$, by Corollary 3.4. Take a maximal $A/X(A)$ -sequence x_1, \dots, x_n and set $B = (x_1, \dots, x_n)A + X(A)$. Further, set $A' = A/X(A)$. From Lemma 3.7 and Lemma 3.6, $\dim(A/B) = \dim(A'/(x_1, \dots, x_n)A') = \dim(A') - n = \dim(A) - n < \dim(A)$. By induction there is a strictly ascending chain $A_2/B \subset \dots \subset A_s/B$ of submodules of A/B such that $\sum_{i=2}^s \text{dep}(A/A_i) = \dim(A/B)$. Set $A_1 = X(A)$, then the submodules A_1, \dots, A_s satisfy the required conditions. \square

4. The completions and embeddings.

Proposition 4.1. *Let T be a finite cover of the Noetherian ring R , I an ideal of R . If we consider R with the I -adic topology, the following conditions are equivalent:*

- (1) $I \subseteq \cap T$;
- (2) the zero ideal and every prime ideal contained in $\cup T$ is closed;
- (3) $f^{-1}(M\widehat{R}) = M$ for all $M \in T$, where \widehat{R} is the I -adic completion of R and $f : R \rightarrow \widehat{R}$ is the natural map.

Proof. (1) \Rightarrow (2). Since $\bigcap_{m=1}^{\infty} I^m = 0$ the zero ideal is closed. If $P \subseteq \cup T$ is a prime ideal, then $P \subseteq M$ for some $M \in T$. Since $\text{Ass}_R(R/P) = \{P\}$, we see that T is a cover of R/P . By Proposition 2.4, $\bigcap_{m=1}^{\infty} (I^m + P) = P$, i.e., P is closed.

(2) \Rightarrow (3). Since $\{0\}$ is closed, we can assume that $R \subseteq \widehat{R}$. Let $M \in T$. By [2, Theorem 21; p. 421], $M\widehat{R}$ is the closure of M in \widehat{R} , hence $M\widehat{R} \cap R$ consists of elements of R which are limits of elements contained in M . Since M is closed we get that $M\widehat{R} \cap R = M$.

(3) \Rightarrow (1). Since $M\widehat{R}$ is closed in \widehat{R} and since the map $f : R \rightarrow \widehat{R}$ is continuous, M is closed in R for all $M \in T$. If $I \not\subseteq \cap T$, then $I \not\subseteq M$ for some $M \in T$. But then we have $I^m + M = R$ for all integer $m > 0$, contradicting the fact that M is closed. \square

Let T be a finite cover of R and set $J = \cap T$ and $S = R - \cup T$. It is immediate from Lemma 2.1 that the map $A \rightarrow A_S$ is injective. Also, the J -adic completion of R is the same as the JR_S -adic completion of R_S . So we have the following result.

Theorem 4.2. *Any Noetherian ring R can be embedded in a complete semi-local Noetherian ring; moreover, if R is irreducible, then R can be embedded in a complete local Noetherian ring.*

If I is an ideal of R , we write $\text{dep}(I)$ to stand for $\text{dep}_I(R)$.

Theorem 4.3. *Let $T = \{M_1, \dots, M_n\}$ be a finite cover of the Noetherian ring R and set $J = \cap T$ and $S = R - \cup T$. Then the J -adic completion \widehat{R} of R is a C-M ring if and only if $\text{dep}(M_i) = \text{ht}(M_i)$, $i = 1, 2, \dots, n$.*

Proof. To prove the theorem, it suffices to show that $\text{ht}(M_i) = \text{ht}(M_i\widehat{R})$ and $\text{dep}(M_i) = \text{dep}(M_i\widehat{R})$, $i = 1, 2, \dots, n$.

(1). The proof of $\text{ht}(M_i) = \text{ht}(M_i\widehat{R})$. Let $B = R_S$, $Q_i = M_iR_S$, and $R_i = B_{Q_i}$. We now regard \widehat{R} as the JB -adic completion of B . From [1, Theorem 8.15], $\widehat{R} = \widehat{R}_1 \times \dots \times \widehat{R}_n$, where \widehat{R}_i is the completion of the local ring R_i . By [2, Theorem 30; p. 433] we have

$$\text{ht}((Q_iR_i)\widehat{R}_i) = \dim(\widehat{R}_i) = \text{ht}(Q_i) = \text{ht}(M_i).$$

Thus

$$\text{ht}(M_i\widehat{R}) = \text{ht}((Q_iR_i)\widehat{R}_i) = \text{ht}(M_i).$$

(2). The proof of $\text{dep}(M_i) = \text{dep}(M_i\widehat{R})$. We may view R as a subring of \widehat{R} . If A is an R -module, let \widehat{A} be the J -adic completion of A , $z(A)$ and $z(\widehat{A})$ the sets of annihilators of A and \widehat{A} respectively. First we have that if $x \notin z(A)$, then $x \notin z(\widehat{A})$. This is because tensoring \widehat{R} over R preserves the monomorphism $A \xrightarrow{x} A$, for \widehat{R} is R -flat. Let $\text{dep}(M_i) = s$ and x_1, \dots, x_s be a maximal regular sequence (on R) contained in M_i . Since $x_{j+1} \notin z(R/(x_1, \dots, x_j))$ implies $x_{j+1} \notin z(\widehat{R}/(x_1, \dots, x_j)\widehat{R})$, we have that x_1, \dots, x_s is a regular sequence on \widehat{R} contained in $M_i\widehat{R}$, so $\text{dep}(M_i\widehat{R}) \geq s$.

On the other hand, since $M_i \subseteq z(R/(x_1, \dots, x_s))$ and since M_i is maximal, there is $x \in R$ such that $M_i = (x_1, \dots, x_s) :_R x$. Thus we have $M_i\widehat{R} = (x_1, \dots, x_s)\widehat{R} :_{\widehat{R}} x$, by [2, Lemma 7; p. 424]. So $M_i\widehat{R} \subseteq z(\widehat{R}/(x_1, \dots, x_s)\widehat{R})$ and hence $\text{dep}(M_i\widehat{R}) = s = \text{dep}(M_i)$. The proof is complete. \square

Corollary 4.4. *Let R be a semi-local Noetherian ring and J the Jacobson radical of R . Then the J -adic completion \widehat{R} of R is a C-M ring if and only*

if R is a C - M ring.

Corollary 4.5. *Any C - M ring can be embedded in a complete semi-local C - M ring.*

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