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**GLOBAL ANALYTIC HYPOELLIPTICITY OF \square_b ON
CIRCULAR DOMAINS**

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Let D be a smoothly bounded pseudoconvex domain in \mathbb{C}^n , $n \geq 2$, with real analytic boundary. In this paper we show that \square_b is globally analytic hypoelliptic if D is either circular satisfying $\sum_{j=1}^n z_j \frac{\partial r}{\partial z_j}(z) \neq 0$ near the boundary bD , where $r(z)$ is a defining function for D , or Reinhardt.

I. Introduction.

Let D be a smoothly bounded pseudoconvex domain in \mathbb{C}^n , $n \geq 2$, with real analytic boundary, and let \mathbb{C}^n be equipped with the standard Euclidean metric. We consider the real analytic regularity problem of the \square_b - equation on the boundary. Namely, given any $f \in C_{p,q}^\omega(bD)$, $0 \leq p \leq n-1$ and $1 \leq q \leq n-1$, let $u = N_b f \in L_{p,q}^2(bD)$ be the solution to the following equation,

$$(1.1) \quad \square_b u = \left(\bar{\partial}_b \bar{\partial}_b^* + \bar{\partial}_b^* \bar{\partial}_b \right) N_b f = f.$$

Then we ask: is $u = N_b f \in C_{p,q}^\omega(bD)$? For the definitions of these notations the reader is referred to Section II.

The existence of the solution $u = N_b f$ is an immediate consequence of the closedness of the range of \square_b which was proved by M.C.Shaw [17] and Boas and M.C.Shaw [1], and independently by Kohn [15]. Since $u = N_b f$ is the canonical solution to the equation (1.1), it is unique. It also follows from Proposition 2.7. Next the real analyticity of the boundary bD implies that $u = N_b f$ is smooth, i.e., $u \in C_{p,q}^\infty(bD)$. For instance see Kohn [14][16]. Therefore, the main concern here is about the real analytic regularity of the solution u . The only result we know so far is that the answer is affirmative when D is of strict pseudoconvexity which is due to Tartakoff [18][19][20] and Treves [21] for $n \geq 3$ and to Geller [13] for $n = 2$.

The purpose of this article is to prove the following main results which presumably yield the first positive result to this problem on weakly pseudoconvex domains.

Theorem 1.2. *Let D be a smoothly bounded pseudoconvex domain with real analytic boundary bD in \mathbb{C}^n , $n \geq 2$. Suppose that D is circular and that $\sum_{j=1}^n z_j \frac{\partial r}{\partial z_j}(z) \neq 0$ near bD , where $r(z)$ is the defining function for D . Then for any $f \in C_{p,q}^\omega(bD)$, $0 \leq p \leq n-1$ and $1 \leq q \leq n-1$, the solution $u = N_b f$ to the $\bar{\square}_b$ -equation is also in $C_{p,q}^\omega(bD)$.*

Here a domain D is called circular if $z \in D$ implies

$$e^{i\theta} \cdot z = (e^{i\theta} z_1, \dots, e^{i\theta} z_n) \in D$$

for any $\theta \in \mathbb{R}$. D is called Reinhardt if $z \in D$ implies $(e^{i\theta_1} z_1, \dots, e^{i\theta_n} z_n) \in D$ for any $\theta_1, \dots, \theta_n \in \mathbb{R}$, and D is called complete Reinhardt if $z \in D$ implies $(\lambda_1 z_1, \dots, \lambda_n z_n) \in D$ for any $\lambda_i \in \mathbb{C}$ with $|\lambda_i| \leq 1$, $i = 1, \dots, n$. Then we also prove

Theorem 1.3. *Let D be a smoothly bounded Reinhardt pseudoconvex domain in \mathbb{C}^n , $n \geq 2$, with real analytic boundary. Then the same assertion as in the Theorem 1.2 holds.*

Hence, in particular, $\bar{\square}_b$ is globally analytically hypoelliptic on any complete Reinhardt domains with real analytic boundary which provides a large class of examples. Next we have the following immediate corollary.

Corollary 1.4. *Let D be a smoothly bounded pseudoconvex domain with real analytic boundary in \mathbb{C}^n , $n \geq 2$. Suppose that either D is Reinhardt or D is circular with $\sum_{j=1}^n z_j \frac{\partial r}{\partial z_j}(z) \neq 0$ near bD , where $r(z)$ is the defining function for D . Then we have*

(i) *The Szegő projection S defined on bD preserves the real analyticity globally, and*

(ii) *The canonical solution w to the $\bar{\partial}_b$ -equation, i.e., $\bar{\partial}_b w = \alpha$, is in $C_{p,q-1}^\omega(bD)$ if the given α is in $C_{p,q}^\omega(bD)$ and satisfies $\bar{\partial}_b \alpha = 0$.*

Here the Szegő projection S is defined to be the orthogonal projection from $L^2(bD)$ onto the closed subspace, denoted by $H^2(bD)$, of square-integrable CR -functions defined on the boundary, and by canonical solution w we mean the solution with minimum L^2 -norm. We remark that statement (i) has been proved by the author before in [5] via a more direct argument, and a special case of (ii), i.e., $n = 2$, is verified by Derridj and Tartakoff in [11].

Now if we combine the above theorems and the main result, i.e., the Theorem B, obtained by the author in Chen [6], then we can conclude the following theorem.

Theorem 1.5. *Let $D \subseteq \mathbb{C}^n$, $n \geq 3$, be a smoothly bounded pseudoconvex domain with real analytic boundary. Then the Szeġo projection S associated with D preserves the real analyticity globally whenever D is defined by*

(i) $D = \left\{ (z_1, \dots, z_n) \in \mathbb{C}^n \mid |f(z_1)|^2 + H(|z_2|^2, \dots, |z_n|^2) < 1 \right\}$, where $f(z_1)$ is holomorphic in z_1 and $H(x_2, \dots, x_n)$ is a polynomial with positive coefficient and $H(0, \dots, 0) = 0$, or

(ii) $D = \left\{ (z_1, \dots, z_n) \in \mathbb{C}^n \mid |f(z_1)|^2 + |g(z_2)|^2 + \sum_{j=3}^n h_j(|z_j|^2) < 1 \right\}$, where $f(z_1)$ and $g(z_2)$ are holomorphic in one variable z_1 or z_2 respectively, and $h_j(x)$ is a polynomial with positive coefficients satisfying $h_j(0) = 0$, $h'_j(0) > 0$ for $3 \leq j \leq n$.

The real analytic regularity of the Bergman projection P , which is defined to be the orthogonal projection from $L^2(D)$ onto the closed subspace $H^2(D)$ of square-integrable holomorphic functions defined on D , on the domains (i) and (ii) defined in Theorem 1.5 has been established in Chen [6].

We should point out that in general the analytic pseudolocality of the Szeġo projection S is false. Counterexamples have been discovered by Christ and Geller [7]. However, so far there is no counterexample to the globally real analytic regularity of S . Meanwhile, a number of positive results of the local analytic hypoellipticity for \square_b have been established on some model pseudoconvex hypersurface by Derridj and Tartakoff. For instance, see [8][9][10].

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II. Proofs of the Theorems 1.2 and 1.3.

Let D be a smoothly bounded pseudoconvex domain with real analytic boundary in \mathbb{C}^n , $n \geq 2$, and let \mathbb{C}^n be equipped with the standard Euclidean metric. Since we assume that the domain D is circular, we can choose a real analytic defining function $r(z)$ for D such that $r(z) = r(e^{i\theta} \cdot z)$ and that $|\nabla r(z)| = 1$ for $z \in bD$. Let $z_0 \in bD$ be a boundary point. We may assume that $\frac{\partial r}{\partial z_n}(z_0) \neq 0$. Hence a local basis for $T^{1,0}(bD)$ near z_0 can be chosen to be

$$L_j = \frac{\partial r}{\partial z_n} \frac{\partial}{\partial z_j} - \frac{\partial r}{\partial z_j} \frac{\partial}{\partial z_n} \quad \text{for } 1 \leq j \leq n-1.$$

Put $X(z) = \sum_{j=1}^n \frac{\partial r}{\partial \bar{z}_j} \frac{\partial}{\partial z_j} - \sum_{j=1}^n \frac{\partial r}{\partial z_j} \frac{\partial}{\partial \bar{z}_j}$. We see that

$$L_1, \dots, L_{n-1}, \bar{L}_1, \dots, \bar{L}_{n-1}$$

and $X(z)$ form a local basis for the complexified tangent space $\mathcal{CT}(bD)$, and $X(z)$ is perpendicular to $T^{1,0}(bD) \oplus T^{0,1}(bD)$. Let w_1, \dots, w_{n-1} be $(1, 0)$ -form dual to L_1, \dots, L_{n-1} respectively. Put $\eta = 2(\partial r - \bar{\partial} r)$. Then it is not hard to see that $w_1, \dots, w_{n-1}, \bar{w}_1, \dots, \bar{w}_{n-1}$ and η form a local basis for the complexified cotangent space $\mathcal{CT}^*(bD)$, and η is dual to $X(z)$ and perpendicular to $T^{*1,0}(bD) \oplus T^{*0,1}(bD)$.

Now for any $\theta \in \mathbb{R}$, define

$$\begin{aligned} \Lambda_\theta : \bar{D} &\rightarrow \bar{D} \\ z &\mapsto e^{i\theta} \cdot z = (e^{i\theta} z_1, \dots, e^{i\theta} z_n). \end{aligned}$$

Put $\zeta = e^{i\theta} \cdot z$, then we obtain by direct computation $\frac{\partial r}{\partial z_k}(z) = e^{i\theta} \frac{\partial r}{\partial \zeta_k}(\zeta)$, $\Lambda_{\theta^*} \left(\frac{\partial}{\partial z_k} \right) = e^{i\theta} \frac{\partial}{\partial \zeta_k}$ and $\Lambda_\theta^*(d\zeta_k) = e^{i\theta} dz_k$ for $1 \leq k \leq n$. It follows that we have

$$(2.1) \quad \Lambda_{\theta^*}(X(z)) = X(\zeta),$$

$$(2.2) \quad \Lambda_{\theta^*}(L_j(z)) = e^{i2\theta} L_j(\zeta), \quad \Lambda_{\theta^*}(\bar{L}_j(z)) = e^{-i2\theta} \bar{L}_j(\zeta), \quad \text{for } 1 \leq j \leq n-1,$$

$$(2.3) \quad \Lambda_\theta^*(\bar{\partial} r(\zeta)) = \bar{\partial} r(z), \quad \Lambda_\theta^*(\partial r(\zeta)) = \partial r(z).$$

This implies that $\Lambda_\theta^* w_i$ is again a $(1, 0)$ -form in $\mathcal{CT}^*(bD)$.

Next we recall the definition of $\bar{\partial}_b$ briefly here. let $f \in C_{p,q}^\infty(bD)$, where $C_{p,q}^\infty(bD)$ denotes the space of tangential (p, q) -forms defined on the boundary with smooth coefficients. Namely, any f in $C_{p,q}^\infty(bD)$ can be expressed in the form

$$f = \sum'_{\substack{|I|=p \\ |J|=q}} f_{IJ} w_I \wedge \bar{w}_J,$$

where $I = (i_1, \dots, i_p)$ and $J = (j_1, \dots, j_q)$ are strictly increasing multiindices of length p and q respectively, and $w_I = w_{i_1} \wedge \dots \wedge w_{i_p}$ and $\bar{w}_J = \bar{w}_{j_1} \wedge \dots \wedge \bar{w}_{j_q}$, and the prime indicates that the summation is carried over only the strictly increasing multiindices. Then consider f as a (p, q) -form in some open neighbourhood U of the boundary, and apply $\bar{\partial}$ to f . We get

$$\bar{\partial} f = F + r(z)G + \bar{\partial} r \wedge H,$$

where F is a $(p, q+1)$ -form involving only the w_i 's and \bar{w}_j 's, and G is a $(p, q+1)$ -form, and H is a (p, q) -form. Then the tangential Cauchy-Riemann operator $\bar{\partial}_b$ is defined to be

$$\bar{\partial}_b f = \pi_{p,q+1} \left(\bar{\partial} f \right) = F \Big|_{bD},$$

where $\pi_{p,q+1}$ maps $\bar{\partial}f$ to the restriction of F on the boundary. For the details the reader is referred to Folland and Kohn [12].

Now the above argument shows Λ_θ^* maps the tangential component to the tangential component and maps the normal component to the normal component. Therefore, if $f \in C_{p,q}^\infty(bD)$ with $1 \leq q \leq n-2$ we obtain

$$\begin{aligned}
\bar{\partial}(\Lambda_\theta^* f(\zeta)) &= \pi_{p,q+1} \circ \bar{\partial} \circ \Lambda_\theta^* f \\
&= \pi_{p,q+1} \circ \Lambda_\theta^* \circ \bar{\partial} f \\
&= \pi_{p,q+1} \circ \Lambda_\theta^* (F + r(\zeta)G + \bar{\partial}r \wedge H) \\
&= \pi_{p,q+1} \circ (\Lambda_\theta^* F + r(z)\Lambda_\theta^* G + \bar{\partial}r \wedge \Lambda_\theta^* H) \\
&= \Lambda_\theta^* F \Big|_{bD} \\
&= \Lambda_\theta^* \circ \pi_{p,q+1} (F + r(\zeta)G + \bar{\partial}r \wedge H) \\
&= \Lambda_\theta^* \circ \pi_{p,q+1} \circ \bar{\partial} f \\
&= \Lambda_\theta^* (\bar{\partial}_b f).
\end{aligned}$$

Hence we have proved the following lemma.

Lemma 2.4. $\bar{\partial}_b \Lambda_\theta^* f = \Lambda_\theta^* \bar{\partial}_b f$ for any $f \in C_{p,q}^\infty(bD)$ with $1 \leq q \leq n-1$.

In general, $\bar{\partial}_b \circ h^* \neq h^* \circ \bar{\partial}_b$ if h is just smooth CR -mapping. Denote by $L_{p,q}^2(bD)$ the space of tangential (p,q) -forms with square-integrable coefficients. Then we have

Lemma 2.5. For any u in $L_{p,q}^2(bD)$, we have $(\Lambda_\theta^* u, v) = (u, \Lambda_{-\theta}^* v)$ for any $\theta \in \mathbb{R}$.

Proof. Put $\zeta = e^{i\theta} \cdot z$, and express u and v in terms of the Euclidean coordinates, we get

$$u(\zeta) = \sum_{\substack{|I|=p \\ |J|=q}}' u_{IJ}(\zeta) d\zeta_I \wedge d\bar{\zeta}_J \text{ and } v(z) = \sum_{\substack{|I|=p \\ |J|=q}}' v_{IJ}(z) dz_I \wedge d\bar{z}_J.$$

Let $d\sigma$ be the surface element defined on bD . We see that $d\sigma$ is invariant under rotation, i.e., $\Lambda_\theta^* d\sigma_\zeta = d\sigma_z$. For instance, see Chen [5]. Hence if we

set $z = e^{-\theta} \cdot \zeta$, we obtain

$$\begin{aligned}
(\Lambda_\theta^* u, v) &= \left(\sum' u_{IJ} (e^{i\theta} \cdot z) e^{i(p-q)\theta} dz_I \wedge d\bar{z}_J, \sum' v_{IJ}(z) dz_I \wedge d\bar{z}_J \right) \\
&= \sum' \int_{bD} u_{IJ} (e^{i\theta} \cdot z) e^{i(p-q)\theta} \overline{v_{IJ}(z)} d\sigma_z \\
&= \sum' \int_{bD} u_{IJ} (\zeta) \cdot \overline{e^{-i(p-q)\theta} v_{IJ} (e^{-i\theta} \cdot \zeta)} d\sigma_\zeta \\
&= (u, \Lambda_{-\theta}^* v) .
\end{aligned}$$

This completes the proof of the lemma. \square

Lemma 2.6. $\bar{\partial}_b^* \Lambda_\theta^* \alpha = \Lambda_\theta^* \bar{\partial}_b^* \alpha$ for any $\alpha \in C_{p,q}^\infty(bD)$ with $1 \leq q \leq n-1$, where $\bar{\partial}_b^*$ is the L^2 -adjoint of $\bar{\partial}_b$.

Proof. Let β be any tangential $(p, q-1)$ -form, i.e., $\beta \in C_{p,q-1}^\infty(bD)$. We have

$$\begin{aligned}
(\bar{\partial}_b^* \Lambda_\theta^* \alpha, \beta) &= (\Lambda_\theta^* \alpha, \bar{\partial}_b \beta) \\
&= (\alpha, \Lambda_{-\theta}^* \bar{\partial}_b \beta) \\
&= (\alpha, \bar{\partial}_b \Lambda_{-\theta}^* \beta) \\
&= (\bar{\partial}_b^* \alpha, \Lambda_{-\theta}^* \beta) \\
&= (\Lambda_\theta^* \bar{\partial}_b^* \alpha, \beta) .
\end{aligned}$$

This proves the lemma. \square

Now denote by $H_{p,q} = \left\{ u \in L_{p,q}^2(bD) \mid \square_b u = 0 \right\}$. We have the following fact.

Proposition 2.7. (i) $H_{p,q} = 0$ for $1 \leq q \leq n-2$, and

(ii) $H_{p,n-1} = \left\{ u \in L_{p,n-1}^2(bD) \mid u \in \text{Dom}(\bar{\partial}_b^*) \text{ and } \bar{\partial}_b^* u = 0 \right\}$.

In general, $H_{p,n-1} \neq 0$. Now let $f \in C_{p,q}^\infty(bD)$, $f \perp H_{p,q}$, for $1 \leq q \leq n-1$ be given, and let $u = N_b f \in C_{p,q}^\infty(bD)$ be the canonical solution to the \square_b -equation,

$$\square_b u = \square_b N_b f = f,$$

where N_b is the so-called boundary Neumann operator. Let T be the vector field generated by the rotation, namely, T is defined by

$$\begin{aligned}
T(z) &= \frac{1}{2} \pi_{z^*} \left(\frac{\partial}{\partial \theta} \Big|_{\theta=0} \right) \\
&= \frac{i}{2} \left(\sum_{j=1}^n z_j \frac{\partial}{\partial z_j} - \sum_{j=1}^n \bar{z}_j \frac{\partial}{\partial \bar{z}_j} \right),
\end{aligned}$$

where π_z is the mapping defined for any $z \in bD$ by

$$\begin{aligned} \pi_z : S^1 &\rightarrow \overline{D} \\ e^{i\theta} &\mapsto e^{i\theta} \cdot z = (e^{i\theta} z_1, \dots, e^{i\theta} z_n). \end{aligned}$$

By our hypotheses stated in the Theorem 1.2, $T(z)$ is tangential and pointing in the bad direction for any $z \in bD$.

From now on, we will assume that f has real analytic coefficients, namely, $f \in C_{p,q}^\omega(bD)$ with $1 \leq q \leq n-1$, and that $f \perp H_{p,n-1}$ if $q = n-1$. Write f as

$$f = \sum'_{I,J} f_{IJ}(z) \omega_I \wedge \bar{\omega}_J.$$

Define Tf by

$$Tf = \sum'_{I,J} Tf_{IJ}(z) \omega_I \wedge \bar{\omega}_J.$$

It is not hard to see that Tf is still a tangential (p,q) -form, i.e., $Tf \in C_{p,q}^\omega(bD)$. Then we have the following key lemma.

Lemma 2.8. $T^k u = T^k N_b f = N_b T^k f$ for any $k \in \mathbb{N}$.

Proof. Since, in general, $H_{p,n-1} \neq 0$, we need to check that if $u \perp H_{p,n-1}$, then $\Lambda_\theta^* u \perp H_{p,n-1}$. So, let $w \in H_{p,n-1}$. By Lemma 2.6 we have $\Lambda_\theta^* w \in H_{p,n-1}$. It follows that

$$(\Lambda_\theta^* u, w) = (u, \Lambda_{-\theta}^* w) = 0.$$

Hence $\Lambda_\theta^* u \perp H_{p,n-1}$. This proves our assertion.

Now by combining Lemma 2.4 and 2.6, we obtain

$$\begin{aligned} \square_b \Lambda_\theta^* N_b f &= \Lambda_\theta^* \square_b N_b f \\ &= \Lambda_\theta^* f \\ &= \square_b N_b \Lambda_\theta^* f. \end{aligned}$$

Therefore, by Proposition 2.7 and our assertion we conclude that

$$(2.9) \quad \Lambda_\theta^* N_b f = N_b \Lambda_\theta^* f \text{ for any } \theta \in \mathbb{R}.$$

So now one can argue as we did in Chen [2] to get $TN_b f = N_b Tf$. Inductively we have $T^k N_b f = N_b T^k f$. This completes the proof of the lemma. \square

Lemma 2.8 enables us to estimate the derivatives of the solution $u = N_b f$ in the bad direction as follows,

$$\|T^k u\| = \|T^k N_b f\| = \|N_b T^k f\| \leq C_0 \|f\|_k \leq CC^k k!,$$

for some constant $C > 0$ and any $k \in \mathbb{N}$, where $\|\cdot\|_k$ is the Sobolev k -norm.

Therefore, what we need to estimate is the mixed derivatives of u , namely, the differentiations involving L_i 's, \bar{L}_i 's and T . For dealing with the $\bar{\partial}$ -Neumann problem we can avail ourselves of the so-called basic estimate to achieve this goal. However, for the $\bar{\partial}_b$ -Neumann problem, in general, the energy norm Q_b does not control the barred terms. But if we add the differentiation in T -direction to the right hand side, then we do have the following estimate,

$$(2.10) \quad \|u\| + \sum_{j=1}^{n-1} \|L_j u\| + \sum_{j=1}^{n-1} \|\bar{L}_j u\| \leq C \left(\|\bar{\partial}_b u\| + \|\bar{\partial}_b^* u\| + \|Tu\| \right),$$

for any $u \in C_{p,q}^\omega(bD)$ with support in some open neighbourhood of z_0 . The estimate (2.10) is essentially proved in [12]. Since we know how to control the T -derivatives of the solution $u = N_b f$, then a standard argument can be used to obtain the estimates of all the other mixed derivatives. For the details the reader is referred to Chen [2][3][4]. This completes the proof of Theorem 1.2.

A similar argument can be applied to prove the Theorem 1.3. Let D be a smoothly bounded Reinhardt pseudoconvex domain with real analytic boundary in \mathbb{C} , $n \geq 2$. Let $z_0 \in bD$ be a boundary point. First one can choose a direction, say z_n , such that $\left(z_n \frac{\partial r}{\partial z_n}\right)(z_0) \neq 0$, where $r(z)$ is the defining function for D . Next we simply consider the rotation in z_n -direction, namely, for each $\theta \in \mathbb{R}$, define

$$\begin{aligned} \Lambda_\theta : \bar{D} &\rightarrow \bar{D} \\ z &\mapsto e^{i\theta} \cdot z = (z_1, \dots, z_{n-1}, e^{i\theta} z_n). \end{aligned}$$

Then by following the proof we present here for circular domains we can show without difficulty that \square_b is globally analytically hypoelliptic on any smoothly bounded Reinhardt pseudoconvex domain with real analytic boundary. Details can be found in Chen [3]. This also completes the proof of the Theorem 1.3.

Finally we make a concluding remark that the method we present here can be used to obtain the Sobolev H^s -regularity for \square_b on any smoothly bounded pseudoconvex domain which is either Reinhardt or circular with $\sum_{j=1}^n z_j \frac{\partial r}{\partial z_j} \neq 0$ near bD , where $r(z)$ is a smooth defining function for D . For instance, see Chen [4].

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