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In this paper, we proved a unicity theorem for meromorphic functions with one sharing pair and a condition on deficiency. An example shows that the condition on deficiency is best possible. This result gives a general answer to the problem due to C.C.Yang (1977).

1. Introduction.

In this paper, by meromorphic function we always mean a function which meromorphic in the plane. Let $f(z)$ be meromorphic. We shall use the following standard notations in Nevanlinna theory:

$$T(r, f), \quad m(r, f), \quad N(r, f), \ldots$$

(see Gross [**5**]). We denote by $S(r, f)$ any function satisfying

$$S(r, f) = o\{T(r, f)\}$$

as $r \to +\infty$, possibly outside a set of finite Lebesgue measure. A meromorphic function $a(z)$ is said to be a small function of f if

$$T(r, a) = S(r, f).$$

In this case, we define

$$\delta(a, f) = 1 - \overline{\lim_{r\to\infty}} \frac{N(r, \frac{1}{f-a})}{T(r, f)},$$

and $a(z)$ is said to be a deficient function of f if $\delta(a, f) > 0$.

Let $g(z)$, $a_1(z)$ and $a_2(z)$ be meromorphic functions. If the two functions $f(z) - a_1(z)$ and $g(z) - a_2(z)$ assume the same zeros with the same multiplicities, then we call that f and g share the pair (a_1, a_2) CM. In particular, if $a_1 = a_2 = a$, then the word "the pair" is replaced by "the value" or "the function" provided that a is a constant or a is a function respectively (cf. Frank-Ohlenroth [**4**], Gundersen [**6**], etc.). In addition, if

$$N(r, (f = a_1)\Delta(g = a_2)) = \min\{S(r, f), \quad S(r, g)\},$$

then we say that f and g almost share the pair (a_1, a_2) CM. Here, $N(r, (f = a_1)\Delta(g = a_2))$ is the counting function of those points which satisfy one of the following three cases: (i) $f = a_1$ but $g \neq a_2$; (ii) $f \neq a_1$ but $g = a_2$; (iii) $f = a_1$ and $g = a_2$ but the multiplicities are not the same.

In 1977, Yang [**9**] proved the following result.

Theorem A. *Suppose that F is a family of the functions which are of the form $\alpha_1(z)e^{\mu(z)} + \alpha_2(z)$, where $\mu(z)$ is an entire functions with finite order, $\alpha_j(z)$ $(j = 1, 2)$ are meromorphic functions of finite order, $\alpha_1 \not\equiv 0$, $\alpha_2 \not\equiv \text{const.}$, the order of α_j $(j = 1, 2)$ is less than the order of μ. Let c_1 and c_2 be two distinct constants, and let $f \in F$, $g \in F$. If f and g share the two values c_1 and c_2 CM, then $f \equiv g$ or*

$$\left(f - \frac{c_2 - c_1\lambda(z)}{1 - \lambda(z)}\right)\left(g + \frac{c_2 - c_1\lambda(z)}{1 - \lambda(z)}\right) = -\frac{(c_2 - c_1)^2\lambda(z)}{(1 - \lambda(z))^2},$$

where $\lambda(z)$ is a nonconstant meromorphic function.

Based on this result, Yang [**9**] proposed the following problem.

Yang's problem. *Whether can we omit the restrictions on the order in the family F?*

It is easy to see from the hypotheses of Theorem A that, if $f = \alpha_1(z)e^{\mu(z)} + \alpha_2(z) \in F$ and $g = \alpha_3(z)e^{\nu(z)} + \alpha_4(z) \in F$, then $N(r, \frac{1}{f-\alpha_2}) = \circ\{T(r, f)\}$ and $N(r, \frac{1}{g-\alpha_4}) = \circ\{T(r, g)\}$. Thus

$$\delta(\alpha_2, f) = \delta(\alpha_4, g) = 1, \tag{1}$$

$$\delta(\infty, f) = \delta(\infty, f) = 1. \tag{2}$$

These observations lead to our main result.

Theorem 1. *Let $f(z)$, $g(z)$, $a(z)$, $b(z)$, $\alpha(z)$ and $\beta(z)$ be meromorphic functions in the plane, where $a(z)$ and $\alpha(z)$ are small functions of f, $b(z)$ and $\beta(z)$ are small functions of $g(z)$, $a(z) \not\equiv \alpha(z)$, $b(z) \not\equiv \beta(z)$. Suppose that f and g share the pair (a, b) CM and*

$$\delta = \delta(\alpha, f) + \delta(\beta, g) + \delta(\infty, f) + \delta(\infty, g) > 3. \tag{3}$$

Then either

$$\frac{f - \alpha}{a - \alpha} = \frac{g - \beta}{b - \beta} \tag{4}$$

or

$$\frac{f-\alpha}{a-\alpha}\frac{g-\beta}{b-\beta}=1. \tag{5}$$

Remark 1. The number 3 in the inequality (3) is sharp.

For example, let P and Q be two nonzero polynomials, $f = e^{2z} - Qe^z$, $g = \frac{e^{2z}}{e^z+\frac{P}{Q}}$. Then one can check that f and g share the pair (P, Q) CM and

$$\delta(0,f)+\delta(0,g)+\delta(\infty,f)+\delta(\infty,g)=\frac{1}{2}+1+1+\frac{1}{2}=3.$$

However, $\frac{f}{P} \not\equiv \frac{g}{Q}$ and $\frac{f}{P}\frac{g}{Q} \not\equiv 1$.

Remark 2. Note that Theorem A needs two shared values. However, in our theorem 1, we only need one shared pair.

Remark 3. The topic on unicity theorem concerning deficiency were studied by Ozawa [**7**], Ueda [**8**] etc. The case that f and g are entire functions and $a(z) = b(z) = 1$ was considered by Yi [**10**].

Remark 4. From the proof of Theorem 1 we see that the word "share" can be replaced by "almost share".

As an application, we obtain the following

Corollary. *The answer to Yang's problem is affirmative.*

2. Some Symbols.

For the sake of convienence, we shall use some symbols introduced by Chuang [**1**] and Chuang-Hua [**2**].

For meromorphic function $f(z)$ and a point z, according as z is a pole of f or not, we denote by $\omega(f,z)$ the multiplicity of z or 0 and by $\overline{\omega}(f,z)$ the value 1 or 0. For three meromorphic functions f, g and h, we divide the set of the poles of f and g on $\{|z| \le r\}$ into five pairwise disjoint subsets as follows:

$$\begin{aligned}
V_1 =:&\quad \{z: \quad f(z)\neq\infty, \quad g(z)=\infty\},\\
V_2 =:&\quad \{z: \quad f(z)=\infty, \quad g(z)\neq\infty\},\\
V_3 =:&\quad \{z: \quad f(z)=\infty, \quad g(z)=\infty, \quad h(z)=\infty\},\\
V_4 =:&\quad \{z: \quad f(z)=\infty, \quad g(z)=\infty, \quad h(z)\neq 0,\infty\},\\
V_5 =:&\quad \{z: \quad f(z)=\infty, \quad g(z)=\infty, \quad h(z)=0\}.
\end{aligned}$$

Furthermore, for each $j \in \{1, ..., 5\}$, we denote by $n_j(f)$ and $n_j(g)$ the number of the poles of f and g in the set V_j respectively, with due count of multiplicity. The corresponding counting functions are denoted by $N_j(f)$ and $N_j(g)$ respectively. Obviously,

$$N(r,f) = N_2(f) + N_3(f) + N_4(f) + N_5(f), \tag{6}$$

$$N(r,g) = N_1(g) + N_3(g) + N_4(g) + N_5(g). \tag{7}$$

3. One Basic Lemma.

For the proof of our results, we need the following lemma which can be found in Chuang-Yang [**3**, p. 39] or Gross [**5**, pp. 70-73].

Lemma 1. *Let f_j $(j = 1, ..., n \geq 2)$ be n linearly independent meromorphic functions. If $f_1 + ... + f_n \equiv 1$, then we have*

$$\begin{aligned} T(r, f_1) \leq & \sum_{j=1}^{n} N\left(r, \frac{1}{f_j}\right) + N(r, W) - N\left(r, \frac{1}{W}\right) \\ & - \sum_{j=2}^{n} N(r, f_j) + S(r, f_1) + ... + S(r, f_n), \end{aligned}$$

where $W = W(z)$ is the Wronskian of $f_1, ..., f_n$.

4. Proof of Theorem 1.

Let

$$F_1 =: \{z : a(z) = \infty\} \cup \{z : b(z) = \infty\} \cup \{z : \alpha(z) = \infty\} \cup \{z : \beta(z) = \infty\},$$
$$F_2 =: \{z : a(z) = \alpha(z)\} \cup \{z : b(z) = \beta(z)\}.$$

Set

$$F =: F_1 \cup F_2,$$

the corresponding counting function is denoted by $N_F(r)$. Put

$$h(z) =: \frac{f(z) - a(z)}{g(z) - b(z)}. \tag{8}$$

Since $a(z)$ and $b(z)$ are small functions of f and g respectively, we know that $h(z) \not\equiv 0, \infty$. Let z_o be a pole of h with $z_o \notin F$. Since f and g share the pair (a, b) CM, we have $z_o \in V_2 \cup V_3$. If $z_o \in V_2$, then $\omega(h, z_o) = \omega(f, z_o)$; If $z_o \in V_3$, then $\omega(h, z_o) = \omega(f, z_o) - \omega(g, z_o)$. Thus

(9)
$$N_2(f) + N_3(f) - N_3(g) - N_F(r) \leq N(r, h) \leq N_2(f) + N_3(f) - N_3(g) + N_F(r).$$

Similarly we have

$$N\left(r, \frac{1}{h}\right) \leq N_1(g) + N_5(g) - N_5(f) + N_F(r). \tag{10}$$

Let

$$f_1 =: \frac{f - \alpha}{a - \alpha}, \quad f_2 =: -\frac{g - \beta}{a - \alpha} h, \quad f_3 =: \frac{b - \beta}{a - \alpha} h.$$

Then

$$N\left(r, \frac{1}{f_1}\right) \leq N\left(r, \frac{1}{f - \alpha}\right) + N_F(r), \tag{11}$$

$$N\left(r, \frac{1}{f_3}\right) \leq N\left(r, \frac{1}{h}\right) + N_F(r), \tag{12}$$

$$N(r, h) - N_F(r) \leq N(r, f_3) \leq N(r, h) + N_F(r). \tag{13}$$

From (8) it is easy to see that any zero of h which is not in the set F is not a zero of f_2. Thus

$$N\left(r, \frac{1}{f_2}\right) \leq N\left(r, \frac{1}{g - \beta}\right) + N_F(r). \tag{14}$$

Now for any pole z_o of f_2 with $z_o \notin F$, we know that z_o is a pole of g or h. If $z_o \in V_1$, then $\omega(g, z_o) = \omega(\frac{1}{h}, z_o)$, and so, $\omega(f_2, z_o) = 0$; If $z_o \in V_2$, then $\omega(f_2, z_o) = \omega(h, z_o) = \omega(f, z_o)$; If $z_o \in V_3$, then $\omega(h, z_o) = \omega(f, z_o) - \omega(g, z_o)$, and so, $\omega(f_2, z_o) = \omega(g, z_o) + \omega(h, z_o) = \omega(f, z_o)$; If $z_o \in V_4$, then $h(z_o) \neq 0, \infty$ and $\omega(f_2, z_o) = \omega(g, z_o)$; If $z_o \in V_5$, then $\omega(\frac{1}{h}, z_o) = \omega(g, z_o) - \omega(f, z_o)$ and $\omega(f_2, z_o) = \omega(g, z_o) - \omega(\frac{1}{h}, z_o) = \omega(f, z_o)$. Combining all these facts we get

$$\begin{aligned} N_2(f) + N_3(f) + N_4(g) + N_5(f) - N_F(r) &\leq N(r, f_2) \\ &\leq N_2(f) + N_3(f) + N_4(g) + N_5(f) + N_F(r). \end{aligned} \tag{15}$$

Next we rewrite (8) in the form

$$f_1 + f_2 + f_3 = 1. \tag{16}$$

Without loss of generality, we suppose that there exists a set I with infinite measure such that

$$T(r,g) \le T(r,f), \quad r \in I. \tag{17}$$

(Otherwise, we only need to consider $T(r,g)$ instead of $T(r,f)$ in the following discussions.) Thus $N_F(r) = S(r,f)$, $r \in I$.

In the sequel, we always let $r \in I$. Now we prove the following lemma.

Lemma 2. *f_1, f_2 and f_3 are linearly dependent.*

Proof. Suppose on the contrary that the f's are linearly independent. By lemma 1,

$$\begin{aligned} T(r,f_1) \le & \sum_{j=1}^{3} N\left(r,\frac{1}{f_j}\right) + N(r,W) - N\left(r,\frac{1}{W}\right) - N(r,f_2) - N(r,f_3) \\ & + S(r,f_1) + S(r,f_2) + S(r,f_3) \end{aligned}$$

where W is the Wronskian of f_1, f_2, f_3, i.e.,

$$W = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = - \begin{vmatrix} f_1' & f_3' \\ f_1'' & f_3'' \end{vmatrix}$$

by (16). Now by (10), (11), (12) and (14),

$$\begin{aligned} \sum_{j=1}^{3} N\left(r,\frac{1}{f_j}\right) \le & N\left(r,\frac{1}{f-\alpha}\right) + N\left(r,\frac{1}{g-\beta}\right) \\ & + N_1(g) + N_5(g) - N_5(f) + 4N_F(r). \end{aligned}$$

In addition, by the inequalities on the left hand sides of (9), (13) and (15),

$$N(r,f_2) + N(r,f_3) \ge 2N_2(f) + 2N_3(f) + N_4(f) + N_5(f) - N_3(g) - 3N_F(r).$$

Combining the three inequalities above we get

$$\begin{aligned} T(r,f_1) \le & N(r,W) - N\left(r,\frac{1}{W}\right) + N\left(r,\frac{1}{f-\alpha}\right) + N\left(r,\frac{1}{g-\beta}\right) \\ & + N_1(g) + N_3(g) + N_5(g) \\ & - 2N_2(f) - 2N_3(f) - N_4(f) - 2N_5(f) \\ & + 7N_F(r) + S(r,f_1) + S(r,f_2) + S(r,f_3) \end{aligned}$$

$$\begin{aligned}
&= N(r,W) - N\left(r,\frac{1}{W}\right) + N\left(r,\frac{1}{f-\alpha}\right) + N\left(r,\frac{1}{g-\beta}\right)\\
&\quad + N_1(g) + N_3(g) + N_4(g) + N_5(g)\\
&\quad - 2N_2(f) - 2N_3(f) - 2N_4(f) - 2N_5(f)\\
&\quad - N_4(g) + N_4(f)\\
&\quad + 7N_F(r) + S(r,f_1) + S(r,f_2) + S(r,f_3).
\end{aligned}$$

Substituting (6) and (7) into the above inequality and using the facts that

$$N_4(f) = N_4(g), \quad T(r,f) = T(r,f_1) + S(r,f),$$

$$T(r,a), \quad T(r,b), \quad T(r,\alpha), \quad T(r,\beta) = S(r,f),$$

$$N_F(r), \quad S(r,f_j) = S(r,f), \quad (j = 1,...,3)$$

we obtain

$$T(r,f) \leq N\left(r,\frac{1}{f-\alpha}\right) + N\left(r,\frac{1}{g-\beta}\right) + N(r,W) - N\left(r,\frac{1}{W}\right)$$

$$+N(r,g) - 2N(r,f) + S(r,f). \tag{18}$$

Next we estimate the term $N(r,W) - N\left(r,\frac{1}{W}\right)$. Since

$$W = -(f_1'f_3'' - f_1''f_3'), \tag{19}$$

from the expressions of f_1 and f_3 we see that the poles of W only occur at the poles of f and the points in F. Let z_0 be a pole of f with $z_0 \notin F$.

If $z_o \in V_2$, then near $z = z_0$,

$$f_1 = \frac{1}{(z-z_0)^{\omega(f,z_0)}}\left\{x + \bigcirc(z-z_0)\right\}, \quad f_3 = \frac{1}{(z-z_0)^{\omega(f,z_0)}}\left\{y + \bigcirc(z-z_0)\right\},$$

where x and y are nonzero constants. If $\omega(f,z_0) \geq 2$, then

$$f_1'f_3'' = \frac{1}{(z-z_0)^{2\omega(f,z_0)+3}}\left\{-\omega(f,z_0)^2(\omega(f,z_0)+1)xy + \bigcirc(z-z_0)\right\},$$

$$f_1''f_3' = \frac{1}{(z-z_0)^{2\omega(f,z_0)+3}}\left\{-\omega(f,z_0)^2(\omega(f,z_0)+1)xy + \bigcirc(z-z_0)\right\},$$

and so,

$$f_1'f_3'' - f_1''f_3' = \bigcirc\left\{\frac{1}{(z-z_0)^{2\omega(f,z_0)+2}}\right\}.$$

If $\omega(f, z_0) = 1$, then

$$f_1' f_3'' = \frac{-2xy}{(z-z_0)^5} + \frac{O(1)}{(z-z_0)^3} + \dots,$$

$$f_1'' f_3' = \frac{-2xy}{(z-z_0)^5} + \frac{O(1)}{(z-z_0)^3} + \dots,$$

and so,

$$f_1' f_3'' - f_1'' f_3' = O\left\{\frac{1}{(z-z_0)^3}\right\}.$$

Thus

$$\omega(W, z_0) \le \begin{cases} 2\omega(f, z_0) + 2, & \text{if } \omega(f, z_0) \ge 2 \\ 3, & \text{if } \omega(f, z_0) = 1 \end{cases}$$
$$\le 3\omega(f, z_0).$$

If $z_0 \in V_3$, then $\omega(g, z_0) \ge 1$ and $\omega(f, z_0) \ge 2$. Thus, by (19),

$$\omega(W, z_0) \le 2\omega(f, z_0) + 3 - \omega(g, z_0)$$
$$\le 2\omega(f, z_0) + 2 \le 3\omega(f, z_0).$$

If $z_0 \in V_4$, then $\omega(f, z_0) = \omega(g, z_0)$, and so, $\omega(f_3, z_0) = 0$. By (19), we get

$$\omega(W, z_0) \le \omega(f, z_0) + 2 \le 3\omega(f, z_0).$$

If $z_0 \in V_5$, and if z_0 is a pole of W, then by (19),

$$\omega(W, z_0) \le \omega(f, z_0) + 2$$
$$\le 3\omega(f, z_0).$$

Combining all the cases above and noting (6), we deduce that

$$N(r, W) \le 3N(r, f) + N_F(r).$$

This and (18) give

$$(20)\quad T(r, f) \le N\left(r, \frac{1}{f-\alpha}\right) + N\left(r, \frac{1}{g-\beta}\right) + N(r, f) + N(r, g) + S(r, f).$$

Now by the definition of deficiency, for $\epsilon = \frac{\delta-3}{8} > 0$, where δ is the sum in (3), there exists $r_o > 0$ such that

$$N\left(r, \frac{1}{f-\alpha}\right) \le (1 - \delta(\alpha, f) + \epsilon)T(r, f),$$

$$N\left(r, \frac{1}{g-\beta}\right) \le (1-\delta(\beta, g)+\epsilon)T(r,g),$$
$$N(r,f) \le (1-\delta(\infty, f)+\epsilon)T(r,f)$$

and

$$N(r,g) \le (1-\delta(\infty, g)+\epsilon)T(r,g)$$

hold for $r \in I$ and $r > r_o$. Substituting all these inequality into (20) and noting (17), we get $\delta \le 3$, which contradicts our hypothesis. This completes the proof of the lemma.

Now by Lemma 2, there exist three constants c_1, c_2 and c_3 with

$$|c_1| + |c_2| + |c_3| \neq 0 \tag{21}$$

and

$$c_1 f_1 + c_2 f_2 + c_3 f_3 = 0. \tag{22}$$

If $c_1 = 0$, then $c_2 c_3 \neq 0$ and $f_2 = -\frac{c_3}{c_2} f_3$. This leads to $g = \frac{c_3}{c_2} b(z) + \left(1 - \frac{c_3}{c_2}\right)\beta(z)$, which contradicts the assumptions that $b(z)$ and $\beta(z)$ are small functions of g. Thus, $c_1 \neq 0$. We may suppose $c_1 = -1$, and (22) reads $f_1 = c_2 f_2 + c_3 f_3$. Combining this and (16) we obtain

$$(1+c_2)f_2 + (1+c_3)f_3 = 1. \tag{23}$$

Next we consider two cases.

(i) $1 + c_2 = 0$. Then $1 + c_3 \neq 0$ and $(1+c_3)f_3 = 1$. It follows from (8) and the definition of f_3 between (10) and (11) that

$$\begin{aligned} f - \frac{c_3 a + \alpha}{1+c_3} &= f - a + a - \frac{c_3 a + \alpha}{1+c_3} \\ &= \left(\frac{1}{1+c_3}\right)\frac{a-\alpha}{b-\beta}(g-b) + a - \frac{c_3 a + \alpha}{1+c_3} \\ &= \left(\frac{1}{1+c_3}\right)\frac{a-\alpha}{b-\beta}(g-\beta). \end{aligned} \tag{24}$$

If $c_3 \neq 0$, then $\frac{c_3 a + \alpha}{1+c_3} \neq \alpha$. By the Nevanlinna "three-functions theorem" we deduce that

$$\begin{aligned} T(r,f) &\le N(r,f) + N\left(r, \frac{1}{f-\alpha}\right) + N\left(r, \frac{1}{f - \frac{c_3 a+\alpha}{1+c_3}}\right) + S(r,f) \\ &= N(r,f) + N\left(r, \frac{1}{f-\alpha}\right) + N\left(r, \frac{1}{g-\beta}\right) + S(r,f). \end{aligned}$$

This is impossible by the same reasoning as in the proof of Lemma 2. Therefore $c_3 = 0$ and (24) reads

$$\frac{f-\alpha}{g-\beta} = \frac{a-\alpha}{b-\beta}.$$

This is what we need.

(ii) $1 + c_2 \neq 0$. It follows from (8), (23) and the definitions of f_2 and f_3 between (10) and (11) that $-(1+c_2)\frac{g-\beta}{a-\alpha} + (1+c_3)\frac{b-\beta}{a-\alpha} = \frac{g-b}{f-a}$, which can be written as

$$f - \frac{c_2 a + \alpha}{1 + c_2} = \left(\frac{c_2 - c_3}{(1+c_2)^2}\right) \frac{(a-\alpha)(b-\beta)}{g - \frac{1+c_3}{1+c_2} b - \frac{c_2 - c_3}{1+c_2}\beta}. \tag{25}$$

If $c_2 \neq 0$, then $\frac{c_2 a+\alpha}{1+c_2} \neq \alpha$. By the "three-functions theorem", we have

$$\begin{aligned} T(r,f) &\leq N(r,f) + N\left(r, \frac{1}{f-\alpha}\right) + N\left(r, \frac{1}{f - \frac{c_2 a+\alpha}{1+c_2}}\right) + S(r,f) \\ &\leq N(r,f) + N\left(r, \frac{1}{f-\alpha}\right) + N(r,g) + S(r,f). \end{aligned}$$

By the same reasoning as in the proof of Lemma 2, we can get a contradiction. Thus $c_2 = 0$, and (25) reads

$$f - \alpha = -c_3 \frac{(a-\alpha)(b-\beta)}{g - (1+c_3)b + c_3\beta}. \tag{26}$$

If $c_3 = -1$, then

$$(f-\alpha)(g-\beta) = (a-\alpha)(b-\beta),$$

as asserted. If $c_3 \neq -1$, then $\frac{\alpha+c_3 a}{1+c_3} \neq \alpha$ and (26) can be written as

$$f - \frac{\alpha + c_3 a}{1+c_3} = -\left(\frac{c_3}{1+c_3}\right) \frac{(a-\alpha)(g-\beta)}{g - (1+c_3)b + c_3\beta}.$$

Thus, the "three-functions theorem" gives

$$\begin{aligned} T(r,f) &\leq N(r,f) + N\left(r, \frac{1}{f-\alpha}\right) + N\left(r, \frac{1}{f - \frac{\alpha+c_3 a}{1+c_3}}\right) + S(r,f) \\ &\leq N(r,f) + N\left(r, \frac{1}{f-\alpha}\right) + N\left(r, \frac{1}{g-\beta}\right) + S(r,f). \end{aligned}$$

By the same reasoning as in the proof of Lemma 2 we obtain a contradiction.

This completes the proof of the theorem. □

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